Geodetic Network Adjustment Examples

Friedhelm Krumm

Geodätisches Institut
Universität Stuttgart

https://www.gis.uni-stuttgart.de

fk193601@mailbox.org

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Contents

1 Introduction

The following numerical examples for the adjustment of Geodetic networks (1D, 2D and 3D) have been prepared at the request of many BSc. and MSc. students who were confronted with my lectures and labs on Geodetic Adjustment over the past years. Most of the examples have been picked from literature, either papers or text books. Reference is given at the beginning of each example.
All networks are processed in a right-handed coordinate system. In most examples from literature, this requires an interchange of x- and y-coordinates. In case of one dimensional networks (height networks) very often planar coordinates x,y have been simulated in order to display the network/levelling lines in the plane.
Whenever possible, also inputfiles (*.adat) for the adjustment program "Adjust" by Charles D. Ghilani (Last access: 25.12.2019) were generated. Together with my own ASCII data files they can be found in corresponding zip-files as mentioned in the example headers. All calulations were performed using MATLAB (R2018b) or OCTAVE (5.1.0).
Although done with great care, mistakes and typing errors are most likely to be present, and I apologize for the caused headaches. In this case or in case of questions/remarks/suggestions, please, drop me an E-Mail. I will be happy to promptly consider your comments.

In order not to overload the examples, a failure of the global test or the indication of a blunder was not always prosecuted, i.e. by using iteratively Baarda's data snooping method. In all examples, a power-of-test $\gamma = 80\%$ was used for data snooping.

Two general remarks with respect to the adjustment procedure have to be added: All problems – no matter if linear or non-linear – have undergone linearization with corresponding iteration until convergence was achieved. Therefore, the used model for parameter adjustment reads $\Delta y = A\Delta x + e$, $\Sigma_y = \sigma_0^2 Q = \sigma_0^2 P^{-1}$. Δy is the $m \times 1$ vector of reduced observations, A the $m \times n$ design matrix, Δx the $n \times 1$ vector of corrections to the (given) approximate unknown parameters and e the $m \times 1$ vector or residuals/inconsistencies. The quantities σ_0^2 and P, respectively, denote the traditional variance of unit weight and the $m \times m$ observational weight matrix, respectively. P is always diagonal except for few cases of dynamic networks with stochastic prior information or GNSS baseline observations with given variance-covariance matrix.
The second remark concerns the datum problem. Without exception, it has been solved using constraints of the kind $D^{\mathsf{T}}\Delta x = 0$ with the consequence that – even in the case of a fixed datum – design matrices, the normal equation matrix and the vector of corrections contain in most cases also columns/rows for datum defininings parameters. Removing corresponding columns and rows from A and Δx was not regarded as reasonable from the practical (programming) point of view.

My sincere thanks go to my colleagues Matthias Roth and Mohammad J. Tourian who helped me a lot with Latex and never gave up to patiently answer related questions.

2 One-dimensional networks (Height networks)

2.1 Ghilani (2010), Ex. 12.6

Ghilani Charles D. (2010): Adjustment Computations. Spatial Data Analysis. Fifth Edition, John Wiley & Sons, Inc., ISBN 978-0-470-46491-5, Ex. 12.6, pp. 218

Available data files: [1D] Ghilani12_6_Height_fix∗.∗

Heights/Coordinates

Point name	H [m]	Easting x [m]	Northing y [m]
A	437.5960 (D)	2200.0000	5800.0000
B	448.1050	3090.1700	8664.8900
C	453.4650	6113.2600	6045.5400
D	444.9420	3614.2100	4385.7900

Datum: fix, (D)...Datum coordinate

Levelled height differences

in	to	h [m]	Lev-Line s [m]	$\lvert\sigma_l\rvert$ [cm/$\sqrt{\text{km}}$]	$\lvert\sigma\rvert$ [cm]	p [−]
A	B	10.5090	1000.00	0.60	0.600 00	27 777.8
	C	15.8810	1000.00	1.20	1.200 00	6944.4
B	C	5.3600	1000.00	0.40	0.400 00	62 500.0
	D	−3.1670	1000.00	0.40	0.400 00	62 500.0
C	D	−8.5230	1000.00	0.50	0.500 00	40 000.0
D	A	−7.3480	1000.00	0.30	0.300 00	111 111.1

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ levelled height differences

A	H_A	H_B	H_C	H_D	Δy
$h_{\text{A,B}}$	−1	1	0	0	0.000
$h_{\text{A,C}}$	−1	0	1	0	1.200
$h_{\text{B,C}}$	0	−1	1	0	0.000
$h_{\text{B,D}}$	0	−1	0	1	−0.400
$h_{\text{C,D}}$	0	0	−1	1	0.000
$h_{\text{D,A}}$	1	0	0	−1	−0.200

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

N	H_{A}	H_{B}	H_{C}	H_{D}	$A^{\mathsf{T}}P\Delta y$
H_{A}	145 833.333 33	−27 777.777 78	−6944.444 44	−111 111.111 11	−305.555 56
H_{B}	−27 777.777 78	152 777.777 78	−62 500.000 00	−62 500.000 00	250.000 00
H_{C}	−6944.444 44	−62 500.000 00	109 444.444 44	−40 000.000 00	83.333 33
H_{D}	−111 111.111 11	−62 500.000 00	−40 000.000 00	213 611.111 11	−27.777 78

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ **of datum constraints**

	H_{A}	H_{B}	H_{C}	H_{D}
D^{T}	1	0	0	0

Least squares solution $\widehat{\Delta x}_{[\mathrm{m}]}$

	H_{A}	H_{B}	H_{C}	H_{D}
$\widehat{\Delta x}^{\mathsf{T}}$	0.000 00	0.003 71	0.003 47	0.001 61

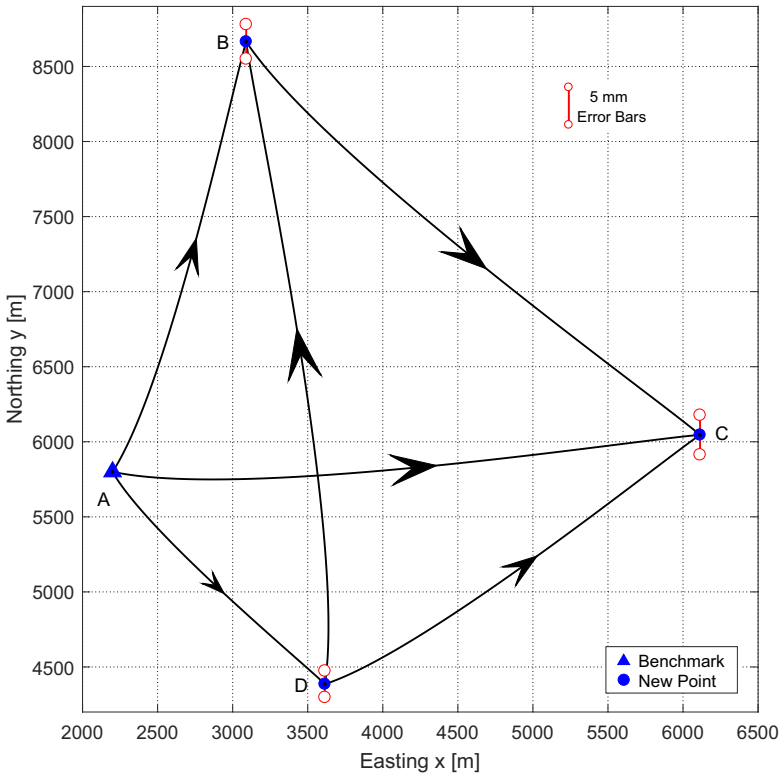
Adjusted heights

Point name	$\widehat{H}$ [m]	$\widehat{H} - H$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]
A	437.5960	0.00	0.00
B	448.1087	3.71	2.30
C	453.4685	3.47	2.64
D	444.9436	1.61	1.76

Adjusted levelled height differences

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
A	B	10.5127	−3.71	2.30	65.49	0.76	3.06	−0.57	3.00	0.55	1.06	0.20	1.06	0.20	1.17
	C	15.8725	8.53	2.64	88.62	0.76	5.27	+0.96	1.48	0.27	0.60	0.11	0.60	0.11	1.16
B	C	5.3598	0.24	2.13	32.94	0.11	2.88	+0.07	5.90	0.15	1.93	0.05	1.93	0.05	0.16
	D	−3.1651	−1.89	1.96	43.26	0.72	2.51	−0.44	4.73	0.82	1.43	0.25	1.43	0.25	1.11
C	D	−8.5249	1.86	2.28	50.92	0.52	2.90	+0.37	4.06	0.51	1.42	0.18	1.42	0.18	0.80
D	A	−7.3476	−0.39	1.76	18.77	0.30	2.86	−0.21	8.60	0.63	2.32	0.17	2.32	0.17	0.47

Network graph



Supplementary information

Observed levelled height differences	:	6
Height unknowns	:	3
Datum defect	:	1
Datum definition	:	fix
Number of datum constraints	:	1
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.6
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.7
Redundancy r	:	3
Redundancy levelled height differences	:	3.00
Weighted square sum of residuals Ω [-]	:	1.27212
(a priori) standard deviation σ_0 [m]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	0.6512
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.6512
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	0.4240 ($k^F_{\alpha_G;r,\infty}$ =4.21)
Global test (Ω/σ_0^2)	:	1.2721 ($k^{\chi^2}_{\alpha_G;r}$ =12.63)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{H} - H\|$ [cm]	:	0.533
Trace height covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{H}}$ [cm^2]	:	0.15319

2.2 Fix network

Available data files: [1D] Krumm_Height_fix∗.∗

Heights/Coordinates

Point name	H [m]	Easting x [m]	Northing y [m]
1	93.4590	481.0000	660.0000
2	107.7590	703.0000	309.0000
3	103.4590	395.0000	299.0000
4	100.4590	140.0000	400.0000
5	110.9560 (D)	957.0000	511.0000

Datum: fix, (D)...Datum coordinate

Levelled height differences

in	to	h [m]	Lev-Line s [m]	$\lvert\sigma_l\rvert$ [cm/$\sqrt{\text{km}}$]	$\lvert\sigma\rvert$ [cm]	p [–]
1	2	14.3010	900.00	0.50	0.474 34	1.111 11
	3	9.9950	800.00	0.50	0.447 21	1.250 00
	4	7.0060	1000.00	0.50	0.500 00	1.000 00
	5	17.5000	1500.00	0.50	0.612 37	0.666 67
3	2	4.2990	500.00	0.50	0.353 55	2.000 00

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[mm]}$ levelled height differences

$\boldsymbol{A}$	H_1	H_2	H_3	H_4	H_5	Δy
$h_{1,2}$	−1	1	0	0	0	1.00
$h_{1,3}$	−1	0	1	0	0	−5.00
$h_{1,4}$	−1	0	0	1	0	6.00
$h_{1,5}$	−1	0	0	0	1	3.00
$h_{3,2}$	0	1	−1	0	0	−1.00

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

N	H_1	H_2	H_3	H_4	H_5	$A^{\mathsf{T}}P\Delta y$
H_1	4.027 78	−1.111 11	−1.250 00	−1.000 00	−0.666 67	−0.002 86
H_2	−1.111 11	3.111 11	−2.000 00	0.000 00	0.000 00	−0.000 89
H_3	−1.250 00	−2.000 00	3.250 00	0.000 00	0.000 00	−0.004 25
H_4	−1.000 00	0.000 00	0.000 00	1.000 00	0.000 00	0.006 00
H_5	−0.666 67	0.000 00	0.000 00	0.000 00	0.666 67	0.002 00

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	H_1	H_2	H_3	H_4	H_5
D^{T}	0	0	0	0	1

Least squares solution $\widehat{\Delta x}_{[m]}$

	H_1	H_2	H_3	H_4	H_5
$\widehat{\Delta x}^{\mathsf{T}}$	−0.003 00	−0.004 86	−0.005 45	0.003 00	0.000 00

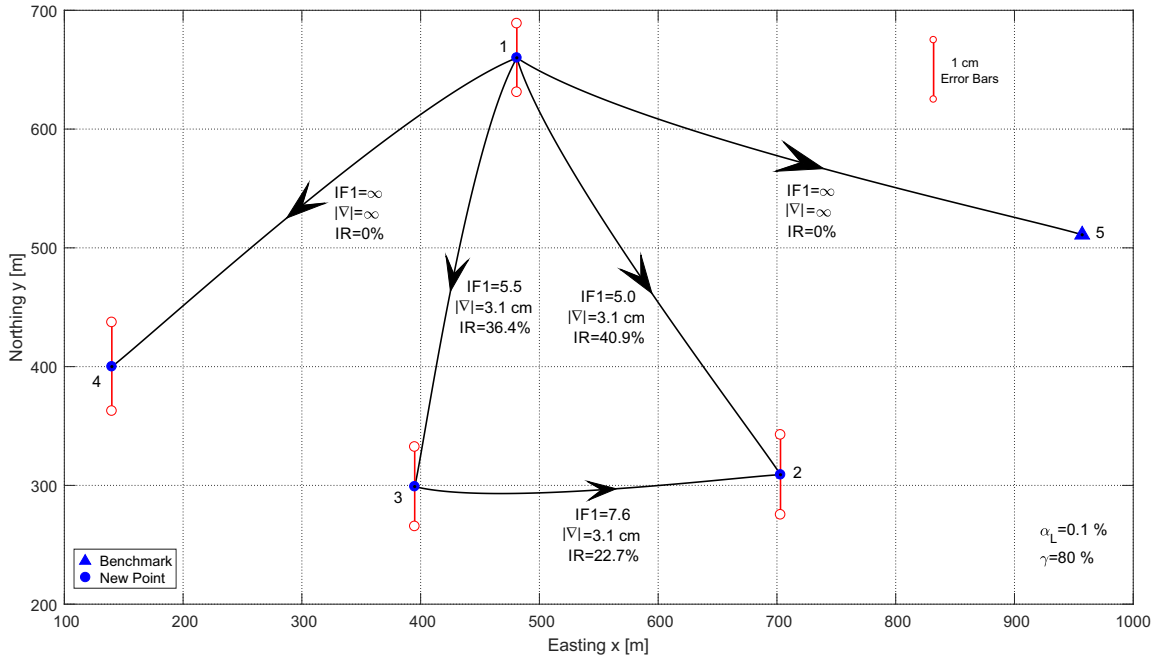
Adjusted heights

Point name	$\widehat{H}$ [m]	$\widehat{H} - H$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]
1	93.4560	−3.00	5.78
2	107.7541	−4.86	6.73
3	103.4535	−5.45	6.69
4	100.4620	3.00	7.46
5	110.9560	0.00	0.00

Adjusted levelled height differences

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	2	14.2981	2.86	3.44	40.91	0.94	3.06	+0.70	4.97	1.13	1.81	0.41	1.81	0.41	1.00
	3	9.9975	−2.55	3.37	36.36	0.94	3.06	−0.70	5.47	1.25	1.95	0.45	1.95	0.45	1.00
	4	7.0060	0.00	4.72	0.00	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
	5	17.5000	0.00	5.78	0.00	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
3	2	4.3006	−1.59	2.93	22.73	0.94	3.06	−0.70	7.62	1.74	2.37	0.54	2.37	0.54	1.00

Network graph



Supplementary information

Observed levelled height differences	:	5
Height unknowns	:	4
Datum defect	:	1
Datum definition	:	fix
Number of datum constraints	:	1
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.1
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	∞
Redundancy r	:	1
Redundancy levelled height differences	:	1.00
Redundancy (Check)	:	1.00
Weighted square sum of residuals Ω [m^2]	:	$2.22727 \cdot 10^{-5}$
(a priori) standard deviation σ_0 [m]	:	$5 \cdot 10^{-3}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$4.71940 \cdot 10^{-3}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.9439
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.8909 ($k^F_{\alpha_G;r,\infty}$ =10.83)
Global test (Ω/σ_0^2)	:	0.8909 ($k^{\chi^2}_{\alpha_G;r}$ =10.83)
Number of outliers (Data snooping)	:	0
Number of outliers (τ-criterion)	:	0
$\|\widehat{H} - H\|$ [cm]	:	0.845
Trace height covariance matrix, tr$\widehat{\Sigma}_{\widehat{H}}$ [cm^2]	:	1.79093
Trace height cofactor matrix, tr$Q_{\widehat{H}}$	:	8.04091

2.3 Dynamic network with covariance matrix

Available data files: [1D] Krumm_Height_dyn∗.∗

Heights

Point name	H [m]
2	107.7541 (D)
3	103.4535 (D)
6	105.6400
7	115.7110
8	112.8850

Datum: dynamic, (D)...Datum coordinate

Variance-Covariance matrix of dynamic heights [m^2]

Σ_H	2	3
2	0.0025	−0.0015
3	−0.0015	0.0036

Levelled height differences

in	to	h [m]	Lev-Line s [m]	$\lvert\sigma_l\rvert$ [cm/$\sqrt{\text{km}}$]	$\lvert\sigma\rvert$ [cm]	p [1/m^2]
2	8	5.1280	700	100	83.666 00	1.428 57
3	6	2.1830	500	100	70.710 68	2
	7	12.2540	500	100	70.710 68	2
6	7	10.0710	800	100	89.442 72	1.250 00
8	7	2.8240	800	100	89.442 72	1.250 00

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[mm]}$ dynamic heights

A	H_2	H_3	H_6	H_7	H_8	Δy
H_2	1	0	0	0	0	0
H_3	0	1	0	0	0	0

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[mm]}$ levelled height differences

A	H_2	H_3	H_6	H_7	H_8	Δy
$h_{2,8}$	−1	0	0	0	1	−2.90
$h_{3,6}$	0	−1	1	0	0	−3.50
$h_{3,7}$	0	−1	0	1	0	−3.50
$h_{6,7}$	0	0	−1	1	0	0.00
$h_{8,7}$	0	0	0	1	−1	−2.00

Normal equation system $N, A^{\mathsf{T}}P\Delta y$ (1. iteration)

N	H_2	H_3	H_6	H_7	H_8	$A^{\mathsf{T}}P\Delta y$
H_2	534.761 90	222.222 22	0.000 00	0.000 00	−1.428 57	0.004 14
H_3	222.222 22	374.370 37	−2.000 00	−2.000 00	0.000 00	0.014 00
H_6	0.000 00	−2.000 00	3.250 00	−1.250 00	0.000 00	−0.007 00
H_7	0.000 00	−2.000 00	−1.250 00	4.500 00	−1.250 00	−0.009 50
H_8	−1.428 57	0.000 00	0.000 00	−1.250 00	2.678 57	−0.001 64

Least squares solution $\widehat{\Delta x}_{[\mathrm{cm}]}$ (1. iteration)

	H_2	H_3	H_6	H_7	H_8
$\widehat{\Delta x}^{\mathsf{T}}$	0.000	0.000	−0.361	−0.377	−0.237

Adjusted heights

Point name	$\widehat{H}_{[\mathrm{m}]}$	$\widehat{H} - H_{[\mathrm{mm}]}$	$\lvert\hat{\sigma}\rvert_{[\mathrm{mm}]}$
2	107.7541	0.00	0.04
3	103.4535	0.00	0.04
6	105.6364	−3.61	0.43
7	115.7072	−3.77	0.39
8	112.8826	−2.37	0.48

Adjusted dynamic heights

Variance component: $\Omega = 0.000,\ r = 0.00,\ \hat{\sigma}_0^2 = 0.00,\ \alpha_G = 0.00‰,\ k^F_{\alpha_G;r,\infty} = 2040.92$

Point name	$\widehat{H}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\widehat{H}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
2	107.7541	−0.003	0.04	0.21	0.00	446.75	−0.14	103.1	0.03	445.79	0.14	445.79	0.14	1.79#
3	103.4535	0.004	0.04	0.27	0.00	474.77	+0.14	91.3	0.03	473.48	0.14	473.48	0.14	1.68*

Adjusted levelled height differences

Variance component: $\Omega = 0.000,\ r = 2.00,\ \hat{\sigma}_0^2 = 0.00,\ \alpha_G = 0.28\%,\ k^F_{\alpha_G;r,\infty} = 5.88$

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
2	8	5.1285	−0.52	0.48	37.43	0.00	565.09	−0.14	5.34	0.00	353.59	0.09	353.59	0.09	1.41#
3	6	2.1829	0.10	0.43	29.84	0.00	534.88	+0.03	6.34	0.00	375.27	0.02	375.27	0.02	0.37
	7	12.2537	0.27	0.39	41.72	0.00	452.35	+0.06	4.88	0.00	263.62	0.04	263.62	0.04	0.82
6	7	10.0708	0.17	0.47	47.75	0.00	534.88	+0.03	4.32	0.00	279.50	0.02	279.50	0.02	0.37
8	7	2.8246	−0.60	0.49	42.78	0.00	565.09	−0.14	4.78	0.00	323.37	0.08	323.37	0.08	1.41*

Supplementary information

Dynamic heights	:	2
Observed levelled height differences	:	5
Height unknowns	:	5
Datum defect	:	0
Datum definition	:	dynamic
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.4
Redundancy r	:	2
Redundancy dynamic heights	:	0.00
Redundancy levelled height differences	:	2.00
Redundancy (Check)	:	2.00
Weighted square sum of residuals Ω [-]	:	$1.04801 \cdot 10^{-6}$
(a priori) standard deviation σ_0 [m]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	$7.23882 \cdot 10^{-4}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.0007
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.0000 ($k^F_{\alpha_G;r,\infty}$ =5.87)
Global test (Ω/σ_0^2)	:	0.0000 ($k^{\chi^2}_{\alpha_G;r}$ =11.73)
Number of outliers (Data snooping)	:	0
Number of outliers (τ-criterion)	:	4
$\|\widehat{H} - H\|$ [cm]	:	0.574
Trace height covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{H}}$ [cm^2]	:	0.00571
Trace height cofactor matrix, $\mathrm{tr}Q_{\widehat{H}}$	:	1.09020

2.4 Lother & Strehle (2007), with 6 different datum definitions

Lother G and Strehle J (2007): Grundlagen der Ausgleichungsrechnung nach der Methode der kleinsten Quadrate. Hochschule München, Fakultät für Geoinformation, Bachelorstudiengang Geoinformatik und Satellitenpositionierung, pp. 11-3

Available data files: [1D] LotherStrehle_Height∗.∗

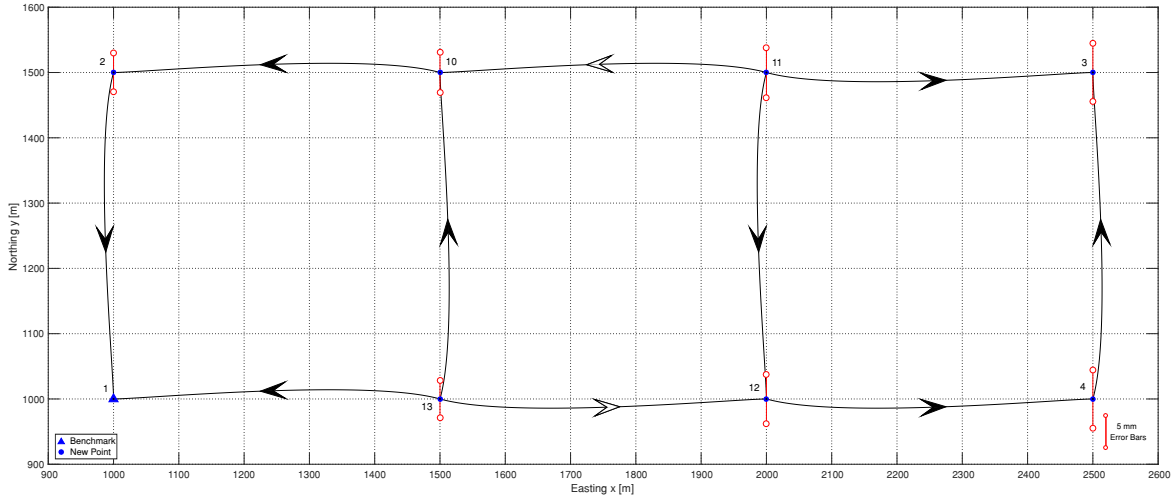
This example consists of 6 different scenarios which differ in the datum definition. Scenario 1 uses point 1 as a benchmark, while in scenario 2 the datum problem is solved by fixing point 4. Problem 3 treats a free network with total-trace minimization of the variance-covariance matrix of estimated point heights. Example 4 is again a fix network but overconstraining the datum definition by fixing four heights. The results of scenario 5 are identical with those of example 4, but using a dynamic approach with high weights for points 1-4. Finally, in example 6, points 1-4 are equipped with an a priori variance-covariance matrix leading to a dynamic datum definition with stochastic prior information.

Heights/coordinates, 6 different datum definitions

Point ID	H [m]	x [m]	y [m]	1	2	3	4	5	6
1	510.369	1000	1000	(D)		(D)	(D)	(D)	(D)
2	508.762	1000	1500			(D)	(D)	(D)	(D)
3	526.174	2500	1500			(D)	(D)	(D)	(D)
4	515.982	2500	1000		(D)	(D)	(D)	(D)	(D)
10	502.163	1500	1500			(D)			
11	501.562	2000	1500			(D)			
12	503.789	2000	1000			(D)			
13	501.984	1500	1000			(D)			
datum type				fix	fix	free	fix	dynamic (high weights)	dynamic (with Σ_H)

(D)...Datum coordinate

Network graph



Levelled height differences

in	to	h [m]	Lev-Line s [m]	$\lvert\sigma_l\rvert$ [cm/$\sqrt{\text{km}}$]	$\lvert\sigma\rvert$ [cm]	p [–]
2	1	1.6035	4620.00	0.15	0.322 41	0.432 90
4	3	10.1910	4120.00	0.15	0.304 47	0.485 44
10	2	6.6025	3980.00	0.15	0.299 25	0.502 51
11	3	24.6060	3080.00	0.15	0.263 25	0.649 35
	12	2.2280	1460.00	0.15	0.181 25	1.369 86
12	4	12.1915	2940.00	0.15	0.257 20	0.680 27
13	1	8.3830	4020.00	0.15	0.300 75	0.497 51
	10	0.1840	1640.00	0.15	0.192 09	1.219 51

Trigonometric height differences

in	to	h [m]	$\lvert\sigma\rvert$ [cm]	p [–]
11	10	0.5995	0.33	0.413 22
13	12	1.8050	0.27	0.617 28

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[mm]}$ levelled height differences

A	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}	Δy
$h_{2,1}$	1	−1	0	0	0	0	0	0	−3.50
$h_{4,3}$	0	0	1	−1	0	0	0	0	−1.00
$h_{10,2}$	0	1	0	0	−1	0	0	0	3.50
$h_{11,3}$	0	0	1	0	0	−1	0	0	−6.00
$h_{11,12}$	0	0	0	0	0	−1	1	0	1.00
$h_{12,4}$	0	0	0	1	0	0	−1	0	−1.50
$h_{13,1}$	1	0	0	0	0	0	0	−1	−2.00
$h_{13,10}$	0	0	0	0	1	0	0	−1	5.00

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[mm]}$ trigonometric height differences

A	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}	Δy
$h_{11,10}$	0	0	0	0	1	−1	0	0	−1.50
$h_{13,12}$	0	0	0	0	0	0	1	−1	0.00

2.4.1 Datum definition 1: Point 1 fixed

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

N	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}	$A^{\mathsf{T}}P\Delta y$
H_1	0.930 41	−0.432 90	0.000 00	0.000 00	0.000 00	0.000 00	0.000 00	−0.497 51	−0.002 51
H_2	−0.432 90	0.935 41	0.000 00	0.000 00	−0.502 51	0.000 00	0.000 00	0.000 00	0.003 27
H_3	0.000 00	0.000 00	1.134 79	−0.485 44	0.000 00	−0.649 35	0.000 00	0.000 00	−0.004 38
H_4	0.000 00	0.000 00	−0.485 44	1.165 71	0.000 00	0.000 00	−0.680 27	0.000 00	−0.000 53
H_{10}	0.000 00	−0.502 51	0.000 00	0.000 00	2.135 25	−0.413 22	0.000 00	−1.219 51	0.003 72
H_{11}	0.000 00	0.000 00	−0.649 35	0.000 00	−0.413 22	2.432 44	−1.369 86	0.000 00	0.003 15
H_{12}	0.000 00	0.000 00	0.000 00	−0.680 27	0.000 00	−1.369 86	2.667 42	−0.617 28	0.002 39
H_{13}	−0.497 51	0.000 00	0.000 00	0.000 00	−1.219 51	0.000 00	−0.617 28	2.334 31	−0.005 10

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ **of datum constraints**

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
D^{T}	1	0	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
$\widehat{\Delta x}^{\mathsf{T}}$	0.000 00	0.005 47	−0.002 61	−0.000 30	0.003 66	0.002 42	0.002 13	0.000 29

Adjusted heights

Point ID	$\widehat{H}_{[\text{m}]}$	$\widehat{H} - H_{[\text{mm}]}$	$\lvert\hat{\sigma}\rvert_{[\text{mm}]}$
1	510.3690	0.00	0.00
2	508.7675	5.47	2.99
3	526.1714	−2.61	4.48
4	515.9817	−0.30	4.43
10	502.1667	3.66	3.10
11	501.5644	2.42	3.84
12	503.7911	2.13	3.74
13	501.9843	0.29	2.88

Adjusted levelled height differences

Variance component: $\Omega/\sigma_0^2 = 2.693,\ r = 2.25,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.20,\ \alpha_G = 0.34\%,\ k^F_{\alpha_G;r,\infty} = 5.31$

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
2	1	1.6015	1.97	2.99	32.96	1.06	2.32	+0.60	5.89	1.52	1.56	0.40	1.56	0.40	0.94
4	3	10.1897	1.30	2.76	36.12	0.71	2.09	+0.36	5.50	0.95	1.34	0.23	1.34	0.23	0.63
10	2	6.6008	1.69	2.87	28.40	1.06	2.32	+0.60	6.56	1.69	1.66	0.43	1.66	0.43	0.94
11	3	24.6070	−0.98	2.55	27.00	0.71	2.09	−0.36	6.79	1.17	1.53	0.26	1.53	0.26	0.63
	12	2.2267	1.29	1.80	22.91	1.49	1.56	+0.56	7.58	2.73	1.21	0.43	1.21	0.43	1.31
12	4	12.1906	0.93	2.51	25.78	0.71	2.09	+0.36	7.01	1.21	1.55	0.27	1.55	0.27	0.63
13	1	8.3847	−1.71	2.88	28.68	1.06	2.32	−0.60	6.52	1.68	1.65	0.43	1.65	0.43	0.94
	10	0.1824	1.63	1.90	23.39	1.75	1.64	+0.70	7.48	3.17	1.26	0.53	1.26	0.53	1.55

Adjusted trigonometric height differences

Variance component: $\Omega/\sigma_0^2 = 1.155,\ r = 0.75,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.54,\ \alpha_G = 0.07\%,\ k^F_{\alpha_G;r,\infty} = 14.18$

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
11	10	0.6022	−2.74	2.78	44.78	1.24	2.04	−0.61	4.59	1.38	1.13	0.34	1.13	0.34	1.10
13	12	1.8068	−1.84	2.56	29.98	1.24	2.04	−0.61	6.32	1.90	1.43	0.43	1.43	0.43	1.10

2.4.2 Datum definition 2: Point 4 fixed

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

Same as for datum definition 1!

