Assessment of aliasing effect of white noise on different solutions in gravity recovery simulations of a GRACE-like mission

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Quality assessment of sub-cycle vs. full cycle solutions in future gravity field mission simulations

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Future Gravity Field Satellite Missions

BMBF Research and Development Programme
• Aliasing
• White noise
• Strips
• Formation fligh
• Filtering
Sampling the Earth from orbit

Solution for 32 days

Aliasing of high-frequency geophysical signals to lower frequencies
Strategies to mitigate the aliasing problem

- Satellite configuration design
  - GRACE, Pendulum, Cartwheel, …

- Alternative recovery strategies
  - Full repeat period solution
  - Sub-cycle solution

- Post processing / Filtering

Input Scenario Output Post-processing

Models (truth) • Orbit design • Error definition • Full cycle • Sub-cycle • Time series • Correlation analysis • Filtering
Orbit design

• GRACE like configuration
• Repeat orbit of 503/32
• Circular, near polar orbit
• Differential orbital elements

Solutions

• Full repeat cycle: 32 days
• Sub-cycle: 7 days
Comparison through

- EWH maps, SH triangle plots, degree
- EOF analysis for 1 year of simulated data
- The solutions from Pendulum and 2 pairs mission formations
Empirical Orthogonal Functions (EOF)

Data decomposition

\[ Z = U \Sigma V^T \]

- \( U \) and \( V \): Orthonormal matrices containing the eigenvectors of \( Z^T Z \)
- \( \Sigma \): Diagonal matrix containing of singular values corresponding to the eigenvectors
- Each mode is represented by a pair of singular vectors of matrices \( U \) and \( V \) and a corresponding singular value in \( \Sigma \)
Kolmogorov-Smirnov (KS) test

• The individual modes of the decomposed data matrix (columns of the eigenvector matrices) tested for the white noisiness (Wouters and Schrama, 2007).

• The cumulative power spectra of each mode is compared to the cumulative power spectrum of ideal white noise through the KS-Test.

• Data matrix is reconstructed by using only the eigenvectors and the singular values of the non-white noise modes.
Data

- Time variable fields (AOHIS): 6 hourly time series of gravity potential spherical harmonic coefficient series

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere</td>
<td>ECMWF ERA-40</td>
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<tr>
<td>Ocean</td>
<td>OMCT</td>
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<tr>
<td>Hydrology</td>
<td>PCR-GLOBWB driven with ECMWF meteorological data</td>
</tr>
<tr>
<td>Ice</td>
<td>ECMWF Operational Analysis</td>
</tr>
<tr>
<td>Solid Earth</td>
<td>DEOS University of Luxembourg</td>
</tr>
<tr>
<td>Tide</td>
<td>Difference of 2 tidal models (EOT08a - GOT4.7)</td>
</tr>
</tbody>
</table>

- Year 2005
- Maximum degree 90
Real world (truth)

Background models (known): 90% of the real world

Solution by 10% +
Diffference of two tidal models

IUGG 2011 – Melbourne, Australia
GRACE $\frac{\beta}{\alpha} = \frac{503}{32}, \, i = 89.5^\circ$

Effect of white noise on the solutions

white noise addition – without white noise
GRACE $\frac{\beta}{\alpha} = \frac{503}{32}$, $i = 89.5^\circ$

Solutions

IUGG 2011 – Melbourne, Australia
EOF Analysis

$C_{lm}$

GRACE $\frac{\beta}{\alpha} = \frac{503}{32}$, $i = 89.5^\circ$
UTU

Order 1

\[ \frac{\beta}{\alpha} = \frac{503}{32}, \quad i = 89.5^\circ \]

After filtering (KS-test)

0.3 Significance level

IUGG 2011 – Melbourne, Australia
Full repeat period

$GRACE \frac{\beta}{\alpha} = \frac{503}{32}, i = 89.5^\circ$

After filtering (KS test)
0.3 Significance level

IUGG 2011 – Melbourne, Australia
Sub-cycle

GRACE $\frac{\beta}{\alpha} = \frac{503}{32}$, $i = 89.5^\circ$

After filtering (KS test)

0.05 Significance level
Pendulum \( \frac{\beta}{\alpha} = \frac{503}{32}, i = 89.5^\circ \)

\[ \Delta a = \Delta e = \Delta i = \Delta \omega = 0, \Delta \Omega = 0.5271^\circ, \Delta M = 0.5271^\circ \Rightarrow \rho_x = \rho_y \approx 62 \text{ km} \]
Full repeat period

GRACE $\frac{\beta}{\alpha} = \frac{503}{32}, \ i = 89.5^\circ$

EWH map

Pendulum $\frac{\beta}{\alpha} = \frac{503}{32}, \ i = 89.5^\circ$

Pendulum vs. GRACE

SH triangle

Degree RMS

IUGG 2011 – Melbourne, Australia
Sub-cycle

GRACE $\frac{\beta}{\alpha} = \frac{503}{32}, i = 89.5^\circ$

EWH map

Pendulum $\frac{\beta}{\alpha} = \frac{503}{32}, i = 89.5^\circ$

SH triangle

Degree RMS

IUGG 2011 – Melbourne, Australia
Full repeat period

GRACE $\frac{\beta}{\alpha} = \frac{503}{32}, i = 89.5^\circ$

EWH map

Two pairs vs. one pair

GRACE $\frac{\beta}{\alpha} = \frac{125}{8}, i = 89.5^\circ$ - PENDULUM $\frac{\beta}{\alpha} = \frac{503}{32}, i = 97^\circ$
Sub-cycle

GRACE $\frac{\beta}{\alpha} = \frac{503}{32}$, $i = 89.5°$

EWH map

Two pairs vs. one pair

GRACE $\frac{\beta}{\alpha} = \frac{125}{8}$, $i = 89.5°$ - PENDULUM $\frac{\beta}{\alpha} = \frac{503}{32}$, $\phi = 97°$

SH triangle

Degree RMS
Conclusion

• Cleaner maps for full repeat period solutions compared to sub-cycle solutions (as expected)
• Possibility of using sub-cycle solutions (by post-processing), esp. for Pendulum and two pairs formations
• Eliminating much of the white noises by „EOF + KS-test“ filter
• Need of much tougher significance level for sub-cycle solution
• Trade-off between eliminating noise and removing geophysical signals, esp. for sub-cycle solution
• Large improvement by Pendulum formation
• Some improvements by using two pairs of satellites in comparison with one pair Pendulum

Thank you
Appendix
Input models

• 10% of AOHIS
Observation equation

\[ \ddot{\rho}(t) - \frac{1}{\rho(t)} (\Delta \dot{X}_1(t))^2 - (\dot{\rho}(t))^2 = e_1 (\nabla V(\dot{X}_2(t)) - \nabla V(\dot{X}_1(t)) \]