Are subcycle solutions meaningful for time variable gravity field analysis for future satellite missions?

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Introduction

Sampling the Earth from a satellite orbit leads to aliasing of high-frequency temporal and spatial geophysical signals in lower frequencies, and thus to aliasing errors in the recovered time-variable gravity field. Aliasing is a limiting factor for current gravity field missions and will remain a fundamental challenge for future missions. In principle for a repeat orbit the aliasing problem can be described, similar to as Heisenger’s uncertainty principle, as a trade-off between spatial and temporal sampling. The larger the repeat period is, the better the spatial sampling, but at the cost of a poorer temporal sampling due to a higher repeat time and vice versa.

This principle of spatial and temporal sampling is illustrated in Figure 1. The minimal spatial and temporal scales which can be recovered by a (pseudo)cycle (RC), where a satellite completes revolutions in (n) nodal days, are:

- Dspace = 2π/(να) (radial)
- Dtime = TRC = n/(nodal days)

with the repeat period TRC = α. The product of both, Dspace and Dtime, can be regarded as a constraint for the orbit heights of interest:

Dspace × Dtime = const

where TRC = α is the orbit period. This corresponds to the Heisenberg principle, which is displayed as a hyperbola. A selected repeat-orbit can be considered as a point on this hyperbola, marked with a blue point. The spatial and temporal scales covered by this repeat orbit are represented by the dark grey area in the upper-right rectangle. Given a repeat orbit, theoretically a higher spatial sampling can only be achieved by a sensor on an interleaved orbit (α-shift), and a better temporal resolution is gained by a second sensor on the same groundtrack (α-shift).

Repeat-Orbits and Subcycles

Let \( n \) be the number of orbits per day. The Fundamental Interval \( S = N \times T \) gives the angular space between two Ascending Node Crossings (ANC) consecutive in time. The sub-interval \( S = n \times T \) is the sampling angle after an entire RC. \( n \times T \) can be written as \( \nu_{\alpha} = \frac{2\pi}{\alpha} \), where \( \nu_{\alpha} \) is the orbital frequency. The integer part of \( n_{\alpha} \) can therefore be written as \( n = \frac{2\pi}{\nu_{\alpha}} \). Within \( S \), the ANX of days and \( n+1 \) days is separated by a distance of \( \nu_{\alpha} \times S \) from the orbit.

A useful graphical tool to represent the relationship between spatial and temporal sampling is the coverage map. Figure 5 shows examples for selected 32 days RC. The X-axis represents the Fundamental Interval at the equator. The Y-axis represents the duration of an entire RC. Each square graphic represents an ANX and shows where it occurs (number of day) and where it falls within \( S \). The orbits can be classified as drifing orbits when \( n < 1 \) or \( n > 1 \), and as drifing orbits in the other cases. The (511/32) RC in Figure 2 represents a drifing orbit, with each track falls next to the previous one, making the coverage map a diagonal line. The sampling of the Fundamental Interval is very progressive. Skipping orbits feature more complex coverage patterns, that persist regardless of large \( S \) and \( D \) the sampling frequency. For a \( S = 1 \), the coverage map is a hexagonal grid. The (511/32) and (532/32) are examples with SCs of 15 days/7 days and pSCs of 2 days/4 days, respectively. Another interesting tool to illustrate the gap evolution. It consists in plotting the width of the minimum/maximum/average (blue/red/black) unobserved gaps for each day. For the drifting orbit (511/32), the largest gap width is reduced as slowly as possible and the smallest gap is immediately 0. The evolution of the gap widths of the skipping orbits is faster, their SC and pSC are `waist’ points.

Simulation and Results

Based on a simplified version of the IAS-1T-arrangement approach using nominal orbits and the Hill equations simulations have been conducted. GRACE-like formations are applied with distances \( d \approx 100 \, \text{km} \). As input all relevant time-variable effects of atmosphere, ocean, hydrology, ice and solid Earth (Arias et al., 2011) and 10% of the 2000/2004 ocean tides (the unknowns) have been adopted. The measurements, sampled with \( \nu_{\alpha} = 5 \), are simulated error-free in order to investigate the pure aliasing effect. As previous studies (e.g. Anselmi et al., 2011) or GRACE showed, a dense spatial orbit sampling is always preferable, especially concerning monthly trends. Thus 32 days repeat orbits provide a dense coverage have been chosen. Additionally, they fulfill (\( \nu \), \( \nu_{\alpha} \)) odd, meaning that the modified (%) symbol with \( \nu_{\alpha} \leq L \) or \( \nu_{\alpha} \\nu_{\alpha} \leq L \) is needed. In order to discard effects of signal attenuation of higher SH-drawings, the measurements have been projected on a resolution orbit height of \( h = 150 \, \text{km} \). The error is represented as geoc-RMS of the difference between the output and the mean of the ACHS-input. From the 32 days RCs the following statements are made:

- The strongly influential are:
  a) the modified (%) symbol must be fulfilled; Solutions below this criteria are of poor quality
  b) in general 4 days + 1 day time are good enough, but do not necessarily approach aliasing errors
- Some geoc-RMS time series (e.g. some for RC for L = 30, or the (511/32) for L = 45/51/37) deviate for certain days from statement b), but these deviations are hardly connected with the SC/pSC as well as with the gap evolution.
- The influence of the SCs or pSCs is hardly pronounced, neither in the error curves for each RC nor when solutions of all RCs are compared at observation periods corresponding to the SC/pSC (even for the spatial sampling is exactly met by the CNP (\( \nu_{\alpha} = 5 \)).

- A homogeneous and smooth gap evolution with avoidance of large unobserved gaps however seems important:
  a) the drifting orbit performs quite worse over all time spans, especially for \( T \geq 10 \, \text{days} \), with very large unobserved gaps
  b) a decreased performance is also apparent for (501/32) (532/32) with large gaps \( T \geq 1 \) to \( T < 5 \) for all y, respectively
  c) the (532/32) with a smooth gap evolution provides for all \( T \) a good performance
- The (503/32) is an exception. It provides very good results for \( T \geq 15 \) days but of large unobserved gaps for \( T > 11 \) days.

Regarding the homogeneity of the groundtrack pattern, it seems that SC/pSC-solutions are the most meaningful sub-R solutions. However, as the simulation shows, sub-RC gap solutions of equal/similar quality are also received for observation periods differing from SC/pSC. It seems that a sufficiently smooth gap evolution and the avoidance of large unobserved gaps is of higher priority than SC/pSC. Nevertheless, more investigations making use of other error measures as well as advanced analysis methods as EOF or canonical correlation analysis (CCA) are necessary.

References


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