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ **of datum constraints**

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
D^{T}	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
$\hat{\boldsymbol{x}}^{\mathsf{T}}$	0.000 30	0.005 77	−0.002 30	0.000 00	0.003 97	0.002 72	0.002 43	0.000 59

Adjusted heights

Point ID	$\widehat{H}_{[\text{m}]}$	$\widehat{H} - H_{[\text{mm}]}$	$\lvert\hat{\sigma}\rvert_{[\text{mm}]}$
1	510.3693	0.30	4.43
2	508.7678	5.77	4.46
3	526.1717	−2.30	2.76
4	515.9820	0.00	0.00
10	502.1670	3.97	3.61
11	501.5647	2.72	2.77
12	503.7914	2.43	2.51
13	501.9846	0.59	3.49

Adjusted levelled height differences

Same as for datum definition 1!

Adjusted trigonometric height differences

Same as for datum definition 1!

2.4.3 Datum definition 3: Free - Total trace minimization

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

Same as for datum definition 1!

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ **of datum constraints**

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
D^{T}	0.353 55	0.353 55	0.353 55	0.353 55	0.353 55	0.353 55	0.353 55	0.353 55

Least squares solution $\widehat{\Delta x}_{[m]}$

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
$\widehat{\Delta x}^{\mathsf{T}}$	−0.001 38	0.004 09	−0.003 99	−0.001 69	0.002 28	0.001 04	0.000 75	−0.001 09

Adjusted heights

Point ID	$\widehat{H}_{[m]}$	$\widehat{H} - H_{[mm]}$	$\lvert\hat{\sigma}\rvert_{[mm]}$
1	510.3676	−1.38	2.67
2	508.7661	4.09	2.69
3	526.1700	−3.99	2.49
4	515.9803	−1.69	2.45
10	502.1653	2.28	1.76
11	501.5630	1.04	1.73
12	503.7897	0.75	1.65
13	501.9829	−1.09	1.67

Adjusted levelled height differences

Same as for datum definition 1!

Adjusted trigonometric height differences

Same as for datum definition 1!

2.4.4 Datum definition 4: Points 1-4 fixed

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

Same as for datum definition 1!

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
D^{T}	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	1	0	0	0	0	0
	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[m]}$

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
$\widehat{\Delta x}^{\mathsf{T}}$	0.000 00	0.000 00	0.000 00	0.000 00	0.001 99	0.002 91	0.002 26	−0.000 55

Adjusted heights (new points only)

Point ID	$\widehat{H}$ [m]	$\widehat{H} - H$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]
10	502.1650	1.99	2.500
11	501.5649	2.91	2.330
12	503.7913	2.26	2.270
13	501.9835	−0.55	2.440

Adjusted levelled height differences

Variance component: $\Omega/\sigma_0^2 = 9.305,\ r = 4.88,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.91,\ \alpha_G = 1.25\%,\ k^F_{\alpha_G;r,\infty} = 2.94$

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
2	1	1.6070	−3.50	0.00	100.00	1.09	1.33	−0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.82
4	3	10.1920	−1.00	0.00	100.00	0.33	1.26	−0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.25
10	2	6.5970	5.49	2.50	59.80	2.37	1.60	+0.92	3.39	1.95	0.64	0.37	0.64	0.37	1.80
11	3	24.6091	−3.09	2.33	55.04	1.58	1.47	−0.56	3.73	1.43	0.66	0.25	0.66	0.25	1.20
	12	2.2264	1.64	2.02	28.50	1.70	1.40	+0.58	6.55	2.69	1.00	0.41	1.00	0.41	1.29
12	4	12.1907	0.76	2.27	55.15	0.40	1.43	+0.14	3.73	0.36	0.64	0.06	0.64	0.06	0.30
13	1	8.3855	−2.55	2.44	62.01	1.08	1.58	−0.41	3.23	0.84	0.60	0.16	0.60	0.16	0.82
	10	0.1815	2.46	2.15	27.78	2.43	1.51	+0.89	6.66	3.92	1.09	0.64	1.09	0.64	1.84

Adjusted trigonometric height differences

Variance component: $\Omega/\sigma_0^2 = 1.114,\ r = 1.12,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.00,\ \alpha_G = 0.12\%,\ k^F_{\alpha_G;r,\infty} = 9.79$

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
11	10	0.6001	−0.59	2.70	61.50	0.23	1.74	−0.10	3.27	0.18	0.67	0.04	0.67	0.04	0.17
13	12	1.8078	−2.81	2.51	50.22	1.47	1.57	−0.56	4.11	1.46	0.78	0.28	0.78	0.28	1.11

2.4.5 Datum definition 5: Dynamic, huge weights for observed points 1-4

Dynamic heights

in	H [m]	$\lvert\sigma\rvert$ [mm]	p [–]
1	510.3690	0.00	10^8
2	508.7620	0.00	10^8
3	526.1740	0.00	10^8
4	515.9820	0.00	10^8

Design matrix A[-] and reduced observation vector Δy[mm] dynamic heights

A	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}	Δy
H_1	1	0	0	0	0	0	0	0	0.00
H_2	0	1	0	0	0	0	0	0	0.00
H_3	0	0	1	0	0	0	0	0	0.00
H_4	0	0	0	1	0	0	0	0	0.00

Normal equation system $N, A^\mathsf{T}P\Delta y$

N	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}	$A^\mathsf{T}P\Delta y$
H_1	10^8	−0.432 90	0.000 00	0.000 00	0.000 00	0.000 00	0.000 00	−0.497 51	−0.002 51
H_2	−0.432 90	10^8	0.000 00	0.000 00	−0.502 51	0.000 00	0.000 00	0.000 00	0.003 27
H_3	0.000 00	0.000 00	10^8	−0.485 44	0.000 00	−0.649 35	0.000 00	0.000 00	−0.004 38
H_4	0.000 00	0.000 00	−0.485 44	10^8	0.000 00	0.000 00	−0.680 27	0.000 00	−0.000 53
H_{10}	0.000 00	−0.502 51	0.000 00	0.000 00	2.135 25	−0.413 22	0.000 00	−1.219 51	0.003 72
H_{11}	0.000 00	0.000 00	−0.649 35	0.000 00	−0.413 22	2.432 44	−1.369 86	0.000 00	0.003 15
H_{12}	0.000 00	0.000 00	0.000 00	−0.680 27	0.000 00	−1.369 86	2.667 42	−0.617 28	0.002 39
H_{13}	−0.497 51	0.000 00	0.000 00	0.000 00	−1.219 51	0.000 00	−0.617 28	2.334 31	−0.005 10

Least squares solution $\widehat{\Delta x}$[m]

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
$\widehat{\Delta x}^\mathsf{T}$	0.000 00	0.000 00	0.000 00	0.000 00	0.001 99	0.002 91	0.002 26	−0.000 55

Adjusted heights

Point ID	$\widehat{H}$ [m]	$\widehat{H} - H$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]
1	510.3690	0.00	0.00
2	508.7620	0.00	0.00
3	526.1740	0.00	0.00
4	515.9820	0.00	0.00
10	502.1650	1.99	2.50
11	501.5649	2.91	2.33
12	503.7913	2.26	2.27
13	501.9835	−0.55	2.44

Adjusted levelled height differences

Same as for datum definition 4!

Adjusted trigonometric height differences

Same as for datum definition 4!

Adjusted dynamic heights

Point ID	$\widehat{H}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\widehat{H}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]
1	510.3690	0.00	0.00	0.00	∞	1.02	+0.00

Point ID	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	∞	0.00	1.02	0.00	1.02	0.00	∞

2.4.6 Datum definition 6: Dynamic, with stochastic prior information for observed points 1-4

Variance-Covariance matrix of dynamic heights [mm^2]

Σ_H	1	2	3	4
1	9.340	8.050	2.050	1.150
2	8.050	8.750	2.100	1.750
3	2.050	2.100	6.110	5.900
4	1.150	1.750	5.900	7.620

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{mm}]}$ dynamic heights

$\boldsymbol{A}$	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}	Δy
H_1	1	0	0	0	0	0	0	0	0.00
H_2	0	1	0	0	0	0	0	0	0.00
H_3	0	0	1	0	0	0	0	0	0.00
H_4	0	0	0	1	0	0	0	0	0.00

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

N	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}	$A^{\mathsf{T}}P\Delta y$
H_1	3.489 42	−2.747 63	−0.806 14	0.769 57	0.000 00	0.000 00	0.000 00	−0.497 51	−0.002 51
H_2	−2.747 63	3.592 52	0.456 97	−0.614 71	−0.502 51	0.000 00	0.000 00	0.000 00	0.003 27
H_3	−0.806 14	0.456 97	4.439 02	−3.027 12	0.000 00	−0.649 35	0.000 00	0.000 00	−0.004 38
H_4	0.769 57	−0.614 71	−3.027 12	3.749 26	0.000 00	0.000 00	−0.680 27	0.000 00	−0.000 53
H_{10}	0.000 00	−0.502 51	0.000 00	0.000 00	2.135 25	−0.413 22	0.000 00	−1.219 51	0.003 72
H_{11}	0.000 00	0.000 00	−0.649 35	0.000 00	−0.413 22	2.432 44	−1.369 86	0.000 00	0.003 15
H_{12}	0.000 00	0.000 00	0.000 00	−0.680 27	0.000 00	−1.369 86	2.667 42	−0.617 28	0.002 39
H_{13}	−0.497 51	0.000 00	0.000 00	0.000 00	−1.219 51	0.000 00	−0.617 28	2.334 31	−0.005 10

Least squares solution $\widehat{\Delta x}_{[\mathrm{m}]}$

	H_1	H_2	H_3	H_4	H_{10}	H_{11}	H_{12}	H_{13}
$\widehat{\Delta x}^{\mathsf{T}}$	0.000 42	0.001 66	−0.000 84	−0.000 23	0.002 56	0.002 75	0.002 21	−0.000 17

Adjusted heights

Point ID	$\widehat{H}_{[\mathrm{m}]}$	$\widehat{H} - H_{[\mathrm{mm}]}$	$\lvert\hat{\sigma}\rvert_{[\mathrm{mm}]}$
1	510.3694	0.42	3.19
2	508.7637	1.66	3.16
3	526.1732	−0.84	2.81
4	515.9818	−0.23	2.98
10	502.1656	2.56	3.50
11	501.5647	2.75	3.34
12	503.7912	2.21	3.32
13	501.9838	−0.17	3.45

Adjusted dynamic heights

Variance component: $\Omega/\sigma_0^2 = 1.250,\ r = 0.93,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.34,\ \alpha_G = 0.09\%,\ k^F_{\alpha_G;r,\infty} = 11.58$

Point name	$\widehat{H}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\widehat{H}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	510.3694	−0.42	3.19	26.95	0.26	2.43	−0.16	16.28	1.04	1.78	0.11	1.78	0.11	0.22
2	508.7637	−1.66	3.16	23.28	1.16	2.53	−0.71	17.28	4.85	1.94	0.55	1.94	0.55	0.96
3	526.1732	0.84	2.81	13.75	0.92	2.75	+0.61	21.12	4.68	2.38	0.53	2.38	0.53	0.76
4	515.9818	0.24	2.98	28.94	0.16	2.12	+0.08	14.08	0.54	1.51	0.06	1.51	0.06	0.13

Adjusted levelled height differences

Variance component: $\Omega/\sigma_0^2 = 6.504,\ r = 4.13,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.58,\ \alpha_G = 0.94\%,\ k^F_{\alpha_G;r,\infty} = 3.31$

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
2	1	1.6058	−2.26	1.48	85.42	0.76	1.44	−0.26	1.71	0.31	0.21	0.04	0.21	0.04	0.63
4	3	10.1914	−0.39	1.42	85.03	0.14	1.36	−0.05	1.73	0.06	0.20	0.01	0.20	0.01	0.12
10	2	6.5981	4.41	2.57	49.23	2.10	1.76	+0.90	4.20	2.13	0.89	0.45	0.89	0.45	1.74
11	3	24.6084	−2.41	2.30	47.17	1.33	1.58	−0.51	4.37	1.41	0.84	0.27	0.84	0.27	1.11
	12	2.2265	1.54	1.86	27.31	1.63	1.43	+0.56	6.74	2.65	1.04	0.41	1.04	0.41	1.35
12	4	12.1906	0.94	2.34	42.73	0.56	1.63	+0.22	4.78	0.65	0.93	0.13	0.93	0.13	0.47
13	1	8.3856	−2.59	2.59	48.89	1.23	1.78	−0.53	4.23	1.26	0.91	0.27	0.91	0.27	1.02
	10	0.1817	2.26	1.98	26.87	2.27	1.53	+0.84	6.82	3.75	1.12	0.62	1.12	0.62	1.89

Adjusted trigonometric height differences

Variance component: $\Omega/\sigma_0^2 = 0.937,\ r = 0.94,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.99,\ \alpha_G = 0.09\%,\ k^F_{\alpha_G;r,\infty} = 11.41$

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
11	10	0.6008	−1.32	2.69	54.15	0.54	1.85	−0.24	3.80	0.50	0.85	0.11	0.85	0.11	0.45
13	12	1.8074	−2.38	2.51	40.27	1.39	1.76	−0.59	5.03	1.69	1.05	0.35	1.05	0.35	1.15

Supplementary information

Datum definition	:	1	2	3	4	5	6
Observed levelled height differences	:	8	8	8	8	8	8
Observed trigonometric height differences	:	2	2	2	2	2	2
Dynamic heights	:	0	0	0	0	4	4
Height unknowns	:	7	7	8	4	8	8
Datum defect	:	1	1	1	1	0	0
Datum definition	:	fix	fix	free	fix	dynamic	dynamic
Number of datum constraints	:	1	1	1	4	0	0
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1	0.1	0.1	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.6	0.6	0.6	1.8	1.8	1.8
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29	3.29	3.29	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1	17.1	17.1	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1	0.1	0.1	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.7	1.7	1.7	2.3	2.3	2.3
Redundancy r	:	3	3	3	6	6	6
Redundancy dynamic heights	:	n.a.	n.a.	n.a.	n.a.	0.00	0.93
Redundancy levelled height differences	:	2.25	2.25	2.25	4.88	4.88	4.13
Redundancy trigonometric height differences	:	0.75	0.75	0.75	1.12	1.12	0.94
Weighted square sum of residuals Ω [mm^2]	:	17.314	17.314	17.314	46.885	46.885	39.109
(a priori) standard deviation σ_0 [mm]	:	2.121	2.121	2.121	2.121	2.121	2.121
(a posteriori) standard deviation $\hat{\sigma}_0$ [mm]	:	2.402	2.402	2.402	2.795	2.795	2.553
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.1325	1.1325	1.1325	1.3178	1.3178	1.2035
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	1.2825	1.2825	1.2825	1.7365	1.7365	1.4485
Critical value $k^F_{\alpha_G;r,\infty}$	:	4.21	4.21	4.21	2.56	2.56	2.56
Number of outliers (Data snooping)	:	0	0	0	0	0	0
Number of outliers (τ-criterion)	:	0	0	0	0	0	0
$\|\widehat{H} - H\|$ [cm]	:	0.779	0.825	0.674	0.422	0.422	0.477
Trace height covariance matrix, tr$\widehat{\Sigma}_{\widehat{H}}$ [mm^2]	:	95.176	86.225	38.186	22.780	22.780	83.320
Trace height cofactor matrix, tr$Q_{\widehat{H}}$	:	16.491	14.940	6.616	2.915	2.915	12.783

2.5 Mittermayer (1971)

Mittermayer E (1971): Eine Verallgemeinerung der Methode der kleinsten Quadrate zur Ausgleichung freier Netze. ZfV 96, pp. 401-410

This network is processed in two versions: In the first scenario, the observational standard deviations depend on the leveling line lengths, and the datum problem is resolved using a free datum choice (total trace minimization). In the second case, the network datum is defined by the benchmarks 1-3, and an unknown scale m in the levelled data as well as an unknown offset a have been added to the problem. While the approximate scale has been set to a reasonable value $m = 1$, for the approximate additive constant a value $a = 4cm$ is used. As a consequence the null hypothesis $H_0 : a = 4cm$ is rejected ($T_F = \frac{(\hat{a}-4_{[cm]})^2}{\hat{\sigma}^2_{\hat{a}}} = 18.8 > k^F_{95\%;1,4} = 7.7$) in favor of the alternative hypothesis $H_A : a = \hat{a}$. Furthermore, in this example, all observation weights are assumed to be $p = 1$.

Heights/Coordinates

Point name	Easting x [m]	Northing y [m]	Free datum: Total trace minimization H [m]	Fix datum H [m]
1	130.2200	173.4000	0.0000 (D)	0.0000 (D)
2	272.8100	218.3500	86.8100 (D)	86.8100 (D)
3	322.3900	126.1000	14.8500 (D)	14.8500 (D)
4	202.0900	157.6200	25.7100 (D)	25.7100
5	283.1900	65.6700	31.2200 (D)	31.2200
6	380.7100	120.3400	42.6300 (D)	42.6300

Datum: free, (D)...Datum coordinate

2.5.1 Free network: Total trace minimization

Available data files: [1D] Mittermayer_Height_free∗.∗

Levelled height differences

in	to	h [m]	Lev-Line s [m]	$\lvert\sigma_l\rvert$ [cm/$\sqrt{\text{km}}$]	$\lvert\sigma\rvert$ [cm]	p [–]
1	2	86.8090	20 400.00	0.10	0.451 66	0.049 02
	4	25.7140	18 700.00	0.10	0.432 43	0.053 48
	5	31.2250	34 800.00	0.10	0.589 92	0.028 74
2	3	−71.9520	15 000.00	0.10	0.387 30	0.066 67
	4	−61.0840	14 200.00	0.10	0.376 83	0.070 42
	6	−44.1780	25 300.00	0.10	0.502 99	0.039 53
3	4	10.8470	12 900.00	0.10	0.359 17	0.077 52
	5	16.3500	9800.00	0.10	0.313 05	0.102 04
5	6	11.4090	19 600.00	0.10	0.442 72	0.051 02

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[\mathrm{cm}]}$ levelled height differences

A	H_1	H_2	H_3	H_4	H_5	H_6	Δy
$h_{1,2}$	−1	1	0	0	0	0	−0.100
$h_{1,4}$	−1	0	0	1	0	0	0.400
$h_{1,5}$	−1	0	0	0	1	0	0.500
$h_{2,3}$	0	−1	1	0	0	0	0.800
$h_{2,4}$	0	−1	0	1	0	0	1.600
$h_{2,6}$	0	−1	0	0	0	1	0.200
$h_{3,4}$	0	0	−1	1	0	0	−1.300
$h_{3,5}$	0	0	−1	0	1	0	−2.000
$h_{5,6}$	0	0	0	0	−1	1	−0.100

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

N	H_1	H_2	H_3	H_4	H_5	H_6	$A^{\mathsf{T}}P\Delta y$
H_1	0.131 23	−0.049 02	0.000 00	−0.053 48	−0.028 74	0.000 00	−0.000 31
H_2	−0.049 02	0.225 63	−0.066 67	−0.070 42	0.000 00	−0.039 53	−0.001 79
H_3	0.000 00	−0.066 67	0.246 23	−0.077 52	−0.102 04	0.000 00	0.003 58
H_4	−0.053 48	−0.070 42	−0.077 52	0.201 42	0.000 00	0.000 00	0.000 33
H_5	−0.028 74	0.000 00	−0.102 04	0.000 00	0.181 80	−0.051 02	−0.001 85
H_6	0.000 00	−0.039 53	0.000 00	0.000 00	−0.051 02	0.090 55	0.000 03

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	H_1	H_2	H_3	H_4	H_5	H_6
D^{T}	0.408 25	0.408 25	0.408 25	0.408 25	0.408 25	0.408 25

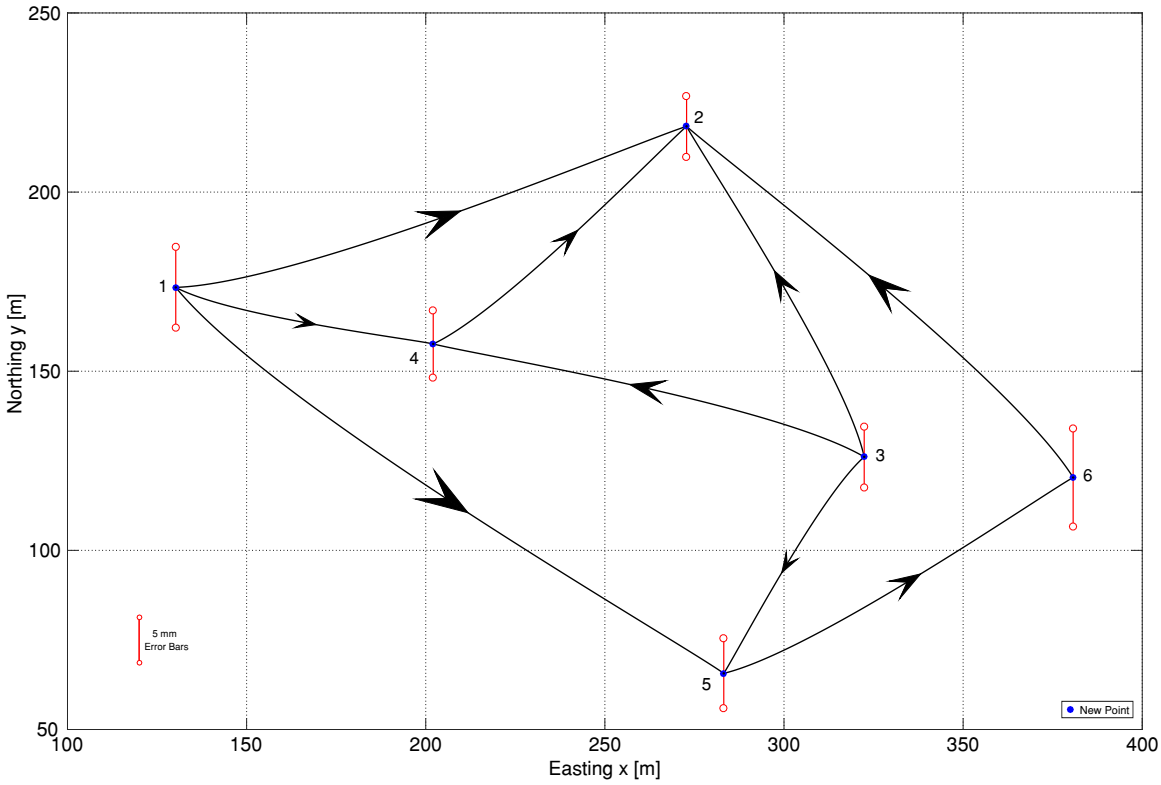
Least squares solution $\widehat{\Delta x}_{[\mathrm{m}]}$

	H_1	H_2	H_3	H_4	H_5	H_6
$\widehat{\Delta x}^{\mathsf{T}}$	−0.003 35	−0.004 38	0.012 68	0.004 11	−0.004 77	−0.004 29

Adjusted heights

Point name	$\widehat{H}_{[\mathrm{m}]}$	$\widehat{H} - H_{[\mathrm{mm}]}$	$\lvert\hat{\sigma}\rvert_{[\mathrm{mm}]}$
1	−0.0034	−3.35	4.51
2	86.8056	−4.38	3.38
3	14.8627	12.68	3.40
4	25.7141	4.11	3.79
5	31.2152	−4.77	3.92
6	42.6257	−4.29	5.45

Network graph



Adjusted levelled height differences

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	2	86.8090	0.02	5.83	51.46	0.01	2.60	0.00	4.01	0.01	1.26	0.00	1.26	0.00	0.00
	4	25.7175	−3.47	5.83	47.00	1.17	2.61	−0.74	4.39	1.24	1.38	0.39	1.38	0.39	0.63
	5	31.2186	6.42	6.80	61.19	1.39	3.12	+1.05	3.29	1.11	1.21	0.41	1.21	0.41	0.75
2	3	−71.9429	−9.06	5.05	50.33	3.30	2.26	−1.80*	4.11	3.27	1.12	0.89	1.12	0.89	1.78
	4	−61.0915	7.51	5.00	48.53	2.86	2.24	+1.55	4.26	2.95	1.15	0.80	1.15	0.80	1.55
	6	−44.1799	1.91	7.05	42.72	0.58	3.18	+0.45	4.78	0.67	1.82	0.26	1.82	0.26	0.31
3	4	10.8514	−4.43	5.11	40.96	1.93	2.32	−1.08	4.96	2.31	1.37	0.64	1.37	0.64	1.04
	5	16.3525	−2.55	5.03	24.71	1.64	2.60	−1.03	7.21	2.86	1.96	0.78	1.96	0.78	0.88
5	6	11.4105	−1.48	6.70	33.10	0.58	3.18	−0.45	5.88	0.83	2.13	0.30	2.13	0.30	0.31

2.5.2 Fix network, with scale and additive constant, unweighted observations

Available data files: [1D] Mittermayer_Height_fix_M∗.∗

Levelled height differences

in	to	$h_{[m]}$	$p_{[-]}$
1	2	86.8090	1
	4	25.7140	1
	5	31.2250	1
2	3	−71.9520	1
	4	−61.0840	1
	6	−44.1780	1
3	4	10.8470	1
	5	16.3500	1
5	6	11.4090	1

Approximate scale: $m = 1.0$ [-]

Approximate additive constant: $a = 40.00$ [mm]

Design matrix $A_{[-,m,-]}$ **and reduced observation vector** $\Delta y_{[cm]}$ **levelled height differences** (1. iteration)

A	H_1	H_2	H_3	H_4	H_5	H_6	m	a	Δy
$h_{1,2}$	−1	1	0	0	0	0	86.8100	1	−4.10
$h_{1,4}$	−1	0	0	1	0	0	25.7100	1	−3.60
$h_{1,5}$	−1	0	0	0	1	0	31.2200	1	−3.50
$h_{2,3}$	0	−1	1	0	0	0	−71.9600	1	−3.20
$h_{2,4}$	0	−1	0	1	0	0	−61.1000	1	−2.40
$h_{2,6}$	0	−1	0	0	0	1	−44.1800	1	−3.80
$h_{3,4}$	0	0	−1	1	0	0	10.8600	1	−5.30
$h_{3,5}$	0	0	−1	0	1	0	16.3700	1	−6.00
$h_{5,6}$	0	0	0	0	−1	1	11.4100	1	−4.10

Normal equation system $N, A^{\mathsf{T}}P\Delta y$ (1. iteration)

N	H_1	H_2	H_3	H_4	H_5	H_6	m	a	$A^{\mathsf{T}}P\Delta y$
H_1	3.000	−1.000	0.000	−1.000	−1.000	0.000	−143.7400	−3.000	0.112 00
H_2	−1.000	4.000	−1.000	−1.000	0.000	−1.000	264.0500	−2.000	0.053 00
H_3	0.000	−1.000	3.000	−1.000	−1.000	0.000	−99.1900	−1.000	0.081 00
H_4	−1.000	−1.000	−1.000	3.000	0.000	0.000	−24.5300	3.000	−0.113 00
H_5	−1.000	0.000	−1.000	0.000	3.000	−1.000	36.1800	1.000	−0.054 00
H_6	0.000	−1.000	0.000	0.000	−1.000	2.000	−32.7700	2.000	−0.079 00
m	−143.740	264.050	−99.190	−24.530	36.180	−32.770	20 551.0972	5.140	−2.155 10
a	−3.000	−2.000	−1.000	3.000	1.000	2.000	5.1400	9.000	−0.360 00

Matrix $D^{\mathsf{T}}{}_{[\text{-,m,-}]}$ of datum constraints

	H_1	H_2	H_3	H_4	H_5	H_6	m	a
D^{T}	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	1	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m,-,m}]}$ (1. iteration)

	H_1	H_2	H_3	H_4	H_5	H_6	m	a
$\widehat{\Delta x}^{\mathsf{T}}$	0.000 00	0.000 00	0.000 00	−0.001 52	−0.007 07	−0.007 69	−0.000 10	−0.036 95

Adjusted heights

Point name	$\widehat{H}$ [m]	$\widehat{H} - H$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]
1	0.0000	0.00	0.00
2	86.8100	0.00	0.00
3	14.8500	0.00	0.00
4	25.7085	−1.52	11.47
5	31.2129	−7.07	10.56
6	42.6223	−7.69	15.80

Adjusted scale ($1 - \alpha_K = 95\%$)

$\widehat{m}$ [−]	$\widehat{m} - m$ [ppm]	$\lvert\hat{\sigma}\rvert$ [ppm]	T_F	$k^F_{\alpha_K;1,4}$
0.999 902 764 05	−97.2	94.2	1.1	7.7

Adjusted additive constant ($1 - \alpha_K = 95\%$)

$\hat{a}$ [mm]	$\hat{a} - a$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	T_F	$k^F_{\alpha_K;1,4}$
3.0548	−36.95	8.5	18.8	7.7

Adjusted levelled height differences

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	2	86.8046	4.39	10.47	34.11	7.51	0.71	+1.29*	3.05	5.54	0.47	0.85	0.00	0.00	0.58
	4	25.7090	4.96	8.10	60.54	6.38	0.53	+0.82*	2.67	4.12	0.21	0.32	0.20	0.31	0.49
	5	31.2129	12.05	8.53	56.32	16.06	0.55	+2.14*	2.91	11.32	0.24	0.93	0.19	0.73	1.25
2	3	−71.9499	−2.05	11.98	13.80	5.52	1.11	−1.49*	7.82	10.46	0.96	1.28	0.00	0.00	0.43
	4	−61.0925	8.52	8.96	51.73	11.85	0.57	+1.65*	2.48	7.11	0.28	0.80	0.15	0.42	0.92
	6	−44.1803	2.33	10.77	30.27	4.24	0.75	+0.77*	5.25	5.39	0.52	0.54	0.59	0.60	0.33
3	4	10.8605	−13.48	7.66	64.73	16.76	0.51	−2.08*	2.50	10.13	0.18	0.73	0.19	0.75	1.30
	5	16.3644	−14.39	8.34	58.22	18.86	0.54	−2.47#	2.94	13.41	0.23	1.03	0.18	0.83	1.46
5	6	11.4113	−2.33	10.77	30.27	4.24	0.75	−0.77*	5.72	5.88	0.52	0.54	0.41	0.42	0.33

Supplementary information

Observed levelled height differences	:	9	9
Height unknowns	:	6	3
Datum defect	:	1	2
Datum definition	:	free	fix
Number of datum constraints	:	1	3
Scale unknown	:	no	yes
Additive constant unknown	:	no	yes
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.9	0.9
Power of test γ [%] (Baarda)	:	80.0	80.0
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.0	2.0
Redundancy r	:	4	4
Redundancy levelled height differences	:	4.00	4.00
Weighted square sum of residuals Ω [mm^2]	:	13.71	665.70
(a priori) standard deviation σ_0 [mm]	:	1	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mm]	:	1.8513	12.9006
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.8513	12.9006
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	3.4274	166.4260
Critical value $k^F_{\alpha_G;r,\infty}$	:	3.38	3.38
Number of outliers (Data snooping)	:	1	9
Number of outliers (τ-criterion)	:	0	0
$\|\widehat{H} - H\|$ [cm]	:	1.579	1.056
Trace height covariance matrix, tr$\widehat{\Sigma}_{\widehat{H}}$ [cm^2]	:	1.02850	4.92842
Trace height cofactor matrix, tr$Q_{\widehat{H}}$	:	30.00858	2.96133

2.6 Niemeier (2008), Ex. 4.4.2 & 7.5.2

Niemeier W (2008): Ausgleichungsrechnung, 2. Auflage. Walter de Gruyter, pp. 153-156/268-269

Available data files: [1D] Niemeier_Height∗.∗

This example consists of 2 different scenarios which differ in the datum definition. Scenario 1 uses point 6 as a benchmark, scenario 2 solves the datum problem using the free network approach (partial-trace minimization of the variance-covariance matrix of estimated point heights)

Heights/Coordinates, 2 different datum definitions

Point name	H [m]	x [m]	y [m]	1	2
1	68.9270	450.7700	430.3100		(D)
2	60.7120	658.1500	704.0300		
3	63.1930	877.9600	302.9600		(D)
4	56.2860	1170.2500	754.0000		
5	44.3240	1650.1800	601.5200		(D)
6	67.2280	1436.4000	230.0000	(D)	
datum type				fix	free

Levelled height differences

in	to	h [m]	Lev-Line s [m]	$\lvert\sigma_l\rvert$ [cm/$\sqrt{\text{km}}$]	$\lvert\sigma\rvert$ [cm]	p [–]
1	2	−8.2060	621.12	0.10	0.078 81	1.610 00
	3	−5.7340	1204.82	0.10	0.109 76	0.830 00
2	3	2.4810	450.45	0.10	0.067 12	2.220 00
	4	−4.4330	800.00	0.10	0.089 44	1.250 00
3	4	−6.9090	1000.00	0.10	0.100 00	1.000 00
	5	−18.8720	1098.90	0.10	0.104 83	0.910 00
	6	4.0350	440.53	0.10	0.066 37	2.270 00
4	5	−11.9620	719.42	0.10	0.084 82	1.390 00
5	6	22.9040	833.33	0.10	0.091 29	1.200 00

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{mm}]}$ levelled height differences

A	H_1	H_2	H_3	H_4	H_5	H_6	Δy
$h_{1,2}$	−1	1	0	0	0	0	9.00
$h_{1,3}$	−1	0	1	0	0	0	0.00
$h_{2,3}$	0	−1	1	0	0	0	0.00
$h_{2,4}$	0	−1	0	1	0	0	−7.00
$h_{3,4}$	0	0	−1	1	0	0	−2.00
$h_{3,5}$	0	0	−1	0	1	0	−3.00
$h_{3,6}$	0	0	−1	0	0	1	0.00
$h_{4,5}$	0	0	0	−1	1	0	0.00
$h_{5,6}$	0	0	0	0	−1	1	0.00

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

N	H_1	H_2	H_3	H_4	H_5	H_6	$A^{\mathsf{T}}P\Delta y$
H_1	2.440 00	−1.610 00	−0.830 00	0.000 00	0.000 00	0.000 00	−0.014 49
H_2	−1.610 00	5.080 00	−2.220 00	−1.250 00	0.000 00	0.000 00	0.023 24
H_3	−0.830 00	−2.220 00	7.230 00	−1.000 00	−0.910 00	−2.270 00	0.004 73
H_4	0.000 00	−1.250 00	−1.000 00	3.640 00	−1.390 00	0.000 00	−0.010 75
H_5	0.000 00	0.000 00	−0.910 00	−1.390 00	3.500 00	−1.200 00	−0.002 73
H_6	0.000 00	0.000 00	−2.270 00	0.000 00	−1.200 00	3.470 00	0.000 00

2.6.1 Fix network

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ **of datum constraints**

	H_1	H_2	H_3	H_4	H_5	H_6
D^{T}	0	0	0	0	0	1

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$

	H_1	H_2	H_3	H_4	H_5	H_6
$\widehat{\Delta x}^{\mathsf{T}}$	−0.003 53	0.003 25	0.000 76	−0.002 18	−0.001 45	0.000 00

Adjusted heights

Point name	$\widehat{H}_{[\text{m}]}$	$\widehat{H} - H_{[\text{mm}]}$	$\lvert\hat{\sigma}\rvert_{[\text{mm}]}$
1	68.9235	−3.53	3.12
2	60.7153	3.25	2.60
3	63.1938	0.76	1.97
4	56.2838	−2.18	2.63
5	44.3226	−1.45	2.30
6	67.2280	0.00	0.00

2.6.2 Free network: Partial trace minimization

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ **of datum constraints**

	H_1	H_2	H_3	H_4	H_5	H_6
D^{T}	0.408 25	0.000 00	0.408 25	0.000 00	0.408 25	0.000 00

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$

	H_1	H_2	H_3	H_4	H_5	H_6
$\widehat{\Delta x}^{\mathsf{T}}$	−0.002 13	0.004 66	0.002 17	−0.000 77	−0.000 04	0.001 40

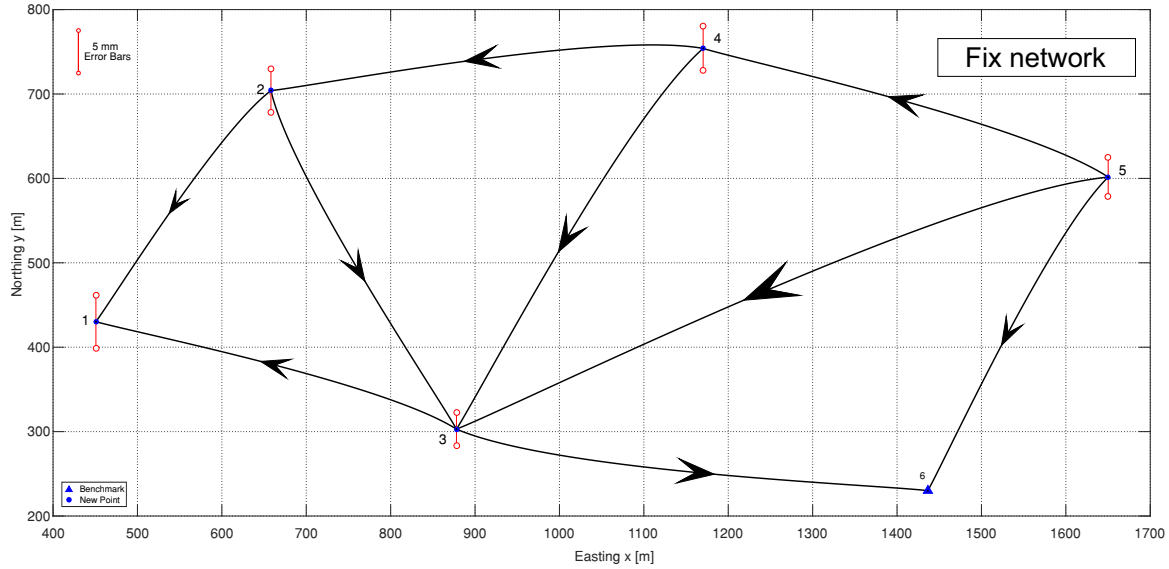
Adjusted heights

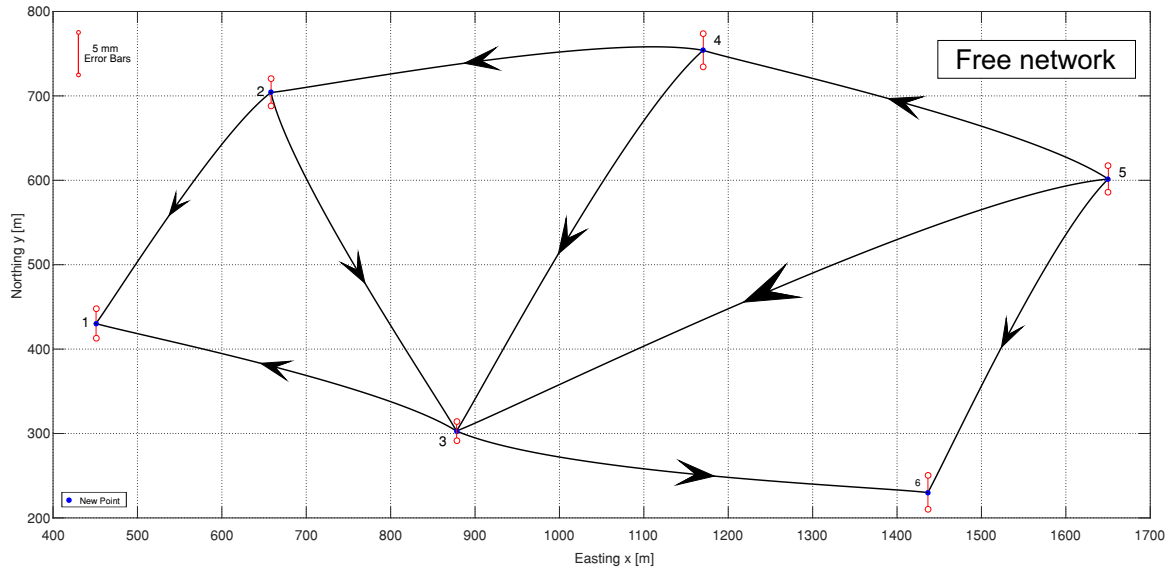
Point name	$\widehat{H}$ [m]	$\widehat{H} - H$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]
1	68.9249	−2.13	1.75
2	60.7167	4.66	1.65
3	63.1952	2.17	1.13
4	56.2852	−0.77	1.94
5	44.3240	−0.04	1.60
6	67.2294	1.40	2.00

Adjusted levelled height differences (both datum definitions)

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	2	−8.2082	2.21	2.26	28.69	5.25	0.61	+0.77*	6.51	8.27	0.43	0.55	0.43	0.55	1.55
	3	−5.7297	−4.30	2.48	55.66	5.25	0.61	−0.77*	3.69	4.68	0.27	0.34	0.27	0.34	1.55
2	3	2.4785	2.49	1.81	36.56	6.13	0.46	+0.68#	5.44	8.08	0.29	0.43	0.29	0.43	1.81
	4	−4.4314	−1.57	2.22	46.29	2.58	0.54	−0.34	4.45	2.78	0.29	0.18	0.29	0.18	0.76
3	4	−6.9099	0.94	2.10	61.90	1.20	0.53	+0.15	3.24	0.94	0.20	0.06	0.20	0.06	0.35
	5	−18.8712	−0.79	2.15	63.46	0.94	0.54	−0.12	3.14	0.72	0.20	0.05	0.20	0.05	0.28
	6	4.0342	0.76	1.97	23.68	2.37	0.56	+0.32	7.42	4.25	0.43	0.25	0.43	0.25	0.70
4	5	−11.9613	−0.73	2.25	38.96	1.38	0.56	−0.19	5.17	1.73	0.34	0.11	0.34	0.11	0.41
5	6	22.9054	−1.45	2.30	44.80	2.37	0.56	−0.32	4.59	2.63	0.31	0.18	0.31	0.18	0.70

Network graph





Supplementary information

Datum definition	:	1	2
Observed levelled height differences	:	9	9
Height unknowns	:	5	6
Datum defect	:	1	1
Datum definition	:	fix	free
Number of datum constraints	:	1	1
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.9	0.9
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.0	2.0
Redundancy r	:	4	4
Redundancy levelled height differences	:	4.00	4.00
Weighted square sum of residuals Ω [mm^2]	:	46.082	46.082
(a priori) standard deviation σ_0 [mm]	:	1	1
(a posteriori) standard deviation $\hat{\sigma}_0$ [mm]	:	3.394	3.394
Ratio $\hat{\sigma}_0/\sigma_0$	:	3.3942	3.3942
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	11.5204	11.5204
Critical value $k^F_{\alpha_G;r,\infty}$	:	3.38	3.38
Number of outliers (Data snooping)	:	3	3
Number of outliers (τ-criterion)	:	0	0
$\|\widehat{H} - H\|$ [cm]	:	0.552	0.579
Trace height covariance matrix, tr$\widehat{\Sigma}_{\widehat{H}}$ [mm^2]	:	0.32554	0.17397
Trace height cofactor matrix, tr$Q_{\widehat{H}}$	:	2.82578	1.51013

2.7 Baumann (1995), Ch. 13.4.2, Final solution

Baumann E (1995): Vermessungskunde, Lehr- und Übungsbuch für Ingenieure. Band 2: Punktbestimmung nach Höhe und Lage. Fünfte, bearbeitete und erweiterte Auflage. Bonn. Chapter 13.4.2, pp. 32

Available data files: [1D] Baumann_Height_fix∗.∗

Heights/Coordinates

Point name	H [m]	Easting x [m]	Northing y [m]
1	199.2950	63.8300	100.0000
2	199.9100	215.9200	112.8600
3	207.6400	280.9900	248.3000
4	226.5780 (D)	278.7200	602.4200
5	218.3800	348.3300	403.4200
6	213.9510 (D)	376.3300	333.0500
7	212.9000	358.9200	278.5700
8	209.1240 (D)	396.0000	232.4100
9	203.7710 (D)	300.6600	137.8300
10	210.8800	456.5300	333.8100
11	211.3800	482.2600	274.0300
12	204.4000	442.1600	123.4600
13	199.8900	511.7700	214.2500
14	197.8620 (D)	574.5700	258.9000

Datum: fix, (D)...Datum coordinate

Levelled height differences

in	to	h [m]	Lev-Line s [m]	$\lvert\sigma_l\rvert$ [cm/$\sqrt{\text{km}}$]	$\lvert\sigma\rvert$ [cm]	p [−]
1	2	0.6235	2500.00	0.10	0.158 11	0.400 00
	2	0.6240	3800.00	0.10	0.194 94	0.263 16
2	3	7.7292	5000.00	0.10	0.223 61	0.200 00
	9	3.8582	1600.00	0.10	0.126 49	0.625 00
3	8	1.4813	1800.00	0.10	0.134 16	0.555 56
5	4	8.2021	3800.00	0.10	0.194 94	0.263 16
6	5	4.4254	900.00	0.10	0.094 87	1.111 11
7	6	1.0502	600.00	0.10	0.077 46	1.666 67
8	7	3.7782	1600.00	0.10	0.126 49	0.625 00
	11	2.2530	1000.00	0.10	0.100 00	1.000 00
9	8	5.3523	2400.00	0.10	0.154 92	0.416 67
	12	0.6374	3000.00	0.10	0.173 21	0.333 33
10	5	7.4945	1800.00	0.10	0.134 16	0.555 56
	7	2.0179	1000.00	0.10	0.100 00	1.000 00
	11	0.4950	1300.00	0.10	0.114 02	0.769 23
12	8	4.7158	2400.00	0.10	0.154 92	0.416 67
13	11	11.4908	1200.00	0.10	0.109 54	0.833 33
	12	4.5218	1700.00	0.10	0.130 38	0.588 24
14	13	2.0246	1200.00	0.10	0.109 54	0.833 33
	13	2.0251	1400.00	0.10	0.118 32	0.714 29

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ levelled height differences

A	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}	Δy
$h_{1,2}$	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0.850
$h_{1,2}$	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0.900
$h_{2,3}$	0	−1	1	0	0	0	0	0	0	0	0	0	0	0	−0.080
$h_{2,9}$	0	−1	0	0	0	0	0	0	1	0	0	0	0	0	−0.280
$h_{3,8}$	0	0	−1	0	0	0	0	1	0	0	0	0	0	0	−0.270
$h_{5,4}$	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0.410
$h_{6,5}$	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	−0.360
$h_{7,6}$	0	0	0	0	0	1	−1	0	0	0	0	0	0	0	−0.080
$h_{8,7}$	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	0.220
$h_{8,11}$	0	0	0	0	0	0	0	−1	0	0	1	0	0	0	−0.300
$h_{9,8}$	0	0	0	0	0	0	0	1	−1	0	0	0	0	0	−0.070
$h_{9,12}$	0	0	0	0	0	0	0	0	−1	0	0	1	0	0	0.840
$h_{10,5}$	0	0	0	0	1	0	0	0	0	−1	0	0	0	0	−0.550
$h_{10,7}$	0	0	0	0	0	0	1	0	0	−1	0	0	0	0	−0.210
$h_{10,11}$	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	−0.500
$h_{12,8}$	0	0	0	0	0	0	0	1	0	0	0	−1	0	0	−0.820
$h_{13,11}$	0	0	0	0	0	0	0	0	0	0	1	0	−1	0	0.080
$h_{13,12}$	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	1.180
$h_{14,13}$	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	−0.340
$h_{14,13}$	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	−0.290

Normal equation system $N, A^{\mathsf{T}}P\Delta y$

N	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}	$A^{\mathsf{T}}P\Delta y$
H_1	0.6632	−0.6632	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.005 77
H_2	−0.6632	1.4882	−0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.6250	0.0000	0.0000	0.0000	0.0000	0.0000	0.007 68
H_3	0.0000	−0.2000	0.7556	0.0000	0.0000	0.0000	0.0000	−0.5556	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.001 34
H_4	0.0000	0.0000	0.0000	0.2632	−0.2632	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.001 08
H_5	0.0000	0.0000	0.0000	−0.2632	1.9298	−1.1111	0.0000	0.0000	0.0000	−0.5556	0.0000	0.0000	0.0000	0.0000	−0.008 13
H_6	0.0000	0.0000	0.0000	0.0000	−1.1111	2.7778	−1.6667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.002 67
H_7	0.0000	0.0000	0.0000	0.0000	0.0000	−1.6667	3.2917	−0.6250	0.0000	−1.0000	0.0000	0.0000	0.0000	0.0000	0.000 61
H_8	0.0000	0.0000	−0.5556	0.0000	0.0000	0.0000	−0.6250	3.0139	−0.4167	0.0000	−1.0000	−0.4167	0.0000	0.0000	−0.003 58
H_9	0.0000	−0.6250	0.0000	0.0000	0.0000	0.0000	0.0000	−0.4167	1.3750	0.0000	0.0000	−0.3333	0.0000	0.0000	−0.004 26
H_{10}	0.0000	0.0000	0.0000	0.0000	−0.5555	0.0000	−1.0000	0.0000	0.0000	2.3248	−0.7692	0.0000	0.0000	0.0000	0.009 00
H_{11}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	−0.7692	2.6026	0.0000	−0.8333	0.0000	−0.006 18
H_{12}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.4167	−0.3333	0.0000	0.0000	1.3382	−0.5882	0.0000	0.013 16
H_{13}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.8333	−0.5882	2.9692	−1.5476	−0.012 51
H_{14}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−1.5476	1.5476	0.004 90

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}
D^{T}	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$

	H_1	H_2	H_3	H_4	H_5	H_6	H_7
$\widehat{\Delta x}^{\mathsf{T}}$	−0.005 77	0.002 93	0.002 55	0.000 00	−0.003 47	0.000 00	0.000 97

	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}
$\widehat{\Delta x}^{\mathsf{T}}$	0.000 00	0.000 00	0.002 57	−0.002 67	0.008 38	−0.003 30	0.000 00

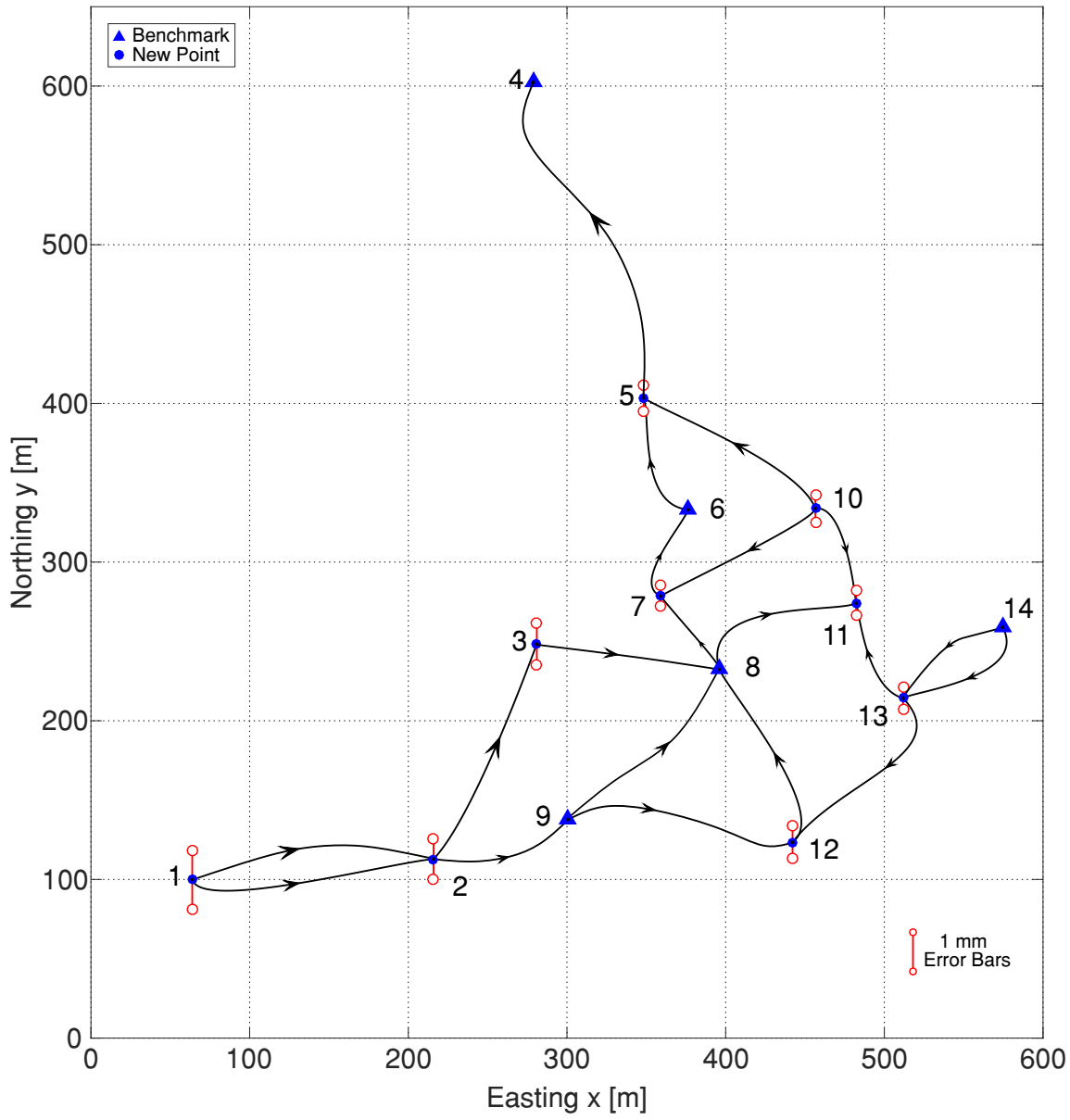
Adjusted heights

Point name	$\widehat{H}_{[\text{m}]}$	$\widehat{H} - H_{[\text{mm}]}$	$\lvert\hat{\sigma}\rvert_{[\text{mm}]}$
1	199.2892	−5.77	0.74
2	199.9129	2.93	0.50
3	207.6426	2.55	0.53
4	226.5780	0.00	0.00
5	218.3765	−3.47	0.33
6	213.9510	0.00	0.00
7	212.9010	0.97	0.27
8	209.1240	0.00	0.00
9	203.7710	0.00	0.00
10	210.8826	2.57	0.35
11	211.3773	−2.67	0.31
12	204.4084	8.38	0.40
13	199.8867	−3.30	0.29
14	197.8620	0.00	0.00

Adjusted levelled height differences

in	to	$\hat{h}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{h}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	2	0.6237	−0.20	0.54	39.68	0.20	1.04	−0.05	5.09	0.25	0.63	0.03	0.63	0.03	0.45
	2	0.6237	0.30	0.54	60.32	0.20	1.04	+0.05	3.35	0.16	0.41	0.02	0.41	0.02	0.45
2	3	7.7296	−0.42	0.63	59.52	0.24	1.20	−0.07	3.41	0.20	0.48	0.03	0.48	0.03	0.55
	9	3.8581	0.13	0.50	19.05	0.24	1.20	+0.07	8.52	0.50	0.97	0.06	0.97	0.06	0.55
3	8	1.4815	−0.15	0.53	21.43	0.24	1.20	−0.07	7.91	0.46	0.94	0.06	0.94	0.06	0.55
5	4	8.2015	0.63	0.33	85.01	0.35	0.87	+0.07	1.74	0.15	0.13	0.01	0.13	0.01	0.79
6	5	4.4255	−0.13	0.33	36.70	0.22	0.65	−0.03	5.43	0.29	0.41	0.02	0.41	0.02	0.49
7	6	1.0500	0.17	0.27	39.81	0.34	0.51	+0.04	5.08	0.42	0.31	0.03	0.31	0.03	0.77
8	7	3.7770	1.23	0.27	77.43	1.11	0.59	+0.16	2.23	0.60	0.13	0.04	0.13	0.04	2.50
	11	2.2533	−0.33	0.31	50.70	0.46	0.58	−0.06	4.07	0.45	0.29	0.03	0.29	0.03	1.04
9	8	5.3530	−0.70	0.00	100.00	0.45	0.64	−0.07	0.00	0.00	0.00	0.00	0.00	0.00	1.02
	12	0.6374	0.02	0.40	72.42	0.01	0.84	0.00	2.55	0.01	0.23	0.00	0.23	0.00	0.03
10	5	7.4940	0.55	0.40	53.70	0.56	0.76	+0.10	3.84	0.52	0.35	0.05	0.35	0.05	1.26
	7	2.0184	−0.49	0.34	39.49	0.78	0.66	−0.12	5.11	0.97	0.40	0.08	0.40	0.08	1.77
	11	0.4948	0.25	0.37	45.61	0.32	0.70	+0.05	4.51	0.35	0.38	0.03	0.38	0.03	0.72
12	8	4.7156	0.18	0.40	65.52	0.14	0.79	+0.03	3.00	0.10	0.27	0.01	0.27	0.01	0.32
13	11	11.4906	0.17	0.34	49.55	0.22	0.64	+0.03	4.17	0.22	0.32	0.02	0.32	0.02	0.49
	12	4.5217	0.12	0.41	48.37	0.13	0.77	+0.02	4.27	0.13	0.40	0.01	0.40	0.01	0.29
14	13	2.0247	−0.10	0.29	65.37	0.11	0.56	−0.01	3.01	0.08	0.19	0.01	0.19	0.01	0.25
	13	2.0247	0.40	0.29	70.32	0.41	0.58	+0.06	2.68	0.26	0.17	0.02	0.17	0.02	0.92

Network graph



Supplementary information

Observed levelled height differences	:	20
Height unknowns	:	9
Datum defect	:	1
Datum definition	:	fix
Number of datum constraints	:	5
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	4.7
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.7
Redundancy r	:	11
Redundancy levelled height differences	:	11.00
Redundancy (Check)	:	11.00
Weighted square sum of residuals Ω [m^2]	:	$2.15296 \cdot 10^{-6}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-3}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$4.42407 \cdot 10^{-4}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.4424
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.1957 ($k^F_{\alpha_G;r,\infty}$ =1.81)
Global test (Ω/σ_0^2)	:	2.1530 ($k^{\chi^2}_{\alpha_G;r}$ =19.90)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{H} - H\|$ [cm]	:	1.250
Trace height covariance matrix, tr$\widehat{\Sigma}_{\widehat{H}}$ [cm^2]	:	0.01723
Trace height cofactor matrix, tr$Q_{\widehat{H}}$	:	8.80115

3 Two-dimensional networks (Planar networks)

This chapter contains numerous examples of planar networks observed by distances (trilateration networks), angles (triangulation networks), directions or combinations of them. Various datum definitions are implemented.

3.1 Trilateration networks

3.1.1 Benning (2011), Ex. 8-2

Benning (2011): Statistik in Geodäsie, Geoinformation und Bauwesen, Wichmann. Ex. 8-2, pp. 256

Available data files: [2D] Benning82_Distance_fix∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
1	0.0000 (D)	1000.0000 (D)
2	1000.0000 (D)	1000.0000 (D)
3	0.0000	0.0000
4	1000.0000	0.0000

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [−]
1	3	1000.0200	10	1
	4	1414.2000	10	1
2	3	1414.2400	10	1
	4	999.9800	10	1
3	4	1000.0000	10	1

Design matrix $A_{[\text{-}]}$ **and reduced observation vector** $\Delta y_{[\text{cm}]}$ **distances** (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	Δy
$s_{1,3}$	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	0.0000	2.000
$s_{1,4}$	−0.7071	0.7071	0.0000	0.0000	0.0000	0.0000	0.7071	−0.7071	−1.356
$s_{2,3}$	0.0000	0.0000	0.7071	0.7071	−0.7071	−0.7071	0.0000	0.0000	2.644
$s_{2,4}$	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	−2.000
$s_{3,4}$	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	1.0000	0.0000	0.000

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4
D^{T}	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	1	0	0	0	0	0
	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4
$\widehat{\Delta x}^{\mathsf{T}}$	0.0000	0.0000	0.0000	0.0000	−0.0096	−0.0226	−0.0070	0.0174

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
3	−0.0096	−0.958	0.901	−0.0226	−2.260	0.637	1.104
4	999.9930	−0.698	0.901	0.0174	1.740	0.637	1.104

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
3	0.97	0.52	129.517 061
4	0.97	0.52	70.483 621

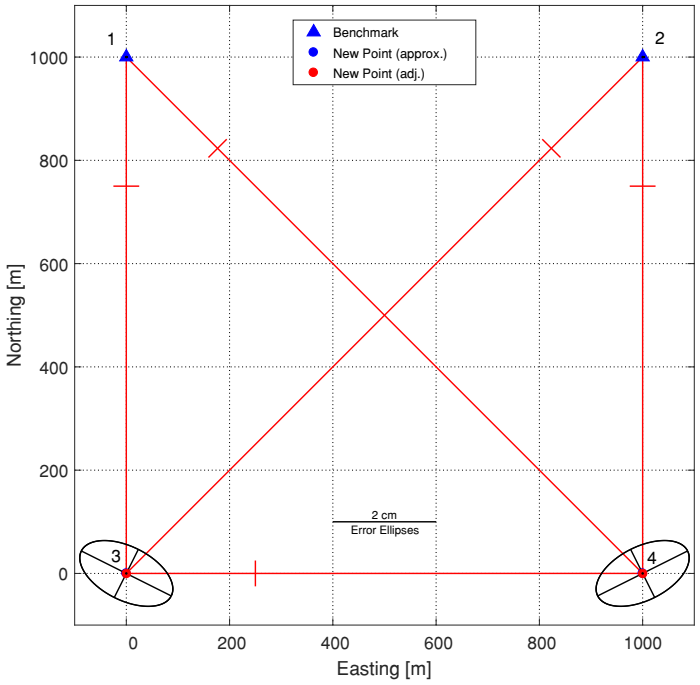
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
3	19.44	10.39	129.517 061
4	19.44	10.39	70.483 621

Adjusted horizontal distances

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	3	1000.0226	−0.260	0.637	14.3	0.7	10.9	−1.821	10.1	1.7	93.7	15.6	93.7	15.6	1.00
	4	1414.1963	0.368	0.582	28.6	0.7	7.7	+1.288	6.5	1.1	55.2	9.2	55.2	9.2	1.00
2	3	1414.2363	0.368	0.582	28.6	0.7	7.7	+1.288	6.5	1.1	55.2	9.2	55.2	9.2	1.00
	4	999.9826	−0.260	0.637	14.3	0.7	10.9	−1.821	10.1	1.7	93.7	15.6	93.7	15.6	1.00
3	4	1000.0026	−0.260	0.637	14.3	0.7	10.9	−1.821	10.1	1.7	93.7	15.6	93.7	15.6	1.00

Network graph



Supplementary information

Observed distances	:	5
Coordinate unknowns	:	4
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	4
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.1
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	∞
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$7 \cdot 10^{-13}$
Redundancy r	:	1
Redundancy distances	:	1.00
Weighted square sum of residuals Ω [m^2]	:	$4.73676 \cdot 10^{-5}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$6.88242 \cdot 10^{-3}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.6882
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.4737 ($k^F_{\alpha_G;r,\infty}$ =10.83)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	3.089
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	2.43606

3.1.2 Benning (2011), Ex. 8-8

Benning (2011): Statistik in Geodäsie, Geoinformation und Bauwesen, Wichmann. Ex. 8-8, pp. 285-291

Available data files: [2D] Benning88_Distance_fix∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
1	3000.0000 (D)	2000.0000 (D)
2	1000.0000 (D)	2000.0000 (D)
3	3000.0000 (D)	2100.0000 (D)
4	1000.0000 (D)	2100.0000 (D)
5	2000.0000 (D)	1000.0000 (D)
6	2000.0000	2000.0000

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
6	1	1000.0000	20	1
	2	1000.0000	20	1
	3	1005.0000	20	1
	4	1005.0000	20	1
	5	1000.0000	20	1

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ distances (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5	x_6	y_6	Δy
$s_{6,1}$	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	0.000
$s_{6,2}$	0.0000	0.0000	−1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.000
$s_{6,3}$	0.0000	0.0000	0.0000	0.0000	0.9950	0.0995	0.0000	0.0000	0.0000	0.0000	−0.9950	−0.0995	1.244
$s_{6,4}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.9950	0.0995	0.0000	0.0000	0.9950	−0.0995	1.244
$s_{6,5}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	1.0000	0.000

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5	x_6	y_6
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0

Least squares solution $\widehat{\Delta x}_{[m]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5	x_6	y_6
$\widehat{\Delta x}^{\mathsf{T}}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0024

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
6	2000.0000	0.000	0.504	1999.9976	−0.243	0.996	1.116

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
6	1.00	0.50	0.000 000

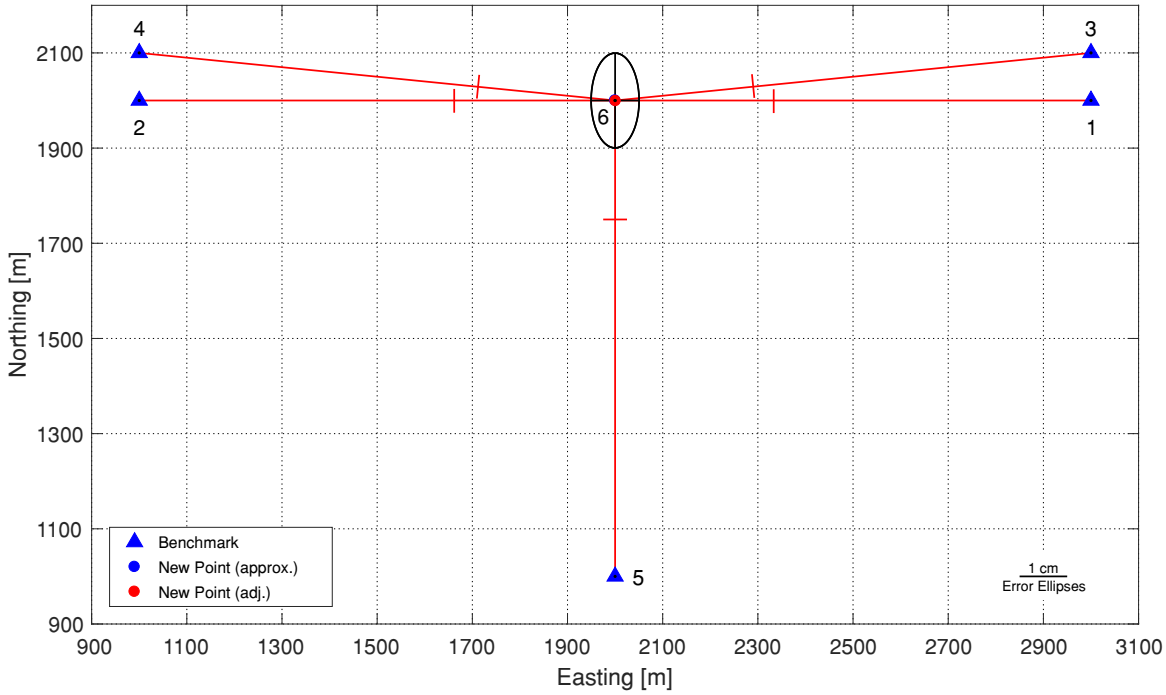
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
6	4.35	2.20	0.000 000

Adjusted horizontal distances

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
6	1	1000.0000	0.000	0.504	74.9	0.0	9.6	0.000	2.4	0.0	24.0	0.0	24.0	0.0	0.00
	2	1000.0000	0.000	0.504	74.9	0.0	9.6	0.000	2.4	0.0	24.0	0.0	24.0	0.0	0.00
	3	1004.9878	1.220	0.511	74.2	0.7	9.6	+1.645	2.4	0.4	24.8	4.3	24.8	4.3	1.41
	4	1004.9878	1.220	0.511	74.2	0.7	9.6	+1.645	2.4	0.4	24.8	4.3	24.8	4.3	1.41
	5	999.9976	0.243	0.996	1.9	0.9	59.3	+12.500	29.4	6.2	581.5	122.6	581.5	122.6	1.73*

Network graph



Supplementary information

Observed distances	:	5
Coordinate unknowns	:	2
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	10
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.6
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.7
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$1.4 \cdot 10^{-12}$
Redundancy r	:	3
Redundancy distances	:	3.00
Weighted square sum of residuals Ω [m^2]	:	$3.03394 \cdot 10^{-4}$
(a priori) standard deviation σ_0 [m]	:	$2 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$1.00564 \cdot 10^{-2}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.5028
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.2528 ($k^F_{\alpha_G;r,\infty}$ =4.21)
Number of outliers (Data snooping)	:	0
Number of outliers (τ-criterion)	:	1
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.243
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	1.24576

3.1.3 Ghilani (2010), Ex. 14.5 & 19.3

Ghilani Charles D. (2010): Adjustment Computations. Spatial Data Analysis. Fifth Edition, John Wiley & Sons, Inc., ISBN 978-0-470-46491-5, Ex. 14.5, pp. 251 & 19.3, pp. 404

Available data files: [2D] Ghilani14_5_Distance_fix*.*

Coordinates

Point name	ID	Easting x [m]	Northing y [m]
Badger	Ba	2 410 000.0000 (D)	390 000.0000 (D)
Bucky	Bu	2 411 820.0000 (D)	386 881.2220 (D)
Wisconsin	Wi	2 415 776.8190	391 043.4610
Campus	Ca	2 416 898.2270	387 602.2940

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
Ba	Ca	7297.5880	10	1
	Wi	5870.3020	10	1
Ca	Bu	5123.7600	10	1
Wi	Bu	5742.8780	10	1
	Ca	3616.4340	10	1

Design matrix A[-] and reduced observation vector Δy[m] distances (1. iteration)

A	x_{Ba}	y_{Ba}	x_{Bu}	y_{Bu}	x_{Wi}	y_{Wi}	x_{Ca}	y_{Ca}	Δy
$s_{Ba,Ca}$	−0.9446	0.3283	0.0000	0.0000	0.0000	0.0000	0.9446	−0.3283	−5.4613
$s_{Ba,Wi}$	−0.9841	−0.1778	0.0000	0.0000	0.9841	0.1778	0.0000	0.0000	−0.0003
$s_{Ca,Bu}$	0.0000	0.0000	−0.9901	−0.1406	0.0000	0.0000	0.9901	0.1406	−5.4051
$s_{Wi,Bu}$	0.0000	0.0000	−0.6890	−0.7248	0.6890	0.7248	0.0000	0.0000	−0.0002
$s_{Wi,Ca}$	0.0000	0.0000	0.0000	0.0000	−0.3098	0.9508	0.3098	−0.9508	−2.8458

Matrix D^T[-] of datum constraints

	x_{Ba}	y_{Ba}	x_{Bu}	y_{Bu}	x_{Wi}	y_{Wi}	x_{Ca}	y_{Ca}
D^T	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	1	0	0	0	0	0
	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}$[m] (1. iteration)

	x_{Ba}	y_{Ba}	x_{Bu}	y_{Bu}	x_{Wi}	y_{Wi}	x_{Ca}	y_{Ca}
$\widehat{\Delta x}^T$	0.0000	0.0000	0.0000	0.0000	0.0848	−0.1652	−5.5314	0.9593

Adjusted coordinates

ID	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
Wi	2 415 776.9044	8.538	14.879	391 043.2945	−16.651	22.061	26.609
Ca	2 416 892.6955	−553.148	10.378	387 603.2551	96.113	27.054	28.977

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
Wi	24.62	10.10	167.642 801
Ca	27.26	9.81	8.468 319

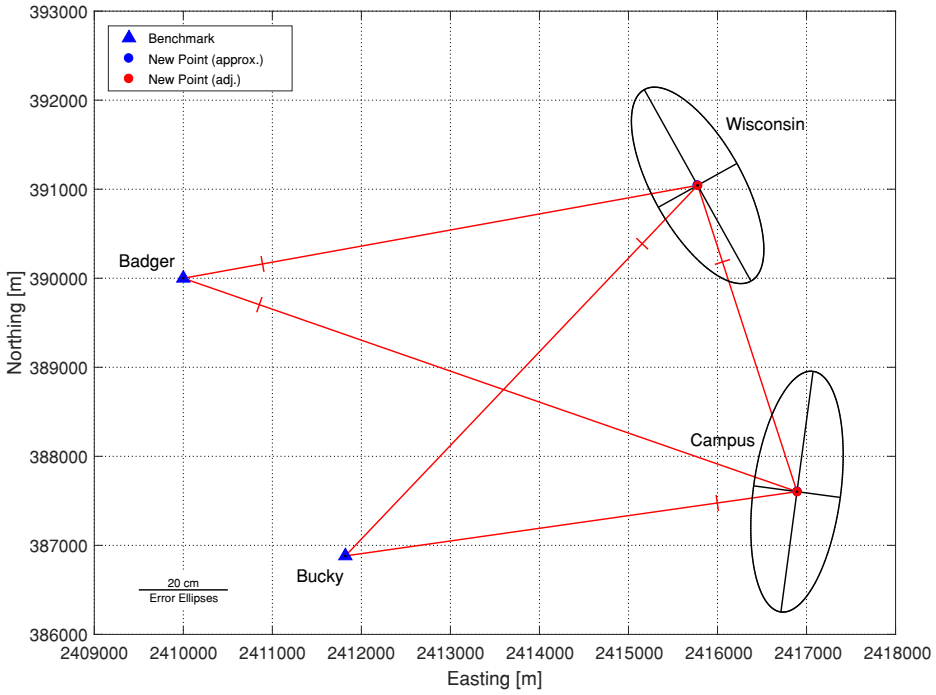
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
Wi	491.75	201.73	167.642 801
Ca	544.60	196.05	8.468 319

Adjusted horizontal distances

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
Ba	Ca	7297.5090	7.901	11.058	33.8	13.6	7.1	+23.377*	5.8	19.0	47.1	154.8	47.1	154.8	1
	Wi	5870.3567	−5.468	12.442	16.2	13.6	10.3	−33.777*	9.4	30.9	86.1	283.1	86.1	283.1	1
Ca	Bu	5123.8239	−6.393	11.993	22.1	13.6	8.8	−28.893*	7.8	25.5	68.4	225.0	68.4	225.0	1
Wi	Bu	5742.8164	6.164	12.112	20.6	13.6	9.1	+29.963#	8.1	26.7	72.4	238.0	72.4	238.0	1
	Ca	3616.4708	−3.675	13.084	7.3	13.6	15.3	−50.258*	14.7	48.4	141.6	465.8	141.6	465.8	1

Network graph



Supplementary information

Observed distances	:	5
Coordinate unknowns	:	4
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	4
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.1
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	∞
Number of iterations (Max=20)	:	20
Redundancy r	:	1
Redundancy distances	:	1.00
Weighted square sum of residuals Ω [m^2]	:	$1.84703 \cdot 10^{-2}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$1.35905 \cdot 10^{-1}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	13.5905
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	184.7027 ($k^F_{\alpha_G;r,\infty}$ =10.83)
Number of outliers (Data snooping)	:	5 (Remove outliers or scale standard deviations by the factor 13.59)
Number of outliers (τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	561.748

3.1.4 Strang & Borre (1997), Ex. 10.1

Strang G and K Borre (1997): Linear Algebra, Geodesy, and GPS. Ex. 10.1, pp. 345

Available data files: [2D] StrangBorre_Distance_fix∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
1	170.7100 (D)	270.7100 (D)
2	100.0000 (D)	100.0000 (D)
3	241.4200 (D)	100.0000 (D)
P	170.7100	170.7100

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
1	P	100.0100	10	1
2	P	100.0200	10	1
3	P	100.0300	10	1

Design matrix A[-] and reduced observation vector Δy[cm] distances (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_P	y_P	Δy
$s_{1,P}$	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−1.0000	1.000
$s_{2,P}$	0.0000	0.0000	−0.7071	−0.7071	0.0000	0.0000	0.7071	0.7071	2.096
$s_{3,P}$	0.0000	0.0000	0.0000	0.0000	0.7071	−0.7071	−0.7071	0.7071	3.096

Matrix D^T[-] of datum constraints

	x_1	y_1	x_2	y_2	x_3	y_3	x_P	y_P
D^T	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	1	0	0	0	0	0
	0	0	0	1	0	0	0	0
	0	0	0	0	1	0	0	0
	0	0	0	0	0	1	0	0

Least squares solution $\widehat{\Delta x}$[m] (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_P	y_P
$\widehat{\Delta x}^T$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0071	0.0134

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
P	170.7029	−0.707	3.303	170.7234	1.336	2.335	4.045

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
P	3.30	2.34	100.004 502

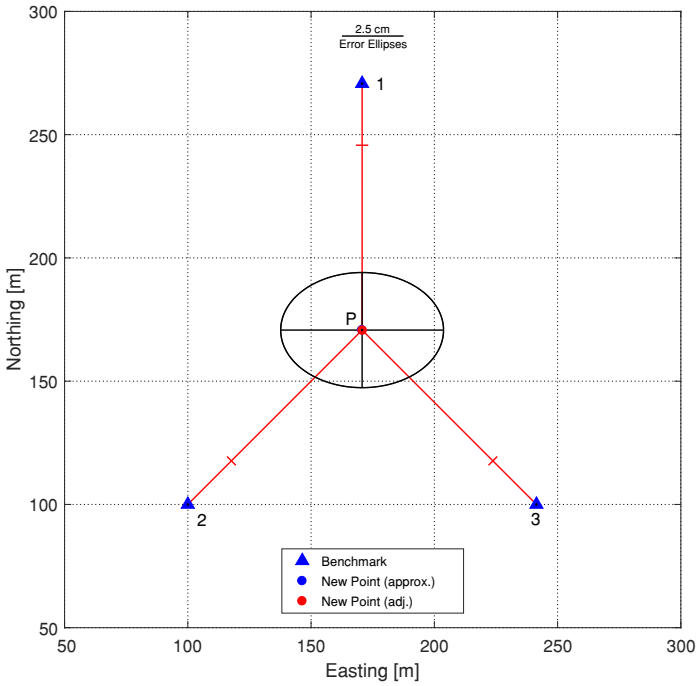
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
P	65.98	46.65	100.004 502

Adjusted horizontal distances

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	P	99.9866	2.336	2.335	50.0	3.3	5.8	+4.671*	4.1	3.3	29.2	23.4	29.2	23.4	1.00
2	P	100.0035	1.652	2.860	25.0	3.3	8.3	+6.605#	7.2	5.7	62.0	49.5	62.0	49.5	1.00
3	P	100.0135	1.651	2.861	25.0	3.3	8.3	+6.607*	7.2	5.7	62.0	49.6	62.0	49.6	1.00

Network graph



Supplementary information

Observed distances	:	3
Coordinate unknowns	:	2
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	6
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.1
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	∞
Number of iterations (Max=20)	:	4
Stop criterion (actual)	:	$5.6 \cdot 10^{-13}$
Redundancy r	:	1
Redundancy distances	:	1.00
Weighted square sum of residuals Ω [m^2]	:	$1.09094 \cdot 10^{-3}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$3.30293 \cdot 10^{-2}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	3.3029
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	10.9094 ($k^F_{\alpha_G;r,\infty}$ =10.83)
Number of outliers (Data snooping)	:	3 (Remove outliers or scale standard deviations by the factor 3.30)
Number of outliers (τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	1.511
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	16.36559

3.1.5 Strang & Borre (1997), Ex. 12.4

Strang G and K Borre (1997): Linear Algebra, Geodesy, and GPS. Ex. 12.4, pp. 411

Available data files: [2D] StrangBorre_Distance_free∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
P	170.7100 (D)	170.7100 (D)
1	170.7100 (D)	270.7100 (D)
2	100.0000 (D)	100.0000 (D)
3	241.4200 (D)	100.0000 (D)

Datum: free, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
1	2	184.7850	10	1
	3	184.8050	10	1
	P	100.0100	10	1
2	3	141.4400	10	1
	P	100.0200	10	1
3	P	100.0300	10	1

Design matrix A[-] and reduced observation vector Δy[cm] distances (1. iteration)

A	x_P	y_P	x_1	y_1	x_2	y_2	x_3	y_3	Δy
$s_{1,2}$	0.0000	0.0000	0.3827	0.9239	−0.3827	−0.9239	0.0000	0.0000	0.998
$s_{1,3}$	0.0000	0.0000	−0.3827	0.9239	0.0000	0.0000	0.3827	−0.9239	2.998
$s_{1,P}$	0.0000	−1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	1.000
$s_{2,3}$	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	1.0000	0.0000	2.000
$s_{2,P}$	0.7071	0.7071	0.0000	0.0000	−0.7071	−0.7071	0.0000	0.0000	2.096
$s_{3,P}$	−0.7071	0.7071	0.0000	0.0000	0.0000	0.0000	0.7071	−0.7071	3.096

Matrix D^T[-] of datum constraints (1. iteration)

	x_P	y_P	x_1	y_1	x_2	y_2	x_3	y_3
D^T	1.000 00	0.000 00	1.000 00	0.000 00	1.000 00	0.000 00	1.000 00	0.000 00
	0.000 00	1.000 00	0.000 00	1.000 00	0.000 00	1.000 00	0.000 00	1.000 00
	−10.355 00	0.000 00	−110.355 00	0.000 00	60.355 00	−70.710 00	60.355 00	70.710 00

Least squares solution $\widehat{\Delta x}$[m] (1. iteration)

	x_P	y_P	x_1	y_1	x_2	y_2	x_3	y_3
$\widehat{\Delta x}^T$	0.0023	0.0085	−0.0068	0.0113	−0.0088	−0.0029	0.0133	−0.0170

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
P	170.7123	0.227	1.079	170.7185	0.853	0.682	1.276
1	170.7032	−0.680	0.810	270.7213	1.133	0.551	0.980
2	99.9912	−0.879	0.641	99.9971	−0.286	0.705	0.953
3	241.4333	1.332	0.640	99.9830	−1.700	0.705	0.953

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
P	1.08	0.68	99.992 733
1	0.81	0.55	99.998 497
2	0.73	0.61	167.417 797
3	0.73	0.61	32.587 207

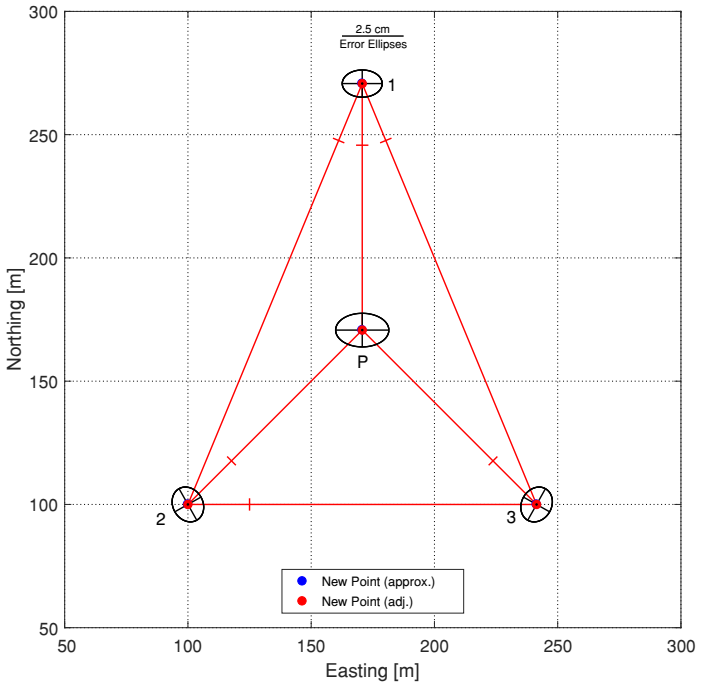
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
P	21.56	13.62	99.992 733
1	16.17	11.01	99.998 497
2	14.65	12.15	167.417 797
3	14.65	12.15	32.587 207

Adjusted horizontal distances

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	2	184.7889	−0.389	1.110	11.0	1.2	12.5	−3.553	11.8	3.4	111.1	31.6	111.1	31.6	1.00
	3	184.8089	−0.390	1.110	11.0	1.2	12.5	−3.552	11.8	3.4	111.1	31.6	111.1	31.6	1.00
	P	100.0028	0.720	0.930	37.4	1.2	6.8	+1.923	5.3	1.5	42.3	12.0	42.3	12.0	1.00
2	3	141.4421	−0.211	1.157	3.2	1.2	23.1	−6.566	22.7	6.5	223.2	63.6	223.2	63.6	1.00
	P	100.0149	0.509	1.061	18.7	1.2	9.6	+2.720	8.6	2.5	77.7	22.1	77.7	22.1	1.00
3	P	100.0249	0.509	1.061	18.7	1.2	9.6	+2.719	8.6	2.5	77.6	22.1	77.6	22.1	1.00

Network graph



Supplementary information

Observed distances	:	6
Coordinate unknowns	:	8
Datum defect	:	3
Datum definition	:	free
Number of datum constraints	:	3
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.1
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	∞
Number of iterations (Max=20)	:	4
Stop criterion (actual)	:	$1.1 \cdot 10^{-13}$
Stop criterion (target)	:	$1 \cdot 10^{-10}$
Redundancy r	:	1
Redundancy distances	:	1.00
Weighted square sum of residuals Ω [m^2]	:	$1.38383 \cdot 10^{-4}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$1.17636 \cdot 10^{-2}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.1764
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	1.3838 ($k^F_{\alpha_G;r,\infty}$ =10.83)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	2.836
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	4.40493

3.1.6 Höpke W (1980), Ex. 35.5

Höpke W (1980): Fehlerlehre und Ausgleichungsrechnung. de Gruyter, Ex. 35.5, pp. 120-123

Coordinates

		Fix datum		Free datum: Total trace minimization	
Point name	ID	Easting x [m]	Northing y [m]	Easting x [m]	Northing y [m]
20	20	3 579 041.4160	5 707 194.4120	3 579 041.4160 (D)	5 707 194.4120 (D)
75	75	3 575 403.2770	5 707 682.6120	3 575 403.2770 (D)	5 707 682.6120 (D)
86	86	3 575 322.0610	5 708 700.9520	3 575 322.0610 (D)	5 708 700.9520 (D)
87	87	3 576 581.7780 (D)	5 709 938.1060 (D)	3 576 581.7780 (D)	5 709 938.1060 (D)
1006	10	3 578 284.2890	5 708 758.6410	3 578 284.2890 (D)	5 708 758.6410 (D)
1011	01	3 577 052.3320	5 708 103.2040	3 577 052.3320 (D)	5 708 103.2040 (D)
1059	02	3 576 852.8940 (D)	5 706 633.6420	3 576 852.8940 (D)	5 706 633.6420 (D)
1087	03	3 576 213.6990	5 709 199.8890	3 576 213.6990 (D)	5 709 199.8890 (D)

(D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
20	75	3670.7350	1	1
	87	3684.7820	1	1
86	20	4012.9210	1	1
	75	1021.5360	1	1
	87	1765.6570	1	1
	10	2962.8320	1	1
	01	1830.6460	1	1
	02	2572.5160	1	1
	03	1021.7680	1	1
10	20	1737.8240	1	1
	75	3075.3730	1	1
	87	2071.1540	1	1
	01	1395.4590	1	1
	02	2562.1340	1	1

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
01	20	2186.8530	1	1
	75	1701.8240	1	1
	87	1894.2630	1	1
	02	1483.0880	1	1
02	20	2259.1670	1	1
	75	1789.4490*	1	1
	87	3315.6300	1	1
03	20	3466.7220	1	1
	75	1720.1300	1	1
	87	824.8630	1	1
	10	2117.1340	1	1
	01	1380.6330	1	1
	02	2644.7890	1	1

* 5 cm typo in original publication

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[m]}$ distances (1. iteration)

A	x_{20}	y_{20}	x_{75}	y_{75}	x_{86}	y_{86}	x_{87}	y_{87}	x_{10}	y_{10}	x_{01}	y_{01}	x_{02}	y_{02}	x_{03}	y_{03}	Δy
$s_{20,75}$	0.9911	−0.1330	−0.9911	0.1330	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0135
$s_{20,87}$	0.6675	−0.7446	0.0000	0.0000	0.0000	0.0000	−0.6675	0.7446	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0078
$s_{86,20}$	0.9269	−0.3754	0.0000	0.0000	−0.9269	0.3754	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0337
$s_{86,75}$	0.0000	0.0000	0.0795	−0.9968	−0.0795	0.9968	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0375
$s_{86,87}$	0.0000	0.0000	0.0000	0.0000	−0.7135	−0.7007	0.7135	0.7007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0305
$s_{86,10}$	0.0000	0.0000	0.0000	0.0000	−0.9998	−0.0195	0.0000	0.0000	0.9998	0.0195	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0423
$s_{86,01}$	0.0000	0.0000	0.0000	0.0000	−0.9452	0.3265	0.0000	0.0000	0.0000	0.0000	0.9452	−0.3265	0.0000	0.0000	0.0000	0.0000	0.0340
$s_{86,02}$	0.0000	0.0000	0.0000	0.0000	−0.5951	0.8037	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5951	−0.8037	0.0000	0.0000	0.1202
$s_{86,03}$	0.0000	0.0000	0.0000	0.0000	−0.8727	−0.4883	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8727	0.4883	0.0261
$s_{10,20}$	0.4357	−0.9001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.4357	0.9001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0062
$s_{10,75}$	0.0000	0.0000	−0.9368	−0.3499	0.0000	0.0000	0.0000	0.0000	0.9368	0.3499	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0243
$s_{10,87}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.8220	0.5695	0.8220	−0.5695	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0006
$s_{10,01}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8828	0.4697	−0.8828	−0.4697	0.0000	0.0000	0.0000	0.0000	−0.0035
$s_{10,02}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5587	0.8294	0.0000	0.0000	−0.5587	−0.8294	0.0000	0.0000	0.0036
$s_{01,20}$	0.9096	−0.4156	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.9096	0.4156	0.0000	0.0000	0.0000	0.0000	−0.0073
$s_{01,75}$	0.0000	0.0000	−0.9690	−0.2471	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9690	0.2471	0.0000	0.0000	0.0000	0.0000	−0.0221
$s_{01,87}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.2484	0.9687	0.0000	0.0000	0.2484	−0.9687	0.0000	0.0000	0.0000	0.0000	−0.0143
$s_{01,02}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1345	0.9909	−0.1345	−0.9909	0.0000	0.0000	0.0546
$s_{02,20}$	0.9687	0.2482	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.9687	−0.2482	0.0000	0.0000	−0.0567
$s_{02,75}$	0.0000	0.0000	−0.8101	0.5862	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8101	−0.5862	0.0000	0.0000	0.1118
$s_{02,87}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0818	0.9967	0.0000	0.0000	0.0000	0.0000	0.0818	−0.9967	0.0000	0.0000	0.0627
$s_{03,20}$	0.8157	−0.5785	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.8157	0.5785	0.0346
$s_{03,75}$	0.0000	0.0000	−0.4711	−0.8821	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4711	0.8821	−0.0192
$s_{03,87}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4462	0.8949	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.4462	−0.8949	−0.0288
$s_{03,10}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9780	−0.2084	0.0000	0.0000	0.0000	0.0000	−0.9780	0.2084	0.0505
$s_{03,01}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6074	−0.7944	0.0000	0.0000	−0.6074	0.7944	0.0450
$s_{03,02}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2417	−0.9704	−0.2417	0.9704	0.1352

3.1.6.1 Fix datum definition

Available data files: [2D] Hoepke_Distance_fix∗.∗

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	x_{20}	y_{20}	x_{75}	y_{75}	x_{86}	y_{86}	x_{87}	y_{87}	x_{10}	y_{10}	x_{01}	y_{01}	x_{02}	y_{02}	x_{03}	y_{03}
D^{T}	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Least squares solution $\widehat{\Delta x}_{[cm]}$ (1. iteration)

	x_{20}	y_{20}	x_{75}	y_{75}	x_{86}	y_{86}	x_{87}	y_{87}
$\widehat{\Delta x}^{\mathsf{T}}$	−6.841	−4.545	−3.959	7.198	−7.050	3.233	0	0

	x_{10}	y_{10}	x_{01}	y_{01}	x_{02}	y_{02}	x_{03}	y_{03}
$\widehat{\Delta x}^{\mathsf{T}}$	−2.576	−3.738	−4.368	0.106	0	−6.397	−5.074	5.593

Adjusted coordinates

ID	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
20	3 579 041.3476	−6.841	0.338	5 707 194.3665	−4.545	0.521	0.621
75	3 575 403.2374	−3.959	0.339	5 707 682.6840	7.198	0.453	0.566
86	3 575 321.9905	−7.050	0.324	5 708 700.9843	3.233	0.405	0.518
10	3 578 284.2632	−2.576	0.325	5 708 758.6036	−3.738	0.427	0.536
01	3 577 052.2883	−4.368	0.361	5 708 103.2051	0.106	0.351	0.503
02	3 576 852.8940	0.000	0.000	5 706 633.5780	−6.397	0.313	0.313
03	3 576 213.6483	−5.074	0.390	5 709 199.9449	5.593	0.331	0.512

Absolute error ellipses

in	$A_{[cm]}$	$B_{[cm]}$	$\phi_{[gon]}$
20	0.55	0.28	26.021 389
75	0.45	0.34	3.907 902
86	0.41	0.32	13.965 659
10	0.43	0.32	197.126 599
01	0.39	0.32	54.871 142
02	0.31	0.00	0.000 000
03	0.39	0.33	90.353 487

Absolute confidence ellipses ($1-\alpha = 95\%$)

in	$A_{[cm]}$	$B_{[cm]}$	$\phi_{[gon]}$
20	1.52	0.76	26.021 389
75	1.24	0.93	3.907 902
86	1.12	0.87	13.965 659
10	1.17	0.89	197.126 599
01	1.05	0.89	54.871 142
02	0.86	0.00	0.000 000
03	1.07	0.90	90.353 487

3.1.6.2 Free datum definition (Total trace minimization)

Available data files: [2D] Hoepke_Distance_free∗.∗

Matrix $D^{T}_{[-]}$ of datum constraints (1. iteration)

	x_{20}	y_{20}	x_{75}	y_{75}	x_{86}	y_{86}	x_{87}	y_{87}
D^{T}	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
	1082.020	2197.448	593.820	−1440.691	−424.520	−1521.907	−1661.674	−262.190

	x_{10}	y_{10}	x_{01}	y_{01}	x_{02}	y_{02}	x_{03}	y_{03}
D^{T}	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
	−482.209	1440.321	173.228	208.364	1642.790	8.926	−923.457	−630.269

Least squares solution $\widehat{\Delta x}_{[cm]}$ (1. iteration)

	x_{20}	y_{20}	x_{75}	y_{75}	x_{86}	y_{86}	x_{87}	y_{87}
$\widehat{\Delta x}^{T}$	−1.178	−0.808	0.833	4.448	−4.074	0.338	0.770	−0.649

	x_{10}	y_{10}	x_{01}	y_{01}	x_{02}	y_{02}	x_{03}	y_{03}
$\widehat{\Delta x}^{T}$	0.298	−1.351	−0.326	0.296	6.663	−6.562	−2.987	4.288

Adjusted coordinates

ID	$\hat{x}_{[m]}$	$\hat{x}-x_{[cm]}$	$\lvert\hat{\sigma}\rvert_{[cm]}$	$\hat{y}_{[m]}$	$\hat{y}-y_{[cm]}$	$\lvert\hat{\sigma}\rvert_{[cm]}$	$\lvert\hat{\sigma}_{2D}\rvert_{[cm]}$
20	3 579 041.4042	−1.178	0.209	5 707 194.4039	−0.808	0.265	0.338
75	3 575 403.2853	0.833	0.232	5 707 682.6565	4.448	0.265	0.352
86	3 575 322.0203	−4.074	0.211	5 708 700.9554	0.338	0.240	0.320
87	3 576 581.7857	0.770	0.279	5 709 938.0995	−0.649	0.226	0.360
10	3 578 284.2920	0.298	0.203	5 708 758.6275	−1.351	0.268	0.336
01	3 577 052.3287	−0.326	0.240	5 708 103.2070	0.296	0.273	0.364
02	3 576 852.9606	6.663	0.247	5 706 633.5764	−6.562	0.212	0.325
03	3 576 213.6691	−2.987	0.241	5 709 199.9319	4.288	0.227	0.331

Adjusted horizontal distances

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
20	75	3670.7356	−0.056	0.334	54.7	0.8	0.6	−0.103	3.8	0.7	2.5	0.5	2.5	0.5	0.15
	87	3684.7780	0.402	0.344	51.9	5.6	0.6	+0.775*	4.0	5.4	2.8	3.7	2.8	3.7	1.13
86	20	4012.9184	0.257	0.299	63.6	3.2	0.5	+0.405	3.1	2.4	1.9	1.5	1.9	1.5	0.65
	75	1021.5364	−0.042	0.387	39.0	0.7	0.7	−0.109	5.2	0.9	4.0	0.7	4.0	0.7	0.14
	87	1765.6541	0.285	0.384	39.9	4.5	0.7	+0.715*	5.1	5.5	3.9	4.3	3.9	4.3	0.91
	10	2962.8331	−0.107	0.310	60.9	1.4	0.5	−0.176	3.3	1.1	2.1	0.7	2.1	0.7	0.28
	01	1830.6476	−0.159	0.320	58.4	2.1	0.5	−0.272	3.5	1.8	2.2	1.1	2.2	1.1	0.42
	02	2572.5152	0.082	0.340	52.8	1.1	0.6	+0.156	3.9	1.1	2.7	0.7	2.7	0.7	0.23
	03	1021.7706	−0.264	0.339	53.3	3.6	0.6	−0.496*	3.9	3.4	2.6	2.3	2.6	2.3	0.73
10	20	1737.8188	0.516	0.389	38.4	8.3	0.7	+1.345*	5.2	10.6	4.1	8.3	4.1	8.3	1.68
	75	3075.3720	0.100	0.324	57.2	1.3	0.5	+0.175	3.6	1.1	2.3	0.8	2.3	0.8	0.27
	87	2071.1547	−0.072	0.365	45.6	1.1	0.6	−0.158	4.5	1.2	3.3	0.9	3.3	0.9	0.22
	01	1395.4603	−0.132	0.338	53.4	1.8	0.6	−0.246	3.9	1.7	2.6	1.1	2.6	1.1	0.36
	02	2562.1381	−0.410	0.369	44.6	6.1	0.6	−0.920*	4.6	6.9	3.4	5.1	3.4	5.1	1.24
01	20	2186.8572	−0.416	0.299	63.7	5.2	0.5	−0.653*	3.1	3.9	1.9	2.4	1.9	2.4	1.05
	75	1701.8246	−0.057	0.339	53.3	0.8	0.6	−0.107	3.9	0.7	2.6	0.5	2.6	0.5	0.16
	87	1894.2654	−0.241	0.337	53.8	3.3	0.6	−0.447	3.8	3.0	2.6	2.1	2.6	2.1	0.66
	02	1483.0919	−0.394	0.342	52.5	5.4	0.6	−0.750*	3.9	5.2	2.7	3.6	2.7	3.6	1.10
02	20	2259.1620	0.502	0.405	33.2	8.7	0.7	+1.510*	5.9	12.3	4.8	10.1	4.8	10.1	1.76
	75	1789.4489	0.006	0.362	46.7	0.1	0.6	+0.012	4.4	0.1	3.2	0.1	3.2	0.1	0.02
	87	3315.6310	−0.101	0.312	60.3	1.3	0.5	−0.167	3.3	1.1	2.1	0.7	2.1	0.7	0.26
03	20	3466.7316	−0.962	0.318	58.8	12.5	0.5	−1.637#	3.5	10.5	2.2	6.8	2.2	6.8	2.53
	75	1720.1298	0.020	0.367	45.2	0.3	0.6	+0.043	4.6	0.3	3.4	0.2	3.4	0.2	0.06
	87	824.8644	−0.139	0.351	49.7	2.0	0.6	−0.281	4.2	2.0	2.9	1.4	2.9	1.4	0.40
	10	2117.1274	0.657	0.334	54.5	8.9	0.6	+1.207*	3.8	8.1	2.6	5.5	2.6	5.5	1.80
	01	1380.6359	−0.290	0.327	56.5	3.9	0.5	−0.513*	3.6	3.4	2.4	2.2	2.4	2.2	0.78
	02	2644.7824	0.656	0.319	58.5	8.6	0.5	+1.121*	3.5	7.2	2.2	4.7	2.2	4.7	1.73

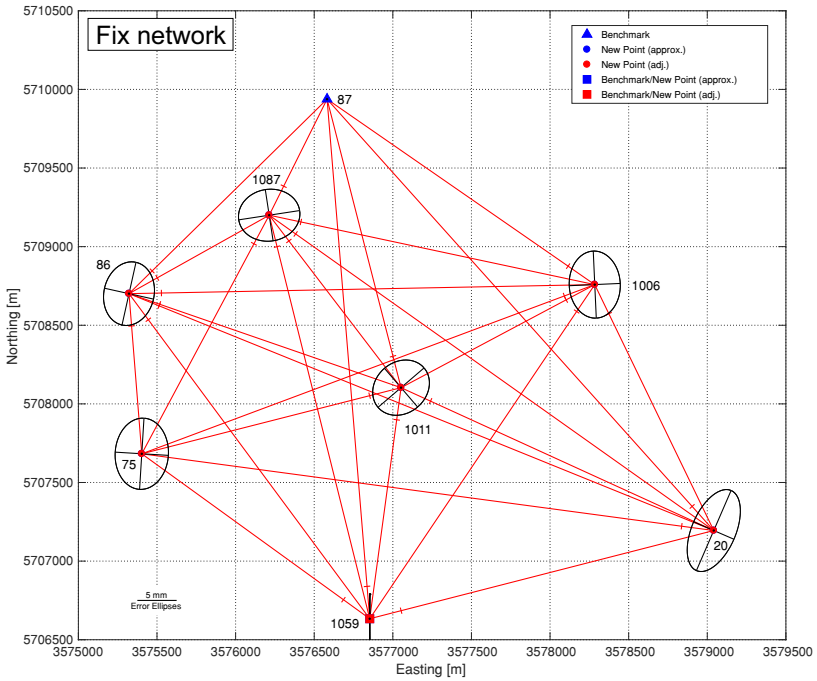
Absolute error ellipses

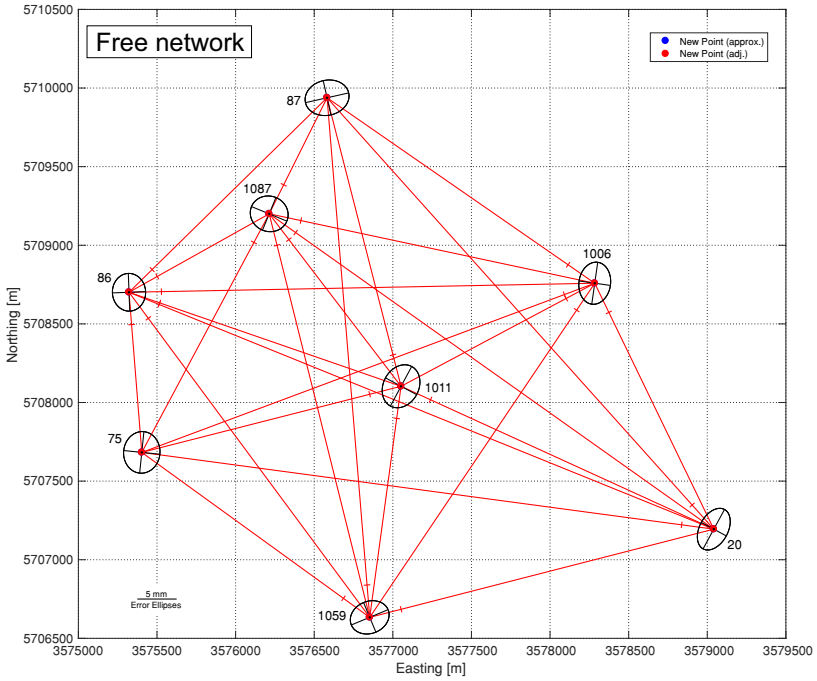
in	A [cm]	B [cm]	ϕ [gon]
20	0.29	0.18	31.680 791
75	0.27	0.23	6.490 442
86	0.24	0.21	197.538 157
87	0.28	0.22	85.856 865
10	0.27	0.20	9.041 268
01	0.29	0.23	31.394 107
02	0.25	0.20	74.872 684
03	0.24	0.22	124.816 630

Absolute confidence ellipses $(1 - \alpha = 95\%)$

in	A [cm]	B [cm]	ϕ [gon]
20	0.78	0.49	31.680 791
75	0.72	0.63	6.490 442
86	0.66	0.58	197.538 157
87	0.77	0.61	85.856 865
10	0.74	0.55	9.041 268
01	0.78	0.62	31.394 107
02	0.69	0.56	74.872 684
03	0.67	0.61	124.816 630

Network graphs





Supplementary information

Observed distances	:	27	27
Coordinate unknowns	:	13	16
Datum defect	:	3	3
Datum definition	:	fix	free
Number of datum constraints	:	3	3
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	6.6	6.6
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.8	2.8
Number of iterations (Max=20)	:	20	20
Stop criterion (actual)	:	$8.4 \cdot 10^{-10}$	$6.4 \cdot 10^{-10}$
Redundancy r	:	14	14
Redundancy distances	:	14.00	14.00
Weighted square sum of residuals Ω [m^2]	:	$3.436 \cdot 10^{-4}$	$3.436 \cdot 10^{-4}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$4.954 \cdot 10^{-3}$	$4.954 \cdot 10^{-3}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	4.9544	4.9544
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	24.5460	24.5460
Critical value $k^F_{\alpha_G;r,\infty}$	:	1.62	1.62
Number of outliers (Data snooping)	:	12	12
			(Remove outliers or scale standard deviations by the factor 4.95)
Number of outliers (τ-criterion)	:	0	0
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	1.8759	0.9295

3.1.7 Weiss et al (2010)

Weiss et al (2010): Detection of erroneous values in the measurement of local geodetic networks. Acta Montanistica Slovaca Rocník 15 (2010), číslo 1, pp. 62-70

Available data files: [2D] WeissEtAl_Distance_fix∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
1	4506.2990 (D)	9001.1230 (D)
2	2798.6220 (D)	9502.4900 (D)
3	3803.9730 (D)	9894.2330 (D)
4	3299.9800	9100.8380
5	3697.8240	9400.5450
6	3080.3700	9775.9000
7	4393.2650	9842.5030
8	4904.5690 (D)	9413.3760 (D)
9	4251.0610	9546.2260

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
1	4	1210.4780	894.427	1.250 00
	5	901.7550	894.427	1.250 00
	7	849.0050	894.427	1.250 00
	9	601.9060	1095.445	0.833 33
2	4	642.4090	1140.175	0.769 23
	5	904.9620	1183.216	0.714 28
	6	392.5500	774.597	1.666 67
	7	1630.4540	1048.809	0.909 09
3	4	939.9410	1095.445	0.833 33
	5	504.9680	1303.840	0.588 23
	7	591.5060	1264.911	0.625 00
	9	566.5550	894.427	1.250 00

Horizontal distances (continued)

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
4	5	498.1070	1264.911	0.625 00
	6	709.9270	1303.840	0.588 23
	8	1634.7470	1140.175	0.769 23
	9	1050.2040	1048.809	0.909 09
5	6	722.6310	1140.175	0.769 23
	7	823.9900	1264.911	0.625 00
	8	1206.8060	1000.000	1.000 00
	9	572.0940	836.660	1.428 57
6	9	1193.0360	836.660	1.428 57
7	8	667.5950	774.597	1.666 67
	9	328.6670	948.683	1.111 11
8	9	666.8740	1140.175	0.769 23

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[cm]}$ distances (1. iteration)

Too large to be displayed !

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5	x_6	y_6	x_7	y_7	x_8	y_8	x_9	y_9
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$ (1. iteration)

	x_4	y_4	x_5	y_5	x_6	y_6	x_7	y_7	x_9	y_9
$\widehat{\Delta x}^{\mathsf{T}}$	−0.0156	−0.0091	−0.0017	−0.0056	−0.0516	−0.0057	−0.0489	0.0588	−0.0115	0.0038

Adjusted coordinates

Point name	$\hat{x}_{[\text{m}]}$	$\hat{x} - x_{[\text{cm}]}$	$\lvert\hat{\sigma}\rvert_{[\text{cm}]}$	$\hat{y}_{[\text{m}]}$	$\hat{y} - y_{[\text{cm}]}$	$\lvert\hat{\sigma}\rvert_{[\text{cm}]}$	$\lvert\hat{\sigma}_{2D}\rvert_{[\text{cm}]}$
4	3299.9644	−1.562	0.752	9100.8289	−0.914	1.121	1.350
5	3697.8223	−0.171	0.670	9400.5394	−0.556	1.207	1.380
6	3080.3184	−5.158	0.924	9775.8943	−0.567	1.193	1.509
7	4393.2160	−4.895	0.817	9842.5618	5.881	0.879	1.200
9	4251.0495	−1.152	0.728	9546.2298	0.376	1.016	1.250

Absolute error ellipses

in	$A_{[\text{cm}]}$	$B_{[\text{cm}]}$	$\phi_{[\text{gon}]}$
4	1.13	0.73	187.863 893
5	1.21	0.67	0.983 837
6	1.21	0.90	182.789 874
7	0.93	0.76	37.619 958
9	1.04	0.70	16.822 936

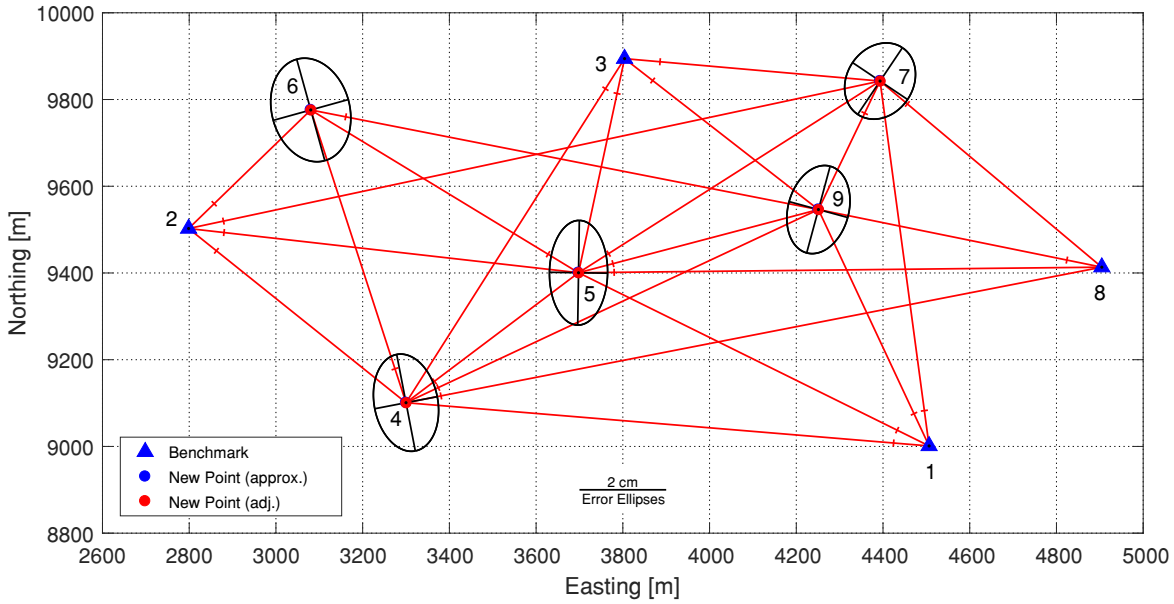
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	$A_{[\text{cm}]}$	$B_{[\text{cm}]}$	$\phi_{[\text{gon}]}$
4	3.10	2.01	187.863 893
5	3.30	1.83	0.983 837
6	3.32	2.46	182.789 874
7	2.53	2.09	37.619 958
9	2.83	1.92	16.822 936

Adjusted horizontal distances

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	4	1210.4480	2.996	0.770	60.5	0.0	475.3	+4.954	3.3	0.0	1879.0	19.6	1879.0	19.6	3.15*
	5	901.7583	−0.333	0.797	57.7	0.0	486.6	−0.577	3.5	0.0	2059.5	2.4	2059.5	2.4	0.36
	7	849.0035	0.145	0.858	50.9	0.0	518.2	+0.286	4.1	0.0	2546.1	1.4	2546.1	1.4	0.17
	9	601.9084	−0.238	0.911	63.1	0.0	569.8	−0.378	3.2	0.0	2102.1	1.4	2102.1	1.4	0.20
2	4	642.3985	1.048	0.985	60.1	0.0	607.5	+1.743	3.4	0.0	2421.0	6.9	2421.0	6.9	0.87
	5	904.9614	0.063	0.677	82.5	0.0	538.2	+0.076	1.9	0.0	940.9	0.1	940.9	0.1	0.04
	6	392.5593	−0.930	0.979	14.7	0.0	835.1	−6.329	10.0	0.1	7123.9	54.0	7123.9	54.0	2.29
	7	1630.4536	0.038	0.851	64.9	0.0	538.1	+0.058	3.0	0.0	1890.6	0.2	1890.6	0.2	0.03
3	4	939.9547	−1.369	0.966	58.5	0.0	591.6	−2.339	3.5	0.0	2453.3	9.7	2453.3	9.7	1.19
	5	504.9765	−0.854	1.191	55.5	0.0	723.2	−1.539	3.7	0.0	3218.8	6.8	3218.8	6.8	0.64
	7	591.5043	0.175	0.804	78.4	0.0	590.2	+0.223	2.2	0.0	1273.1	0.5	1273.1	0.5	0.11
	9	566.5542	0.083	0.760	61.5	0.0	471.4	+0.134	3.3	0.0	1816.3	0.5	1816.3	0.5	0.09
4	5	498.1138	−0.679	1.018	65.5	0.0	646.1	−1.037	3.0	0.0	2231.9	3.6	2231.9	3.6	0.48
	6	709.8998	2.719	1.327	44.7	0.0	805.5	+6.078	4.6	0.0	4451.4	33.6	4451.4	33.6	2.28
	8	1634.7604	−1.344	0.734	77.9	0.0	533.8	−1.725	2.2	0.0	1179.9	3.8	1179.9	3.8	0.98
	9	1050.2118	−0.780	0.938	57.3	0.0	572.4	−1.361	3.6	0.0	2442.2	5.8	2442.2	5.8	0.72
5	6	722.6357	−0.468	1.106	49.8	0.0	667.9	−0.940	4.2	0.0	3356.1	4.7	3356.1	4.7	0.42
	7	823.9880	0.198	1.023	65.1	0.0	647.8	+0.304	3.0	0.0	2261.0	1.1	2261.0	1.1	0.14
	8	1206.8150	−0.898	0.671	76.0	0.0	474.0	−1.182	2.3	0.0	1137.7	2.8	1137.7	2.8	0.75
	9	572.0891	0.485	0.793	52.0	0.0	479.3	+0.933	4.0	0.0	2298.9	4.5	2298.9	4.5	0.59
6	9	1193.0453	−0.927	0.924	34.9	0.0	585.0	−2.654	5.6	0.0	3806.7	17.3	3806.7	17.3	1.37
7	8	667.5944	0.059	0.766	47.9	0.0	462.6	+0.123	4.3	0.0	2411.6	0.6	2411.6	0.6	0.08
	9	328.6701	−0.307	0.999	40.9	0.0	613.2	−0.751	5.0	0.0	3626.5	4.4	3626.5	4.4	0.37
8	9	666.8867	−1.271	0.702	79.8	0.0	527.5	−1.594	2.1	0.0	1067.4	3.2	1067.4	3.2	0.91

Network graph



Supplementary information

Observed distances	:	24
Coordinate unknowns	:	10
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	8
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	6.6
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.8
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$8.6 \cdot 10^{-11}$
Redundancy r	:	14
Redundancy distances	:	14.00
Weighted square sum of residuals Ω [m^2]	:	$2.62343 \cdot 10^{-3}$
(a priori) standard deviation σ_0 [m]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$1.36890 \cdot 10^{-2}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.0137
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	0.0002 ($k^F_{\alpha_G;r,\infty}$ =1.62)
Number of outliers (Data snooping)	:	0
Number of outliers (τ-criterion)	:	1
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	9.516
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	9.00750

3.2 Direction networks

3.2.1 Grossmann W (1969)

Grossmann W (1969): Grundzüge der Ausgleichungsrechnung. 3. extended Edition, Springer 1969, pp. 170

Available data files: [2D] Grossmann_Direction_fix*.*

Coordinates

Point name	Easting x [m]	Northing y [m]
A	9498.2600 (D)	78 594.9100 (D)
B	10 367.5900 (D)	75 913.2500 (D)
C	9300.4300 (D)	75 306.8000 (D)
D	7115.0900 (D)	75 723.6800 (D)
E	7206.6500 (D)	78 907.8800 (D)
F	6633.2700 (D)	76 701.5700 (D)
P	8401.8800	76 607.8500

Datum: fix, (D)...Datum coordinate

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
A	B	0.000 000	2.5	1	180.040 089
	P	52.059 600	2.5	1	
	E	128.601 900	2.5	1	
C	B	0.000 000	2.5	1	67.105 085
	D	244.892 300	2.5	1	
	P	294.415 700	2.5	1	
D	E	0.000 000	2.5	1	1.823 937
	P	59.849 300	2.5	1	
	C	110.181 500	2.5	1	
	F	369.033 000	2.5	1	
P	A	0.000 000	2.5	1	32.098 791
	B	89.521 900	2.5	1	
	C	129.425 600	2.5	1	
	E	337.390 800	2.5	1	

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
$r_{\text{A,B}}$	21.4822	6.9640	−21.4822	−6.9640	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$r_{\text{A,P}}$	24.5610	−13.5518	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$r_{\text{A,E}}$	−3.7246	−27.2718	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.7246	27.2718
$r_{\text{C,B}}$	0.0000	0.0000	25.6256	−45.0929	−25.6256	45.0929	0.0000	0.0000	0.0000	0.0000
$r_{\text{C,D}}$	0.0000	0.0000	0.0000	0.0000	−5.3620	−28.1085	5.3620	28.1085	0.0000	0.0000
$r_{\text{C,P}}$	0.0000	0.0000	0.0000	0.0000	−33.1293	−22.8803	0.0000	0.0000	0.0000	0.0000
$r_{\text{D,E}}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−19.9766	0.5744	19.9766	−0.5744
$r_{\text{D,P}}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−23.0917	33.6069	0.0000	0.0000
$r_{\text{D,C}}$	0.0000	0.0000	0.0000	0.0000	−5.3620	−28.1085	5.3620	28.1085	0.0000	0.0000
$r_{\text{D,F}}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−52.3842	−25.8104	0.0000	0.0000
$r_{\text{P,A}}$	24.5610	−13.5518	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$r_{\text{P,B}}$	0.0000	0.0000	−10.1736	−28.7913	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$r_{\text{P,C}}$	0.0000	0.0000	0.0000	0.0000	−33.1293	−22.8803	0.0000	0.0000	0.0000	0.0000
$r_{\text{P,E}}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	21.7935	11.3252

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (continued)

A	x_F	y_F	x_P	y_P	ω_A	ω_C	ω_D	ω_P	Δy
$r_{\text{A,B}}$	0.0000	0.0000	0.0000	0.0000	−1	0	0	0	−2.74
$r_{\text{A,P}}$	0.0000	0.0000	−24.5610	13.5518	−1	0	0	0	1.74
$r_{\text{A,E}}$	0.0000	0.0000	0.0000	0.0000	−1	0	0	0	1.00
$r_{\text{C,B}}$	0.0000	0.0000	0.0000	0.0000	0	−1	0	0	3.84
$r_{\text{C,D}}$	0.0000	0.0000	0.0000	0.0000	0	−1	0	0	−2.73
$r_{\text{C,P}}$	0.0000	0.0000	33.1293	22.8803	0	−1	0	0	−1.11
$r_{\text{D,E}}$	0.0000	0.0000	0.0000	0.0000	0	0	−1	0	−6.13
$r_{\text{D,P}}$	0.0000	0.0000	23.0917	−33.6069	0	0	−1	0	−0.70
$r_{\text{D,C}}$	0.0000	0.0000	0.0000	0.0000	0	0	−1	0	5.32
$r_{\text{D,F}}$	52.3842	25.8104	0.0000	0.0000	0	0	−1	0	1.50
$r_{\text{P,A}}$	0.0000	0.0000	−24.5610	13.5518	0	0	0	−1	0.84
$r_{\text{P,B}}$	0.0000	0.0000	10.1736	28.7913	0	0	0	−1	−2.96
$r_{\text{P,C}}$	0.0000	0.0000	33.1293	22.8803	0	0	0	−1	2.50
$r_{\text{P,E}}$	0.0000	0.0000	−21.7935	−11.3252	0	0	0	−1	−0.38

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	x_F	y_F	x_P	y_P	ω_A	ω_C	ω_D	ω_P
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[m,mgon]}$ (1. iteration)

	x_P	y_P	ω_A	ω_C	ω_D	ω_P
$\widehat{\Delta x}^{\mathsf{T}}$	−0.0163	0.0093	0.175	−0.109	−0.172	0.137

Adjusted coordinates

Point name	$\hat{x}_{[m]}$	$\hat{x} - x_{[cm]}$	$\lvert\hat{\sigma}\rvert_{[cm]}$	$\hat{y}_{[m]}$	$\hat{y} - y_{[cm]}$	$\lvert\hat{\sigma}\rvert_{[cm]}$	$\lvert\hat{\sigma}_{2D}\rvert_{[cm]}$
P	8401.8637	−1.625	6.422	76 607.8593	0.925	8.345	10.530

Absolute error ellipses

in	$A_{[cm]}$	$B_{[cm]}$	$\phi_{[gon]}$
P	8.64	6.02	176.492 420

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	$A_{[cm]}$	$B_{[cm]}$	$\phi_{[gon]}$
P	25.80	17.98	176.492 420

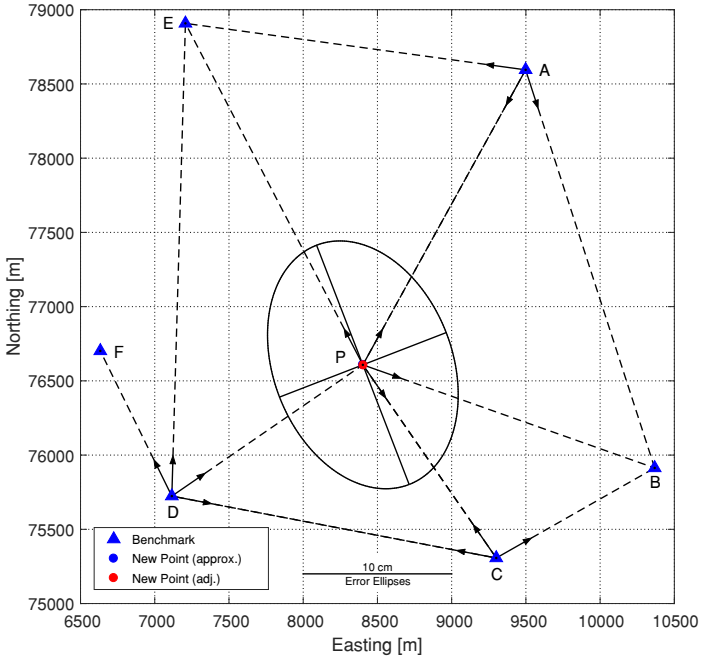
Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
A	180.040 264	0.17	2.33
C	67.104 976	−0.11	2.37
D	1.823 765	−0.17	2.11
P	32.098 928	0.14	2.23

Adjusted directions

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
A	B	0.002 565	180.042 829	−2.565	2.334	63.19	1.29	13.00	−4.060	2819	113.6	0.97	0.30	211.8	66.2	0.0	0.0	0.84
	P	52.058 207	232.098 471	1.393	2.644	52.77	0.77	14.22	+2.639	2269	49.6	2.12	0.39	239.4	44.4	105.7	19.6	0.50
	E	128.600 727	308.640 991	1.173	2.334	63.19	0.59	13.00	+1.856	2313	42.6	0.97	0.14	173.8	24.8	0.0	0.0	0.38
C	B	399.996 270	67.101 246	3.730	2.372	62.00	1.89	13.12	+6.015	1227	71.9	1.13	0.52	96.1	44.1	0.0	0.0	1.23
	D	244.895 139	312.000 115	−2.839	2.372	62.00	1.44	13.12	−4.579	2225	99.2	1.13	0.40	174.2	60.8	0.0	0.0	0.94
	P	294.416 590	361.521 566	−0.890	2.774	48.01	0.51	14.91	−1.854	1581	22.1	2.58	0.32	192.5	23.9	103.6	12.9	0.33
D	E	0.006 297	1.830 062	−6.297	2.111	69.90	3.01	12.36	−9.009	3186	315.1	1.12	0.81	186.1	135.7	0.0	0.0	1.96
	P	59.849 483	61.673 248	−0.183	3.239	29.12	0.14	19.14	−0.627	1561	4.5	5.19	0.17	332.7	10.9	287.2	9.4	0.09
	C	110.176 350	112.000 115	5.150	2.111	69.90	2.46	12.36	+7.367	2225	180.0	1.12	0.67	130.0	77.5	0.0	0.0	1.60
	F	369.031 670	370.855 435	1.330	2.111	69.90	0.64	12.36	+1.903	1090	22.8	1.12	0.17	63.7	9.8	0.0	0.0	0.41
P	A	399.999 544	32.098 471	0.456	2.458	59.19	0.24	13.43	+0.771	2269	16.3	2.14	0.12	195.4	11.2	91.8	5.3	0.15
	B	89.524 824	121.623 752	−2.924	2.323	63.56	1.47	12.96	−4.601	2085	95.8	1.75	0.62	154.7	54.9	83.8	29.7	0.95
	C	129.422 639	161.521 566	2.961	2.869	44.40	1.78	15.50	+6.670	1581	73.6	3.43	1.48	214.1	92.1	122.9	52.9	1.16
	E	337.391 294	369.490 222	−0.494	2.909	42.85	0.30	15.78	−1.153	2592	20.1	3.58	0.26	367.2	26.8	123.3	9.0	0.20

Network graph



Supplementary information

Observed directions	:	14
Orientation unknowns	:	4
Coordinate unknowns	:	2
Datum defect	:	4
Datum definition	:	fix
Number of datum constraints	:	12
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	2.8
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.5
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$1.2 \cdot 10^{-11}$
Redundancy r	:	8
Redundancy directions	:	8.00
Weighted square sum of residuals Ω [mgon^2]	:	118.41
(a priori) standard deviation σ_0 [mgon]	:	2.5
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	3.8473
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.5389
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	2.3683 ($k^F_{\alpha_G;r,\infty}$ =2.15)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	1.870
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	110.89

3.2.2 Lother & Strehle (2007), with 7 different datum definitions

G. Lother und J. Strehle (2007): Grundlagen der Ausgleichungsrechnung nach der Methode der kleinsten Quadrate. Hochschule München, Fakultät für Geoinformation, Bachelorstudiengang Geoinformatik und Satellitenpositionierung, pp. 11-17

Available data files: [2D] LotherStrehle_Direction∗.∗

This example consists of 7 different scenarios which differ in the datum definition. Scenario 1 uses points 10 and 20 as benchmarks, while in scenario 2 the datum problem is solved by fixing points 30 and 40. Problem 3 treats a free network with total-trace minimization of the variance-covariance matrix of estimated point coordinates. Example 4 is again a free network but using partial-trace minimization with respect to points 10-30, only. Solution 5 is obtained by fixing points 20-40 which is equivalent to case 6 (dynamic solution), where these points are treated as being observed with huge weights. Scenario 7 displays the situation where a constant standard deviation of 1cm is assigned to all four points leading to a dynamic datum definition using stochastic prior information.

Directions with approximate orientations

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
10	20	0.000 000	1	1	40.330 800
	30	59.669 400	1	1	
	40	103.319 500	1	1	
20	10	0.000 000	1	1	240.330 900
	30	352.679 200	1	1	
	40	359.179 900	1	1	
30	20	0.000 000	1	1	393.010 400
	40	217.100 200	1	1	
	10	306.990 800	1	1	
40	10	0.000 000	1	1	343.648 800
	20	55.862 200	1	1	
	30	66.465 000	1	1	

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}	Δy
$r_{10,20}$	−70.2320	51.5885	70.2320	−51.5885	0.0000	0.0000	0.0000	0.0000	−1	0	0	0	−1.34
$r_{10,30}$	0.0000	127.9890	0.0000	0.0000	0.0000	−127.9890	0.0000	0.0000	−1	0	0	0	0.20
$r_{10,40}$	70.9456	86.7275	0.0000	0.0000	0.0000	0.0000	−70.9456	−86.7275	−1	0	0	0	1.25
$r_{20,10}$	−70.2320	51.5885	70.2320	−51.5885	0.0000	0.0000	0.0000	0.0000	0	−1	0	0	−1.24
$r_{20,30}$	0.0000	0.0000	106.8272	11.7790	−106.8272	−11.7790	0.0000	0.0000	0	−1	0	0	1.40
$r_{20,40}$	0.0000	0.0000	67.1134	0.5155	0.0000	0.0000	−67.1134	−0.5155	0	−1	0	0	−0.26
$r_{30,20}$	0.0000	0.0000	106.8272	11.7790	−106.8272	−11.7790	0.0000	0.0000	0	0	−1	0	1.70
$r_{30,40}$	0.0000	0.0000	0.0000	0.0000	172.5370	−27.6425	−172.5370	27.6425	0	0	−1	0	−2.87
$r_{30,10}$	0.0000	127.9890	0.0000	0.0000	0.0000	−127.9890	0.0000	0.0000	0	0	−1	0	1.20
$r_{40,10}$	70.9456	86.7275	0.0000	0.0000	0.0000	0.0000	−70.9456	−86.7275	0	0	0	−1	−0.25
$r_{40,20}$	0.0000	0.0000	67.1134	0.5155	0.0000	0.0000	−67.1134	−0.5155	0	0	0	−1	−0.06
$r_{40,30}$	0.0000	0.0000	0.0000	0.0000	172.5370	−27.6425	−172.5370	27.6425	0	0	0	−1	0.33

3.2.2.1 Datum definition 1: Points 10 and 20 fixed

Coordinates

Point name	Easting x [m]	Northing y [m]
10	1000.0000 (D)	1000.0000 (D)
20	1432.4820 (D)	1588.7760 (D)
30	1497.4020	1000.0000
40	1439.7670	640.2580

Datum: fix, (D)...Datum coordinate

Matrix $D^{\mathsf{T}}{}_{[\cdot]}$ of datum constraints

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\mathrm{m,mgon}]}$ (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
$\widehat{\Delta x}^{\mathsf{T}}$	0.0000	0.0000	0.0000	0.0000	−0.0251	−0.0169	−0.0217	0.0002	1.194	1.482	1.636	0.950

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
30	1497.3769	−2.513	1.211	999.9831	−1.692	1.107	1.641
40	1439.7453	−2.172	1.664	640.2582	0.023	1.344	2.139

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
30	1.40	0.86	56.376 422
40	1.75	1.23	128.618 453

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
30	5.20	3.21	56.376 422
40	6.52	4.59	128.618 453

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
10	40.331 994	1.19	1.10
20	240.332 382	1.48	1.09
30	393.012 036	1.64	1.40
40	343.649 750	0.95	1.40

Adjusted directions

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
10	20	0.000 147	40.332 142	−0.147	1.098	25.00	0.29	8.26	−0.590	731	1.7	5.33	0.38	71.1	5.1	0.0	0.0	0.23
	30	59.670 171	100.002 165	−0.771	1.097	25.05	1.54	8.26	−3.078	497	6.0	5.33	1.99	48.3	18.0	40.3	15.0	1.22
	40	103.318 582	143.650 576	0.918	1.097	25.07	1.83	8.25	+3.664	568	8.2	5.32	2.36	55.2	24.5	46.0	20.4	1.45
20	10	399.999 759	240.332 142	0.241	1.087	26.48	0.47	8.03	+0.909	731	2.8	5.09	0.58	67.7	7.7	0.0	0.0	0.37
	30	352.679 205	193.011 587	−0.005	0.941	44.94	0.01	6.16	−0.011	592	0.0	2.87	0.01	31.6	0.1	27.5	0.1	0.01
	40	359.180 136	199.512 518	−0.236	0.828	57.30	0.31	5.46	−0.411	949	3.5	1.67	0.13	34.7	2.6	18.9	1.4	0.25
30	20	399.999 551	393.011 587	0.449	1.081	27.23	0.86	7.92	+1.648	592	4.2	4.97	1.03	53.6	11.2	24.1	5.0	0.68
	40	217.101 320	210.113 356	−1.120	1.098	25.01	2.24	8.26	−4.477	364	6.4	5.33	2.89	35.5	19.2	23.6	12.8	1.77
	10	306.990 129	300.002 165	0.671	1.080	27.46	1.28	7.89	+2.443	497	5.2	4.94	1.53	44.7	13.8	23.3	7.2	1.01
40	10	0.000 826	343.650 576	−0.826	1.081	27.24	1.58	7.92	−3.033	568	7.4	4.97	1.90	51.4	19.7	26.8	10.3	1.25
	20	55.862 768	399.512 518	−0.568	0.886	51.14	0.79	5.78	−1.111	949	8.5	2.28	0.44	42.1	8.1	1.9	0.4	0.63
	30	66.463 606	10.113 356	1.394	0.997	38.08	2.26	6.70	+3.661	364	8.0	3.58	1.96	23.7	13.0	16.6	9.1	1.78

3.2.2.2 Datum definition 2: Points 30 and 40 fixed

Coordinates

Point name	Easting x [m]	Northing y [m]
10	1000.0000	1000.0000
20	1432.4820	1588.7760
30	1497.4020 (D)	1000.0000 (D)
40	1439.7670 (D)	640.2580 (D)

Datum: fix, (D)...Datum coordinate

Matrix $D^{\mathsf{T}}{}_{[\cdot]}$ of datum constraints

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
D^{T}	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\mathrm{m,mgon}]}$ (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
$\widehat{\Delta x}^{\mathsf{T}}$	0.0013	0.0178	0.0231	0.0453	0.0000	0.0000	0.0000	0.0000	1.308	1.596	1.750	1.063

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
10	1000.0013	0.128	1.757	1000.0178	1.781	1.095	2.070
20	1432.5051	2.307	1.323	1588.8213	4.527	3.311	3.566

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
10	1.78	1.05	113.394 745
20	3.32	1.30	195.259 095

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
10	6.64	3.93	113.394 745
20	12.37	4.86	195.259 095

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
10	40.332 108	1.31	1.40
20	240.332 496	1.60	1.36
30	393.012 150	1.75	1.10
40	343.649 863	1.06	1.00

Adjusted directions

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
10	20	0.000 147	40.332 255	−0.147	1.098	25.00	0.29	8.26	−0.590	731	1.7	5.33	0.38	71.1	5.1	47.5	3.4	0.23
	30	59.670 171	100.002 279	−0.771	1.097	25.05	1.54	8.26	−3.078	497	6.0	5.33	1.99	48.3	18.0	24.4	9.1	1.22
	40	103.318 582	143.650 689	0.918	1.097	25.07	1.83	8.25	+3.664	568	8.2	5.32	2.36	55.2	24.5	27.2	12.1	1.45
20	10	399.999 759	240.332 255	0.241	1.087	26.48	0.47	8.03	+0.909	731	2.8	5.09	0.58	67.7	7.7	48.1	5.4	0.37
	30	352.679 205	193.011 701	−0.005	0.941	44.94	0.01	6.16	−0.011	592	0.0	2.87	0.01	31.6	0.1	17.9	0.0	0.01
	40	359.180 136	199.512 631	−0.236	0.828	57.30	0.31	5.46	−0.411	949	3.5	1.67	0.13	34.7	2.6	9.9	0.7	0.25
30	20	399.999 551	393.011 701	0.449	1.081	27.23	0.86	7.92	+1.648	592	4.2	4.97	1.03	53.6	11.2	44.5	9.3	0.68
	40	217.101 320	210.113 469	−1.120	1.098	25.01	2.24	8.26	−4.477	364	6.4	5.33	2.89	35.5	19.2	0.0	0.0	1.77
	10	306.990 129	300.002 279	0.671	1.080	27.46	1.28	7.89	+2.443	497	5.2	4.94	1.53	44.7	13.8	36.9	11.4	1.01
40	10	0.000 826	343.650 689	−0.826	1.081	27.24	1.58	7.92	−3.033	568	7.4	4.97	1.90	51.4	19.7	46.4	17.8	1.25
	20	55.862 768	399.512 631	−0.568	0.886	51.14	0.79	5.78	−1.111	949	8.5	2.28	0.44	42.1	8.1	15.4	3.0	0.63
	30	66.463 606	10.113 469	1.394	0.997	38.08	2.26	6.70	+3.661	364	8.0	3.58	1.96	23.7	13.0	0.0	0.0	1.78

3.2.2.3 Datum definition 3: Free - Total trace minimization

Coordinates

Point name	Easting x [m]	Northing y [m]
10	1000.0000 (D)	1000.0000 (D)
20	1432.4820 (D)	1588.7760 (D)
30	1497.4020 (D)	1000.0000 (D)
40	1439.7670 (D)	640.2580 (D)

Datum: free, (D)...Datum coordinate

Matrix $D^{\mathsf{T}}{}_{[\cdot]}$ of datum constraints (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
D^{T}	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	0	0	0	0
	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0	0	0	0
	57.258	−342.413	−531.518	90.069	57.258	154.989	417.000	97.354	−1	−1	−1	−1
	−342.413	−57.258	90.069	531.518	154.989	−57.258	97.354	−417.000	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\mathrm{m,mgon}]}$ (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
$\widehat{\Delta x}^{\mathsf{T}}$	0.0101	−0.0035	0.0013	0.0105	−0.0109	−0.0100	−0.0004	0.0030	−0.147	0.141	0.295	−0.391

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
10	1000.0101	1.009	0.594	999.9965	−0.351	0.584	0.833
20	1432.4833	0.126	0.324	1588.7865	1.046	0.603	0.684
30	1497.3911	−1.093	0.407	999.9900	−0.995	0.771	0.872
40	1439.7666	−0.042	0.409	640.2610	0.301	0.615	0.738

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
10	0.68	0.48	148.332 399
20	0.61	0.31	9.803 275
30	0.78	0.38	13.670 875
40	0.62	0.40	187.024 398

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
10	2.54	1.78	148.332 399
20	2.27	1.17	9.803 275
30	2.92	1.41	13.670 875
40	2.32	1.48	187.024 398

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
10	40.330 653	−0.15	0.89
20	240.331 041	0.14	0.79
30	393.010 695	0.30	0.86
40	343.648 409	−0.39	0.88

Adjusted directions

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
10	20	0.000 147	40.330 801	−0.147	1.098	25.00	0.29	8.26	−0.590	731	1.7	5.33	0.38	71.1	5.1	33.9	2.4	0.23
	30	59.670 171	100.000 824	−0.771	1.097	25.05	1.54	8.26	−3.078	497	6.0	5.33	1.99	48.3	18.0	29.9	11.2	1.22
	40	103.318 582	143.649 235	0.918	1.097	25.07	1.83	8.25	+3.664	568	8.2	5.32	2.36	55.2	24.5	31.4	14.0	1.45
20	10	399.999 759	240.330 801	0.241	1.087	26.48	0.47	8.03	+0.909	731	2.8	5.09	0.58	67.7	7.7	37.1	4.2	0.37
	30	352.679 205	193.010 246	−0.005	0.941	44.94	0.01	6.16	−0.011	592	0.0	2.87	0.01	31.6	0.1	14.3	0.0	0.01
	40	359.180 136	199.511 177	−0.236	0.828	57.30	0.31	5.46	−0.411	949	3.5	1.67	0.13	34.7	2.6	4.9	0.4	0.25
30	20	399.999 551	393.010 246	0.449	1.081	27.23	0.86	7.92	+1.648	592	4.2	4.97	1.03	53.6	11.2	22.0	4.6	0.68
	40	217.101 320	210.112 015	−1.120	1.098	25.01	2.24	8.26	−4.477	364	6.4	5.33	2.89	35.5	19.2	20.8	11.3	1.77
	10	306.990 129	300.000 824	0.671	1.080	27.46	1.28	7.89	+2.443	497	5.2	4.94	1.53	44.7	13.8	28.7	8.9	1.01
40	10	0.000 826	343.649 235	−0.826	1.081	27.24	1.58	7.92	−3.033	568	7.4	4.97	1.90	51.4	19.7	29.9	11.4	1.25
	20	55.862 768	399.511 177	−0.568	0.886	51.14	0.79	5.78	−1.111	949	8.5	2.28	0.44	42.1	8.1	3.9	0.7	0.63
	30	66.463 606	10.112 015	1.394	0.997	38.08	2.26	6.70	+3.661	364	8.0	3.58	1.96	23.7	13.0	14.1	7.7	1.78

3.2.2.4 Datum definition 4: Free - Partial trace minimization

Coordinates

Point name	Easting x [m]	Northing y [m]
10	1000.0000 (D)	1000.0000 (D)
20	1432.4820 (D)	1588.7760 (D)
30	1497.4020 (D)	1000.0000 (D)
40	1439.7670	640.2580

Datum: free, (D)...Datum coordinate

Matrix $D^{\mathsf{T}}{}_{[\cdot]}$ of datum constraints (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
D^{T}	1.000	0.000	1.000	0.000	1.000	0.000	0.000	0.000	0	0	0	0
	0.000	1.000	0.000	1.000	0.000	1.000	0.000	0.000	0	0	0	0
	57.258	−342.413	−531.518	90.069	57.258	154.989	0.000	0.000	−1	−1	−1	−1
	−342.413	−57.258	90.069	531.518	154.989	−57.258	0.000	0.000	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\mathrm{m,mgon}]}$ (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
$\widehat{\Delta x}^{\mathsf{T}}$	0.0114	−0.0017	0.0004	0.0097	−0.0118	−0.0080	−0.0009	0.0066	−0.173	0.115	0.269	−0.417

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
10	1000.0114	1.145	0.533	999.9983	−0.174	0.330	0.627
20	1432.4824	0.040	0.277	1588.7857	0.972	0.448	0.527
30	1497.3902	−1.185	0.571	999.9920	−0.798	0.522	0.773
40	1439.7661	−0.093	0.899	640.2646	0.661	1.350	1.622

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
10	0.53	0.33	103.695 785
20	0.45	0.28	196.704 724
30	0.66	0.41	56.375 082
40	1.37	0.87	187.024 355

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
10	1.99	1.23	103.695 785
20	1.67	1.03	196.704 724
30	2.45	1.51	56.375 082
40	5.09	3.25	187.024 355

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
10	40.330 627	−0.17	0.89
20	240.331 015	0.12	0.81
30	393.010 669	0.27	0.97
40	343.648 383	−0.42	1.11

Adjusted directions

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
10	20	0.000 147	40.330 775	−0.147	1.098	25.00	0.29	8.26	−0.590	731	1.7	5.33	0.38	71.1	5.1	24.0	1.7	0.23
	30	59.670 171	100.000 798	−0.771	1.097	25.05	1.54	8.26	−3.078	497	6.0	5.33	1.99	48.3	18.0	26.6	9.9	1.22
	40	103.318 582	143.649 209	0.918	1.097	25.07	1.83	8.25	+3.664	568	8.2	5.32	2.36	55.2	24.5	42.9	19.1	1.45
20	10	399.999 759	240.330 775	0.241	1.087	26.48	0.47	8.03	+0.909	731	2.8	5.09	0.58	67.7	7.7	26.6	3.0	0.37
	30	352.679 205	193.010 220	−0.005	0.941	44.94	0.01	6.16	−0.011	592	0.0	2.87	0.01	31.6	0.1	16.0	0.0	0.01
	40	359.180 136	199.511 151	−0.236	0.828	57.30	0.31	5.46	−0.411	949	3.5	1.67	0.13	34.7	2.6	11.9	0.9	0.25
30	20	399.999 551	393.010 220	0.449	1.081	27.23	0.86	7.92	+1.648	592	4.2	4.97	1.03	53.6	11.2	20.7	4.3	0.68
	40	217.101 320	210.111 988	−1.120	1.098	25.01	2.24	8.26	−4.477	364	6.4	5.33	2.89	35.5	19.2	26.7	14.5	1.77
	10	306.990 129	300.000 798	0.671	1.080	27.46	1.28	7.89	+2.443	497	5.2	4.94	1.53	44.7	13.8	22.0	6.8	1.01
40	10	0.000 826	343.649 209	−0.826	1.081	27.24	1.58	7.92	−3.033	568	7.4	4.97	1.90	51.4	19.7	30.7	11.7	1.25
	20	55.862 768	399.511 151	−0.568	0.886	51.14	0.79	5.78	−1.111	949	8.5	2.28	0.44	42.1	8.1	0.4	0.1	0.63
	30	66.463 606	10.111 988	1.394	0.997	38.08	2.26	6.70	+3.661	364	8.0	3.58	1.96	23.7	13.0	15.5	8.5	1.78

3.2.2.5 Datum definition 5: Points 20, 30 and 40 fixed

Coordinates

Point name	Easting x [m]	Northing y [m]
10	1000.0000	1000.0000
20	1432.4820 (D)	1588.7760 (D)
30	1497.4020 (D)	1000.0000 (D)
40	1439.7670 (D)	640.2580 (D)

Datum: fix, (D)...Datum coordinate

Matrix $D^{\mathsf{T}}{}_{[\cdot]}$ of datum constraints

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
D^{T}	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\mathrm{m,mgon}]}$ (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
$\widehat{\Delta x}^{\mathsf{T}}$	0.0142	0.0031	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.243	−0.244	0.123	0.420

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
10	1000.0142	1.421	1.290	1000.0031	0.309	1.158	1.733

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
10	1.37	1.06	135.638 467

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
10	4.39	3.41	135.638 467

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
10	40.331 043	0.24	1.39
20	240.330 656	−0.24	1.02
30	393.010 523	0.12	1.06
40	343.649 220	0.42	1.02

Adjusted directions

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
10	20	0.000 260	40.331 303	−0.260	1.308	34.82	0.44	7.00	−0.747	731	3.0	3.95	0.42	52.4	5.6	18.4	2.0	0.27
	30	59.669 353	100.000 396	0.047	1.040	58.79	0.06	5.39	+0.081	497	0.4	1.51	0.02	17.4	0.3	10.8	0.2	0.04
	40	103.319 288	143.650 331	0.212	1.311	34.58	0.36	7.03	+0.614	568	1.9	3.98	0.35	41.0	3.6	14.6	1.3	0.22
20	10	0.000 648	240.331 303	−0.648	1.227	42.62	0.99	6.33	−1.519	731	7.4	3.10	0.75	41.7	10.0	26.2	6.3	0.61
	30	352.678 048	193.008 704	1.152	1.016	60.66	1.48	5.31	+1.899	592	10.7	1.30	0.47	19.4	7.0	0.0	0.0	0.91
	40	359.180 404	199.511 060	−0.504	1.016	60.66	0.65	5.31	−0.832	949	7.5	1.30	0.20	31.1	4.9	0.0	0.0	0.40
30	20	399.998 181	393.008 704	1.819	1.058	57.38	2.40	5.46	+3.171	592	16.9	1.66	0.97	21.6	12.6	0.0	0.0	1.48
	40	217.102 946	210.113 469	−2.746	1.058	57.38	3.63	5.46	−4.786*	364	15.7	1.66	1.46	13.3	11.7	0.0	0.0	2.24
	10	306.989 873	300.000 396	0.927	1.360	29.51	1.71	7.61	+3.141	497	7.2	4.64	1.91	41.9	17.3	33.1	13.7	1.05
40	10	0.001 111	343.650 331	−1.111	1.230	42.40	1.71	6.35	−2.620	568	9.9	3.13	1.29	32.6	13.5	20.6	8.5	1.05
	20	55.861 840	399.511 060	0.360	1.017	60.60	0.46	5.31	+0.594	949	5.4	1.31	0.15	31.2	3.5	0.0	0.0	0.29
	30	66.464 249	10.113 469	0.751	1.017	60.60	0.96	5.31	+1.239	364	4.3	1.31	0.31	12.0	2.8	0.0	0.0	0.60

3.2.2.6 Datum definition 6: Dynamic, huge weights for observed points, equivalent to case 5

Coordinates

Point name	Easting x [m]	Northing y [m]
10	1000.0000	1000.0000
20	1432.4820 (D)	1588.7760 (D)
30	1497.4020 (D)	1000.0000 (D)
40	1439.7670 (D)	640.2580 (D)

Datum: dynamic, (D)...Datum coordinate

Dynamic coordinates

Point name	Easting x [m]	$\lvert\sigma\rvert$ [cm]	p [rad²/m²]	Northing y [m]	$\lvert\sigma\rvert$ [cm]	p [rad²/m²]
20	1432.4820	0.00	$1.0{\cdot}10^{8}$	1588.7760	0.00	$1.0{\cdot}10^{8}$
30	1497.4020	0.00	$1.0{\cdot}10^{8}$	1000.0000	0.00	$1.0{\cdot}10^{8}$
40	1439.7670	0.00	$1.0{\cdot}10^{8}$	640.2580	0.00	$1.0{\cdot}10^{8}$

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[mm]}$ dynamic coordinates (1. iteration)

A	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	Δy
x_{20}	0	0	1	0	0	0	0	0	0.00
y_{20}	0	0	0	1	0	0	0	0	0.00
x_{30}	0	0	0	0	1	0	0	0	0.00
y_{30}	0	0	0	0	0	1	0	0	0.00
x_{40}	0	0	0	0	0	0	1	0	0.00
y_{40}	0	0	0	0	0	0	0	1	0.00

Least squares solution $\widehat{\Delta x}_{[m,mgon]}$ (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
$\widehat{\Delta x}^{\mathsf{T}}$	0.0142	0.0031	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.243	−0.244	0.123	0.420

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
10	1000.0142	1.421	1.290	1000.0031	0.309	1.158	1.733
20	1432.4820	0.000	0.000	1588.7760	0.000	0.000	0.000
30	1497.4020	0.000	0.000	1000.0000	0.000	0.000	0.000
40	1439.7670	0.000	0.000	640.2580	0.000	0.000	0.000

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
10	1.37	1.06	135.638 467
20	0.00	0.00	195.241 994
30	0.00	0.00	5.833 430
40	0.00	0.00	12.452 641

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
10	4.39	3.41	135.638 467
20	0.00	0.00	195.241 994
30	0.00	0.00	5.833 430
40	0.00	0.00	12.452 641

Adjusted dynamic coordinates

Variance component: $\Omega/\sigma_0^2 = 0.000$, $r = 0.00$, $\hat{\sigma}_0^2/\sigma_0^2 = 6.04$, $\alpha_G = 0.00‰$, $k^F_{\alpha_G;r,\infty} = \infty$

Point name	$\hat{x}$ [m]	$\hat{e}_x$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	$\hat{y}$ [m]	$\hat{e}_y$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
20	1432.4820	0.00	0.00				0.00	∞	4.24	∞	∞	∞	4.24	∞	4.24	∞	∞
20				1588.7760	0.00	0.00	0.00	∞	10.72	∞	∞	∞	10.72	∞	10.72	∞	∞
30	1497.4020	0.00	0.00				0.00	∞	1.64	∞	∞	∞	1.64	∞	1.64	∞	∞
30				1000.0000	0.00	0.00	0.00	∞	4.05	∞	∞	∞	4.05	∞	4.05	∞	∞
40	1439.7670	0.00	0.00				0.00	∞	2.65	∞	∞	∞	2.65	∞	2.65	∞	∞
40				640.2580	0.00	0.00	0.00	∞	5.98	∞	∞	∞	5.98	∞	5.98	∞	∞

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 15.755$, $r = 6.00$, $\hat{\sigma}_0^2/\sigma_0^2 = 2.63$, $\alpha_G = 1.77\%$, $k^F_{\alpha_G;r,\infty} = 2.56$

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
10	20	0.000 260	40.331 303	−0.260	1.308	34.82	0.44	7.00	−0.747	731	3.0	3.95	0.42	52.4	5.6	18.4	2.0	0.27
	30	59.669 353	100.000 396	0.047	1.040	58.79	0.06	5.39	+0.081	497	0.4	1.51	0.02	17.4	0.3	10.8	0.2	0.04
	40	103.319 288	143.650 331	0.212	1.311	34.58	0.36	7.03	+0.614	568	1.9	3.98	0.35	41.0	3.6	14.6	1.3	0.22
20	10	0.000 648	240.331 303	−0.648	1.227	42.62	0.99	6.33	−1.519	731	7.4	3.10	0.75	41.7	10.0	26.2	6.3	0.61
	30	352.678 048	193.008 704	1.152	1.016	60.66	1.48	5.31	+1.899	592	10.7	1.30	0.47	19.4	7.0	0.0	0.0	0.91
	40	359.180 404	199.511 060	−0.504	1.016	60.66	0.65	5.31	−0.832	949	7.5	1.30	0.20	31.1	4.9	0.0	0.0	0.40
30	20	399.998 181	393.008 704	1.819	1.058	57.38	2.40	5.46	+3.171	592	16.9	1.66	0.97	21.6	12.6	0.0	0.0	1.48
	40	217.102 946	210.113 469	−2.746	1.058	57.38	3.63	5.46	−4.786*	364	15.7	1.66	1.46	13.3	11.7	0.0	0.0	2.24
	10	306.989 873	300.000 396	0.927	1.360	29.51	1.71	7.61	+3.141	497	7.2	4.64	1.91	41.9	17.3	33.1	13.7	1.05
40	10	0.001 111	343.650 331	−1.111	1.230	42.40	1.71	6.35	−2.620	568	9.9	3.13	1.29	32.6	13.5	20.6	8.5	1.05
	20	55.861 840	399.511 060	0.360	1.017	60.60	0.46	5.31	+0.594	949	5.4	1.31	0.15	31.2	3.5	0.0	0.0	0.29
	30	66.464 249	10.113 469	0.751	1.017	60.60	0.96	5.31	+1.239	364	4.3	1.31	0.31	12.0	2.8	0.0	0.0	0.60

3.2.2.7 Datum definition 7: Dynamic, with stochastic prior information for observed points

Coordinates

Point name	Easting x [m]	Northing y [m]
10	1000.0000 (D)	1000.0000 (D)
20	1432.4820 (D)	1588.7760 (D)
30	1497.4020 (D)	1000.0000 (D)
40	1439.7670 (D)	640.2580 (D)

Datum: dynamic, (D)...Datum coordinate

Dynamic coordinates

Point name	Easting x [m]	$\lvert\sigma\rvert$ [cm]	p [rad²/m²]	Northing y [m]	$\lvert\sigma\rvert$ [cm]	p [rad²/m²]
10	1000.0000	1	$2.4674{\cdot}10^{-6}$	1000.0000	1	$2.4674{\cdot}10^{-6}$
20	1432.4820	1	$2.4674{\cdot}10^{-6}$	1588.7760	1	$2.4674{\cdot}10^{-6}$
30	1497.4020	1	$2.4674{\cdot}10^{-6}$	1000.0000	1	$2.4674{\cdot}10^{-6}$
40	1439.7670	1	$2.4674{\cdot}10^{-6}$	640.2580	1	$2.4674{\cdot}10^{-6}$

Design matrix $A_{[-]}$ **and reduced observation vector** $\Delta y_{[mm]}$ **dynamic coordinates** (1. iteration)

A	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	Δy
x_{10}	1	0	0	0	0	0	0	0	0.00
y_{10}	0	1	0	0	0	0	0	0	0.00
x_{20}	0	0	1	0	0	0	0	0	0.00
y_{20}	0	0	0	1	0	0	0	0	0.00
x_{30}	0	0	0	0	1	0	0	0	0.00
y_{30}	0	0	0	0	0	1	0	0	0.00
x_{40}	0	0	0	0	0	0	1	0	0.00
y_{40}	0	0	0	0	0	0	0	1	0.00

Least squares solution $\widehat{\Delta x}_{[m,mgon]}$ (1. iteration)

	x_{10}	y_{10}	x_{20}	y_{20}	x_{30}	y_{30}	x_{40}	y_{40}	ω_{10}	ω_{20}	ω_{30}	ω_{40}
$\widehat{\Delta x}^{\mathsf{T}}$	0.0065	−0.0009	0.0008	0.0059	−0.0086	−0.0054	0.0012	0.0003	−0.004	0.153	0.051	−0.433

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
10	1000.0065	0.655	0.828	999.9991	−0.089	0.821	1.165
20	1432.4828	0.081	0.942	1588.7819	0.594	0.984	1.362
30	1497.3934	−0.857	0.657	999.9946	−0.539	0.773	1.014
40	1439.7682	0.122	0.846	640.2583	0.033	0.892	1.229

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
10	0.85	0.80	145.008 808
20	0.99	0.94	8.913 470
30	0.78	0.65	10.737 818
40	0.89	0.85	194.164 105

Absolute confidence ellipses ($1-\alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
10	2.53	2.39	145.008 808
20	2.94	2.81	8.913 470
30	2.32	1.95	10.737 818
40	2.66	2.53	194.164 105

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
10	40.330 796	0.00	1.13
20	240.331 053	0.15	1.09
30	393.010 451	0.05	1.11
40	343.648 367	−0.43	1.13

Adjusted dynamic coordinates

Variance component: $\Omega/\sigma_0^2 = 1.836,\ r = 3.01,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.61,\ \alpha_G = 0.55\%,\ k^F_{\alpha_G;r,\infty} = 4.20$

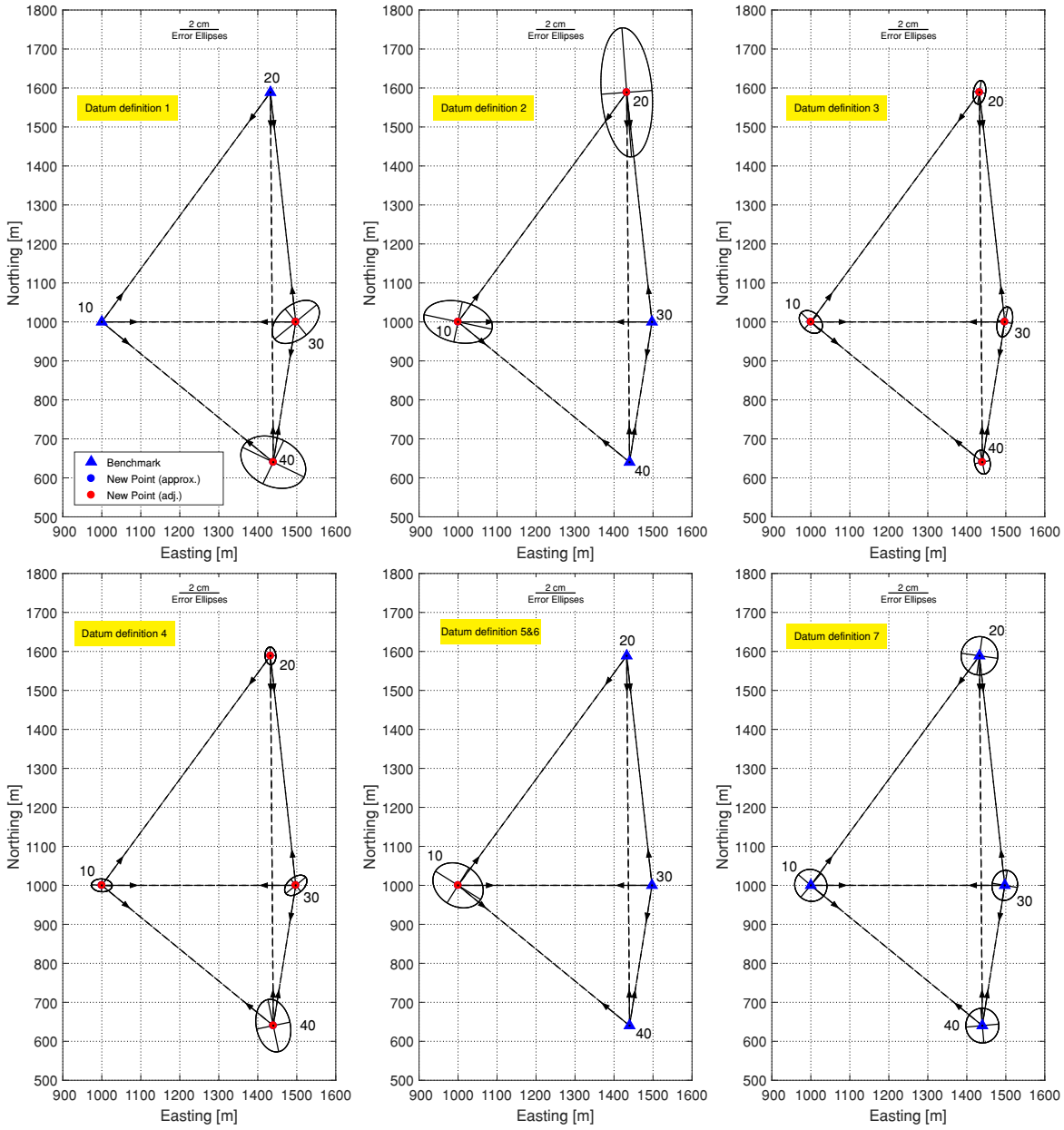
Point name	$\hat{x}$ [m]	$\hat{e}_x$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	$\hat{y}$ [m]	$\hat{e}_y$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
10	1000.0000	−6.55	8.28				40.61	1.03	6.48	−1.61	5.00	1.24	3.85	0.96	3.85	0.96	0.96
10				1000.0000	0.89	8.21	41.63	0.14	6.40	+0.21	4.89	0.16	3.74	0.12	3.74	0.12	0.13
20	1432.4820	−0.81	9.42				23.14	0.17	8.59	−0.35	7.53	0.31	6.60	0.27	6.60	0.27	0.16
20				1588.7760	−5.94	9.84	15.98	1.49	10.34	−3.72	9.48	3.41	8.69	3.12	8.69	3.12	1.38
30	1497.4020	8.57	6.57				62.56	1.08	5.22	+1.37	3.20	0.84	1.96	0.51	1.96	0.51	1.01
30				1000.0000	5.39	7.73	48.25	0.78	5.95	+1.12	4.28	0.80	3.08	0.58	3.08	0.58	0.72
40	1439.7670	−1.22	8.46				37.95	0.20	6.71	−0.32	5.28	0.25	4.16	0.20	4.16	0.20	0.18
40				640.2580	−0.33	8.92	31.09	0.06	7.41	−0.11	6.15	0.09	5.11	0.07	5.11	0.07	0.06

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 7.391,\ r = 4.99,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.48,\ \alpha_G = 1.30\%,\ k^F_{\alpha_G;r,\infty} = 2.89$

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
10	20	0.000 590	40.331 386	−0.590	0.854	36.71	0.97	6.82	−1.607	731	6.8	3.73	0.88	49.5	11.7	18.1	4.3	0.91
	30	59.669 779	100.000 576	−0.379	0.841	38.62	0.61	6.65	−0.982	497	3.0	3.52	0.52	31.9	4.7	17.9	2.6	0.57
	40	103.318 531	143.649 327	0.969	0.864	35.21	1.63	6.96	+2.753	568	8.6	3.91	1.54	40.3	15.9	19.8	7.8	1.52
20	10	0.000 333	240.331 386	−0.333	0.810	43.14	0.51	6.29	−0.771	731	3.8	3.05	0.37	41.0	5.0	17.0	2.1	0.47
	30	352.678 785	193.009 838	0.415	0.738	52.72	0.57	5.69	+0.787	592	3.9	2.13	0.29	25.0	3.5	8.9	1.2	0.53
	40	359.179 982	199.511 035	−0.082	0.681	59.77	0.11	5.34	−0.137	949	1.2	1.40	0.04	32.0	0.8	3.2	0.1	0.10
30	20	399.999 387	393.009 838	0.613	0.879	32.94	1.07	7.20	+1.860	592	5.7	4.18	1.08	44.9	11.6	16.5	4.3	0.99
	40	217.101 488	210.111 939	−1.288	0.903	29.24	2.38	7.64	−4.405	364	7.4	4.67	2.70	30.9	17.8	17.6	10.1	2.22
	10	306.990 125	300.000 576	0.675	0.866	35.00	1.14	6.98	+1.930	497	5.3	3.93	1.09	35.5	9.8	20.7	5.7	1.06
40	10	0.000 960	343.649 327	−0.960	0.844	38.23	1.55	6.68	−2.511	568	8.6	3.56	1.34	36.8	13.8	18.0	6.7	1.45
	20	55.862 668	399.511 035	−0.468	0.726	54.27	0.64	5.61	−0.863	949	7.0	1.97	0.30	38.2	5.9	2.4	0.4	0.59
	30	66.463 572	10.111 939	1.428	0.811	42.96	2.18	6.30	+3.324	364	8.2	3.07	1.62	20.6	10.9	11.4	6.0	2.03

Network graphs



Supplementary information

Datum definition	:	1	2	3	4	5	6	7
Dynamic coordinates	:	0	0	0	0	0	6	8
Observed directions	:	12	12	12	12	12	12	12
Orientation unknowns	:	4	4	4	4	4	4	4
Coordinate unknowns	:	4	4	8	8	2	8	8
Datum defect	:	4	4	4	4	4	0	0
Datum definition	:	fix	fix	free	free	fix	dynamic	dynamic
Number of datum constraints	:	4	4	4	4	6	0	0
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.9	0.9	0.9	0.9	1.8	1.8	2.8
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29	3.29	3.29	3.29	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1	17.1	17.1	17.1	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.0	2.0	2.0	2.0	2.3	2.3	2.5
Redundancy r	:	4	4	4	4	6	6	8
Redundancy dynamic coordinates	:	0.00	0.00	0.00	0.00	0.00	0.00	3.01
Redundancy directions	:	4.00	4.00	4.00	4.00	6.00	6.00	4.99
Weighted square sum of residuals Ω [mgon^2]	:	6.4265	6.4265	6.4265	6.4265	15.755	15.755	9.2272
(a priori) standard deviation σ_0 [mgon]	:	1	1	1	1	1	1	1
(a posteriori) standard deviation $\hat{\sigma}_0$ [mgon]	:	1.2675	1.2675	1.2675	1.2675	1.6204	1.6204	1.074
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.2675	1.2675	1.2675	1.2675	1.6204	1.6204	1.0740
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	1.6066	1.6066	1.6066	1.6066	2.6258	2.6258	1.1534
Critical value $k^F_{\alpha_G;r,\infty}$	:	3.38	3.38	3.38	3.38	2.56	2.56	2.15
Number of outliers (Data snooping)	:	0	0	0	0	1	1	0
Number of outliers (τ-criterion)	:	0	0	0	0	0	0	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	3.728	5.385	2.128	2.184	1.454	1.454	1.355
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	7.26625	16.99917	2.46623	3.89963	3.00444	3.00444	5.75309

3.3 Triangulation networks

3.3.1 Ghilani (2010), Ex. 15.4

Ghilani Charles D. (2010): Adjustment Computations. Spatial Data Analysis. Fifth Edition, John Wiley & Sons, Inc., ISBN 978-0-470-46491-5, Ex. 15.4, pp. 271

Available data files: [2D] Ghilani15_4_Angle_fix*.*

Coordinates

Point name	Easting x [m]	Northing y [m]
R	865.4000 (D)	4527.1500 (D)
S	2432.5500 (D)	2047.2500 (D)
T	2865.2200 (D)	27.1500 (D)
U	6861.3500	3727.5900

Datum: fix, (D)...Datum coordinate

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]
R	U	S	55.682 098 8	1	1
S	R	U	112.792 284 0	1	1
	U	T	109.653 395 1	1	1
T	S	U	65.870 679 0	1	1

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ angles (1. iteration)

A	x_R	y_R	x_S	y_S	x_T	y_T	x_U	y_U	Δy
$\alpha_{R,U,S}$	16.9540	1.1610	−18.3451	−11.5930	0.0000	0.0000	1.3911	10.4320	−0.0734
$\alpha_{S,R,U}$	−18.3451	−11.5930	13.5775	24.1587	0.0000	0.0000	4.7676	−12.5657	−0.0660
$\alpha_{S,U,T}$	0.0000	0.0000	34.8996	−6.1119	−30.1320	−6.4537	−4.7676	12.5657	−0.2126
$\alpha_{T,S,U}$	0.0000	0.0000	−30.1320	−6.4537	22.1900	15.0303	7.9420	−8.5766	−6.2453

Matrix $D^T_{[\text{-}]}$ of datum constraints

	x_R	y_R	x_S	y_S	x_T	y_T	x_U	y_U
D^T	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	1	0	0	0	0	0
	0	0	0	1	0	0	0	0
	0	0	0	0	1	0	0	0
	0	0	0	0	0	1	0	0

Least squares solution $\widehat{\Delta x}_{[\mathrm{cm}]}$ (1. iteration)

	x_R	y_R	x_S	y_S	x_T	y_T	x_U	y_U
$\widehat{\Delta x}^T$	0	0	0	0	0	0	−62.40	−11.50

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
U	6860.7260	−62.397	37.817	3727.4751	−11.494	17.809	41.801

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
U	40.25	11.27	76.767 706

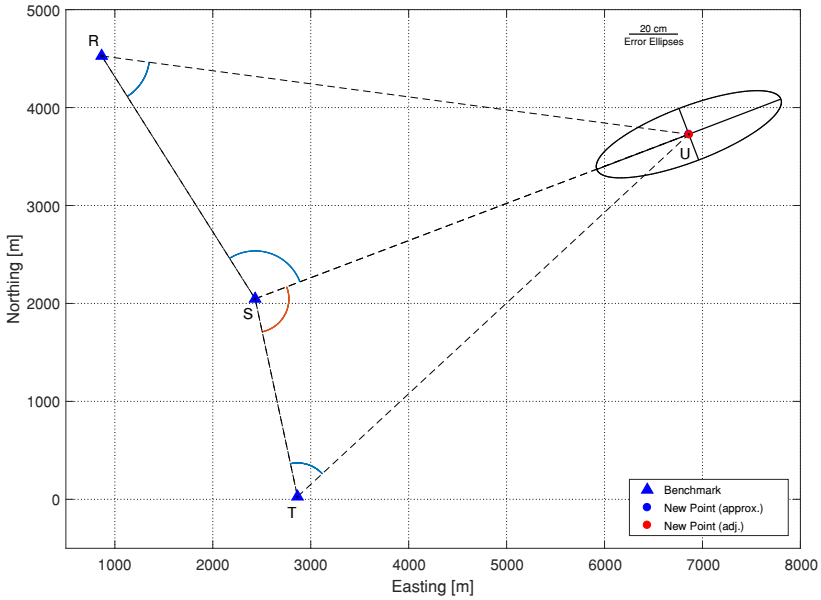
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
U	248.14	69.46	76.767 706

Adjusted horizontal angles

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
R	U	S	55.680 105	1.994	2.275	27.81	3.78	7.84	+7.17#	6.66	6.09	5.7	5.2	1.41
S	R	U	112.790 819	1.465	1.515	68.00	1.78	5.01	+2.15	2.83	1.22	1.6	0.7	0.66
	U	T	109.655 138	−1.743	1.515	68.00	2.11	5.01	−2.56	2.83	1.45	1.6	0.8	0.79
T	S	U	65.872 954	−2.275	2.138	36.20	3.78	6.87	−6.28*	5.49	5.02	4.4	4.0	1.41

Network graph



Supplementary information

Observed angles	:	4
Coordinate unknowns	:	2
Datum defect	:	4
Datum definition	:	fix
Number of datum constraints	:	6
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.4
Number of iterations (Max=20)	:	4
Stop criterion (actual)	:	$4.9 \cdot 10^{-12}$
Redundancy r	:	2
Redundancy angles	:	2.00
Weighted square sum of residuals Ω [mgon^2]	:	14.336
(a priori) standard deviation σ_0 [mgon]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	2.6773
Ratio $\hat{\sigma}_0/\sigma_0$	:	2.6773
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	7.1681 ($k^F_{\alpha_G;r,\infty}$ =5.87)
Number of outliers (Data snooping)	:	2 (Remove outliers or scale standard deviations by the factor 2.68)
Number of outliers (τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	63.447
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	1747.29

3.3.2 Ghilani (2010), Ex. 15.5

Ghilani Charles D. (2010): Adjustment Computations. Spatial Data Analysis. Fifth Edition, John Wiley & Sons, Inc., ISBN 978-0-470-46491-5, Ex. 15.5, pp. 277

Available data files: [2D] Ghilani15_5_Angle_fix∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
P	1303.5990 (D)	1458.6150 (D)
Q	1636.4360 (D)	1310.4680 (D)
R	1503.3950 (D)	888.3620 (D)
S	1506.2620 (D)	785.0610 (D)
U	1000.0300	999.9600

Datum: fix, (D)...Datum coordinate

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
U	P	Q	33.880 555 6	1.5432	$1.701\,81\cdot10^{9}$
	Q	R	42.787 345 7	1.8519	$1.181\,81\cdot10^{9}$
	R	S	11.665 740 7	1.8519	$1.181\,81\cdot10^{9}$

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ angles (1. iteration)

A	x_P	y_P	x_Q	y_Q	x_R	y_R	x_S	y_S	x_U	y_U	Δy
$\alpha_{U,P,Q}$	−96.5193	63.8830	39.4225	−80.7990	0.0000	0.0000	0.0000	0.0000	57.0968	16.9160	−0.06
$\alpha_{U,Q,R}$	0.0000	0.0000	−39.4225	80.7990	−26.7259	−120.5476	0.0000	0.0000	66.1484	39.7486	−0.05
$\alpha_{U,R,S}$	0.0000	0.0000	0.0000	0.0000	26.7259	120.5476	−45.2332	−106.5547	18.5073	−13.9929	−2.10

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_P	y_P	x_Q	y_Q	x_R	y_R	x_S	y_S	x_U	y_U
D^{T}	1	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	1	0	0

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (1. iteration)

	x_P	y_P	x_Q	y_Q	x_R	y_R	x_S	y_S	x_U	y_U
$\widehat{\Delta x}^{\mathsf{T}}$	0	0	0	0	0	0	0	0	−3.111	6.530

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
U	999.9989	−3.110	2.057	1000.0253	6.530	4.268	4.738

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
U	4.62	1.06	174.372 354

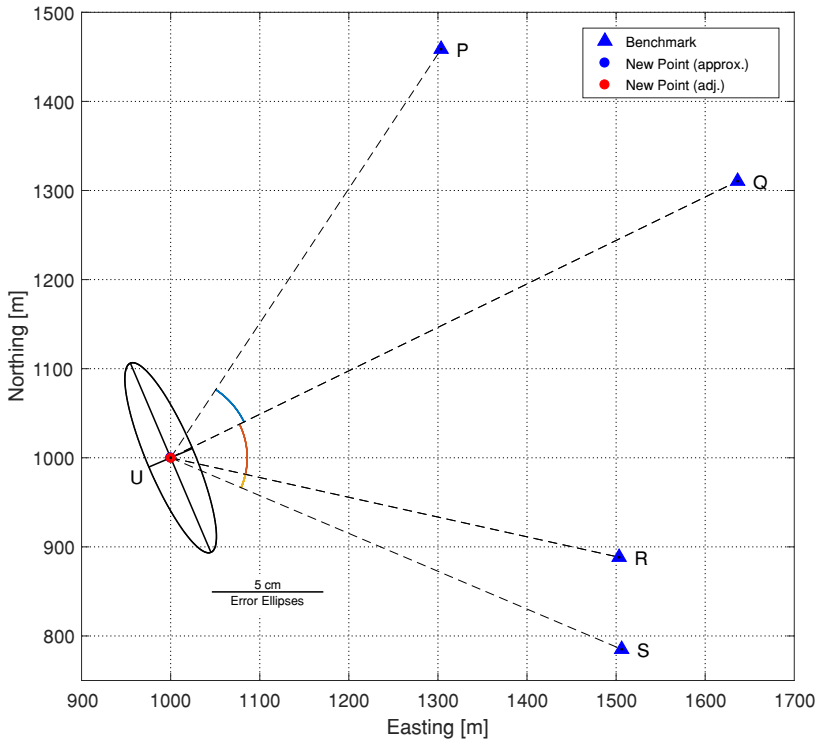
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
U	92.21	21.25	174.372 354

Adjusted horizontal angles

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
U	P	Q	33.879 947	0.609	0.704	42.80	0.60	9.75	+1.42	4.78	0.70	5.6	0.8	1.00
	Q	R	42.787 933	−0.587	0.950	27.62	0.60	14.56	−2.12	6.69	0.98	10.5	1.5	1.00
	R	S	11.666 348	−0.607	0.937	29.58	0.60	14.07	−2.05	6.38	0.93	9.9	1.4	1.00

Network graph



Supplementary information

Observed angles	:	3
Coordinate unknowns	:	2
Datum defect	:	4
Datum definition	:	fix
Number of datum constraints	:	8
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.1
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	∞
Number of iterations (Max=20)	:	4
Stop criterion (actual)	:	$5 \cdot 10^{-13}$
Redundancy r	:	1
Redundancy angles	:	1.00
Weighted square sum of residuals Ω [-]	:	$3.63606 \cdot 10^{-1}$
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	$6.02998 \cdot 10^{-1}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.6030
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.3636 ($k^F_{\alpha_G;r,\infty}$ =10.83)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	7.233
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	22.444

3.4 Distance-Direction networks

3.4.1 Benning (2011), Ex. 8-3

Benning (2011): Statistik in Geodäsie, Geoinformation und Bauwesen, Wichmann. Ex. 8-3, pp. 258

Available data files: [2D] Benning83_DistanceDirection_fix∗.∗

In order to demonstrate the impact of an unknown scale factor in the distance observations, the example has been processed first without, in the second run including the scale factor. Although the F-test indicates a non-significant deviation from one ($T_F = 2.7 < k^F_{1-\alpha_K;1,4} = 7.7$), the results are rather different.

Coordinates

Point name	Easting x [m]	Northing y [m]
1	0.0000 (D)	1000.0000 (D)
2	1000.0000 (D)	1000.0000 (D)
3	0.0000	0.0000
4	1000.0000	0.0000

Datum: fix, (D)...Datum coordinate

Approximate orientations

Point name	ω [gon]
1	150.000 000
2	200.000 000
3	0.000 000

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
1	3	1000.0200	10	1
	4	1414.2000	10	1
2	3	1414.2400	10	1
	4	999.9800	10	1
3	4	1000.0000	10	1

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [m²/rad²]	ω [gon]
1	4	0.000 000	1	405 284.734 57	150
	3	50.001 000	1	405 284.734 57	
2	4	0.000 000	1	405 284.734 57	200
	3	49.998 000	1	405 284.734 57	
3	1	0.000 000	1	405 284.734 57	0
	2	49.999 000	1	405 284.734 57	
	4	99.997 000	1	405 284.734 57	

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3	Δy
$r_{1,4}$	31.8310	31.8310	0.0000	0.0000	0.0000	0.0000	−31.8310	−31.8310	−1	0	0	0.00
$r_{1,3}$	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0.0000	0.0000	−1	0	0	1.00
$r_{2,4}$	0.0000	0.0000	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0	−1	0	0.00
$r_{2,3}$	0.0000	0.0000	31.8310	−31.8310	−31.8310	31.8310	0.0000	0.0000	0	−1	0	−2.00
$r_{3,1}$	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0.0000	0.0000	0	0	−1	0.00
$r_{3,2}$	0.0000	0.0000	31.8310	−31.8310	−31.8310	31.8310	0.0000	0.0000	0	0	−1	−1.00
$r_{3,4}$	0.0000	0.0000	0.0000	0.0000	0.0000	63.6620	0.0000	−63.6620	0	0	−1	−3.00

3.4.1.1 No scale factor in distance observations

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ distances (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	Δy
$s_{1,3}$	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	0.0000	2.000
$s_{1,4}$	−0.7071	0.7071	0.0000	0.0000	0.0000	0.0000	0.7071	−0.7071	−1.356
$s_{2,3}$	0.0000	0.0000	0.7071	0.7071	−0.7071	−0.7071	0.0000	0.0000	2.644
$s_{2,4}$	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	−2.000
$s_{3,4}$	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	1.0000	0.0000	0.000

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3
D^{T}	1	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m,mgon}]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3
$\widehat{\Delta x}^{\mathsf{T}}$	0	0	0	0	−0.0101	−0.0231	−0.0096	0.0163	−0.286	1.097	0.571

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
3	−0.0101	−1.009	0.563	−0.0231	−2.314	0.409	0.695
4	999.9904	−0.959	0.570	0.0163	1.633	0.395	0.694

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
3	0.62	0.32	132.301 779
4	0.62	0.32	70.695 639

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
3	2.11	1.08	132.301 779
4	2.10	1.08	70.695 639

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
1	149.999 714	−0.29	0.44
2	200.001 097	1.10	0.44
3	0.000 571	0.57	0.41

3.4.1.2 Including an unknown scale factor in distance observations

Design matrix $A_{[-,m]}$ and reduced observation vector $\Delta y_{[cm]}$ distances (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	m	Δy
$s_{1,3}$	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	0.0000	1000.0000	2.000
$s_{1,4}$	−0.7071	0.7071	0.0000	0.0000	0.0000	0.0000	0.7071	−0.7071	1414.2136	−1.356
$s_{2,3}$	0.0000	0.0000	0.7071	0.7071	−0.7071	−0.7071	0.0000	0.0000	1414.2136	2.644
$s_{2,4}$	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	1000.0000	−2.000
$s_{3,4}$	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	1.0000	0.0000	1000.0000	0.000

Matrix $D^{\mathsf{T}}{}_{[-,m]}$ of datum constraints

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3	m
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[m,mgon,-]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3	m
$\widehat{\Delta x}^{\mathsf{T}}$	0	0	0	0	−0.0048	−0.0098	−0.0166	0.0298	−0.556	1.449	0.542	$1.166 \cdot 10^{-5}$

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
3	−0.0048	−0.482	0.584	−0.0098	−0.983	0.888	1.063
4	999.9834	−1.661	0.654	0.0298	2.976	0.891	1.106

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
3	0.92	0.53	20.872 545
4	0.97	0.53	168.927 254

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
3	3.43	1.98	20.872 545
4	3.61	1.99	168.927 254

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
1	149.999 444	−0.56	0.41
2	200.001 449	1.45	0.44
3	0.000 542	0.54	0.35

Adjusted scale ($1 - \alpha_K = 95\%$)

$\widehat{m}$ [−]	$\widehat{m} - m$ [ppm]	$\lvert\hat{\sigma}\rvert$ [ppm]	T_F	$k^F_{\alpha_K;1,4}$
1.000 011 662 68	11.7	7.1	2.7	7.7

Adjusted directions (No scale unknown)

Variance component: $\Omega/\sigma_0^2 = 0.497,\ r = 3.42,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.15,\ \alpha_G = 0.69\%,\ k^F_{\alpha_G;r,\infty} = 3.80$

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	3	50.000 928	200.000 642	0.072	0.345	43.16	0.11	6.29	+0.166	1000	1.1	1.64	0.04	56.2	1.5	13.5	0.4	0.24
	4	0.000 072	149.999 786	−0.072	0.345	43.16	0.11	6.29	−0.166	1414	1.6	1.64	0.04	79.4	2.1	0.0	0.0	0.24
2	3	49.998 487	249.999 584	−0.487	0.347	42.39	0.75	6.35	−1.149	1414	10.8	1.75	0.32	81.2	14.7	0.9	0.2	1.64
	4	399.999 513	200.000 611	0.487	0.347	42.39	0.75	6.35	+1.149	1000	7.6	1.75	0.32	57.4	10.4	14.5	2.6	1.64
3	1	0.000 071	0.000 642	−0.071	0.302	56.42	0.09	5.50	−0.125	1000	1.1	1.76	0.04	37.7	0.9	10.8	0.2	0.21
	2	49.999 013	49.999 584	−0.013	0.277	63.30	0.02	5.19	−0.021	1414	0.3	0.95	0.00	42.3	0.2	4.7	0.0	0.04
	4	99.996 916	99.997 488	0.084	0.320	51.18	0.12	5.78	+0.164	1000	1.3	2.27	0.06	44.3	1.3	18.8	0.5	0.26

Adjusted horizontal distances (No scale unknown)

Variance component: $\Omega/\sigma_0^2 = 0.550,\ r = 1.58,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.35,\ \alpha_G = 0.20\%,\ k^F_{\alpha_G;r,\infty} = 7.18$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	3	1000.0231	−0.314	0.409	20.3	0.7	9.2	−1.550	8.2	1.4	73.2	12.4	73.2	12.4	1.53
	4	1414.1952	0.476	0.360	38.0	0.8	6.7	+1.253	5.3	1.0	41.6	7.8	41.6	7.8	1.69
2	3	1414.2371	0.294	0.348	42.1	0.5	6.4	+0.700	4.9	0.5	36.9	4.1	36.9	4.1	0.99
	4	999.9837	−0.367	0.395	25.3	0.7	8.2	−1.452	7.1	1.3	61.4	10.8	61.4	10.8	1.60
3	4	1000.0005	−0.050	0.376	32.4	0.1	7.3	−0.153	6.0	0.1	49.1	1.0	49.1	1.0	0.19

Adjusted directions (with scale unknown)

Variance component: $\Omega/\sigma_0^2 = 0.493$, $r = 2.94$, $\hat{\sigma}_0^2/\sigma_0^2 = 0.17$, $\alpha_G = 0.53\%$, $k^F_{\alpha_G;r,\infty} = 4.28$

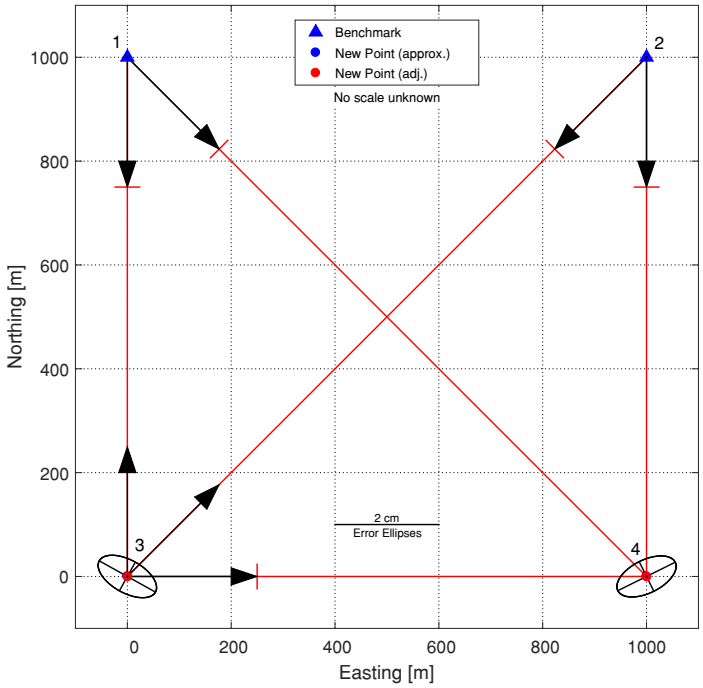
in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	4	0.000 137	149.999 581	−0.137	0.301	42.14	0.21	6.37	−0.325	1414	3.0	1.78	0.09	81.8	4.2	4.5	0.2	0.53
	3	50.000 863	200.000 307	0.137	0.301	42.14	0.21	6.37	+0.325	1000	2.2	1.78	0.09	57.8	3.0	18.9	1.0	0.53
2	4	399.999 608	200.001 057	0.392	0.306	40.22	0.62	6.52	+0.974	1000	6.2	2.04	0.30	61.2	9.1	25.3	3.8	1.56
	3	49.998 392	249.999 840	−0.392	0.306	40.22	0.62	6.52	−0.974	1414	8.7	2.04	0.30	86.5	12.9	7.5	1.1	1.56
3	1	399.999 765	0.000 307	0.235	0.322	34.07	0.40	7.08	+0.690	1000	3.7	4.04	0.39	73.3	7.1	41.1	4.0	1.02
	2	49.999 298	49.999 840	−0.298	0.297	43.88	0.45	6.24	−0.680	1414	6.6	2.98	0.32	77.8	8.5	18.5	2.0	1.14
	4	99.996 937	99.997 479	0.063	0.277	51.08	0.09	5.78	+0.124	1000	1.0	2.28	0.05	44.4	0.9	18.8	0.4	0.22

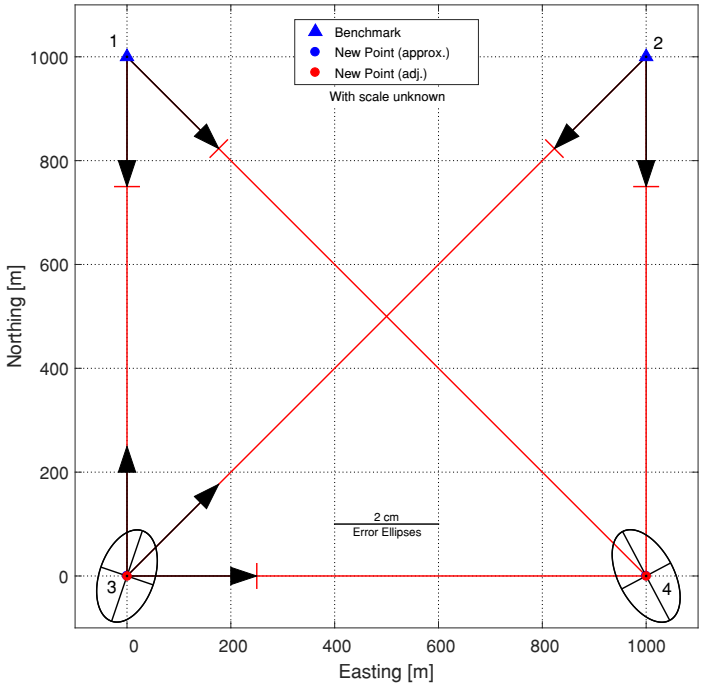
Adjusted horizontal distances (with scale unknown)

Variance component: $\Omega/\sigma_0^2 = 0.135$, $r = 1.06$, $\hat{\sigma}_0^2/\sigma_0^2 = 0.13$, $\alpha_G = 0.11\%$, $vk^F_{\alpha_G;r,\infty} = 10.24$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	3	1000.0215	−0.150	0.368	13.8	0.4	11.1	−1.084	9.4	0.9	95.9	9.3	146.7	14.3	1.02
	4	1414.1973	0.273	0.336	28.2	0.5	7.8	+0.971	5.1	0.6	55.9	7.0	6.3	0.8	1.30
2	3	1414.2404	−0.042	0.365	15.0	0.1	10.7	−0.279	8.0	0.2	90.6	2.4	50.7	1.3	0.27
	4	999.9819	−0.190	0.359	17.8	0.5	9.8	−1.068	8.1	0.9	80.6	8.8	129.0	14.1	1.14
3	4	999.9999	0.012	0.328	31.5	0.0	7.4	+0.039	5.4	0.0	50.5	0.3	63.2	0.3	0.06

Network graphs





Supplementary information

Observed directions	:	7	7
Observed distances	:	5	5
Orientation unknowns	:	3	3
Coordinate unknowns	:	4	4
Datum defect	:	3	4
Datum definition	:	fix	fix
Number of datum constraints	:	4	4
Scale unknown	:	no	yes
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	1.3	0.9
Power of test γ [%] (Baarda)	:	80.0	80.0
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.2	2.0
Number of iterations (Max=20)	:	3	3
Stop criterion (actual)	:	$1.1 \cdot 10^{-12}$	$8.2 \cdot 10^{-12}$
Stop criterion (target)	:	$1 \cdot 10^{-10}$	$1 \cdot 10^{-10}$
Redundancy r	:	5	4
Redundancy directions	:	3.42	2.94
Redundancy distances	:	1.58	1.06
Redundancy (Check)	:	5.00	4.00
Weighted square sum of residuals Ω [m^2]	:	$1.04634 \cdot 10^{-4}$	$6.27657 \cdot 10^{-5}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-2}$	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$4.57458 \cdot 10^{-3}$	$3.96124 \cdot 10^{-3}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.4575	0.3961
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	0.2093	0.1569
Critical value $k^F_{\alpha_G;r,\infty}$	:	2.89	3.38
Number of outliers (Data snooping)	:	0	0
Number of outliers (τ-criterion)	:	0	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	3.155	3.580
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.96492	2.35207
Trace coordinate cofactor matrix, tr$Q_{\widehat{X}}$	:	4.61094	14.98953

3.4.2 Benning (2011), Ex. 8-5

Benning (2011): Statistik in Geodäsie, Geoinformation und Bauwesen, Wichmann. Ex. 8-5, pp. 273

Available data files: [2D] Benning85_DistanceDirection_free∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
1	0.0000 (D)	1000.0000 (D)
2	1000.0000 (D)	1000.0000 (D)
3	0.0000 (D)	0.0000 (D)
4	1000.0000 (D)	0.0000 (D)

Datum: free, (D)...Datum coordinate

Approximate orientations

Point name	ω [gon]
1	150
2	200
3	0

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
1	3	1000.0200	10	1
	4	1414.2000	10	1
2	3	1414.2400	10	1
	4	999.9800	10	1
3	4	1000.0000	10	1

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [m^2/rad^2]	ω [gon]
1	4	0.000 000	1	405 284.734 57	150
	3	50.001 000	1	405 284.734 57	
2	4	0.000 000	1	405 284.734 57	200
	3	49.998 000	1	405 284.734 57	
3	1	0.000 000	1	405 284.734 57	0
	2	49.999 000	1	405 284.734 57	
	4	99.997 000	1	405 284.734 57	

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3	Δy
$r_{1,4}$	31.8310	31.8310	0.0000	0.0000	0.0000	0.0000	−31.8310	−31.8310	−1	0	0	0.00
$r_{1,3}$	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0.0000	0.0000	−1	0	0	1.00
$r_{2,4}$	0.0000	0.0000	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0	−1	0	0.00
$r_{2,3}$	0.0000	0.0000	31.8310	−31.8310	−31.8310	31.8310	0.0000	0.0000	0	−1	0	−2.00
$r_{3,1}$	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0.0000	0.0000	0	0	−1	0.00
$r_{3,2}$	0.0000	0.0000	31.8310	−31.8310	−31.8310	31.8310	0.0000	0.0000	0	0	−1	−1.00
$r_{3,4}$	0.0000	0.0000	0.0000	0.0000	0.0000	63.6620	0.0000	−63.6620	0	0	−1	−3.00

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ distances (1. iteration)

A	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	Δy
$s_{1,3}$	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	0.0000	2.000
$s_{1,4}$	−0.7071	0.7071	0.0000	0.0000	0.0000	0.0000	0.7071	−0.7071	−1.356
$s_{2,3}$	0.0000	0.0000	0.7071	0.7071	−0.7071	−0.7071	0.0000	0.0000	2.644
$s_{2,4}$	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	−2.000
$s_{3,4}$	0.0000	0.0000	0.0000	0.0000	−1.0000	0.0000	1.0000	0.0000	0.000

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3
D^{T}	1	0	1	0	1	0	1	0	0	0	0
	0	1	0	1	0	1	0	1	0	0	0
	−500	−500	−500	500	500	−500	500	500	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m,mgon}]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	ω_1	ω_2	ω_3
$\widehat{\Delta x}^{\mathsf{T}}$	0.0018	0.0031	0.0135	−0.0014	−0.0076	−0.0184	−0.0077	0.0167	−0.267	1.738	0.831

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
1	0.0018	0.180	0.354	1000.0031	0.312	0.214	0.413
2	1000.0135	1.346	0.382	999.9986	−0.142	0.203	0.432
3	−0.0076	−0.757	0.180	−0.0184	−1.838	0.194	0.265
4	999.9923	−0.769	0.193	0.0167	1.668	0.197	0.276

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
1	0.36	0.21	89.732 495
2	0.38	0.20	103.498 066
3	0.20	0.18	16.900 809
4	0.21	0.17	153.509 110

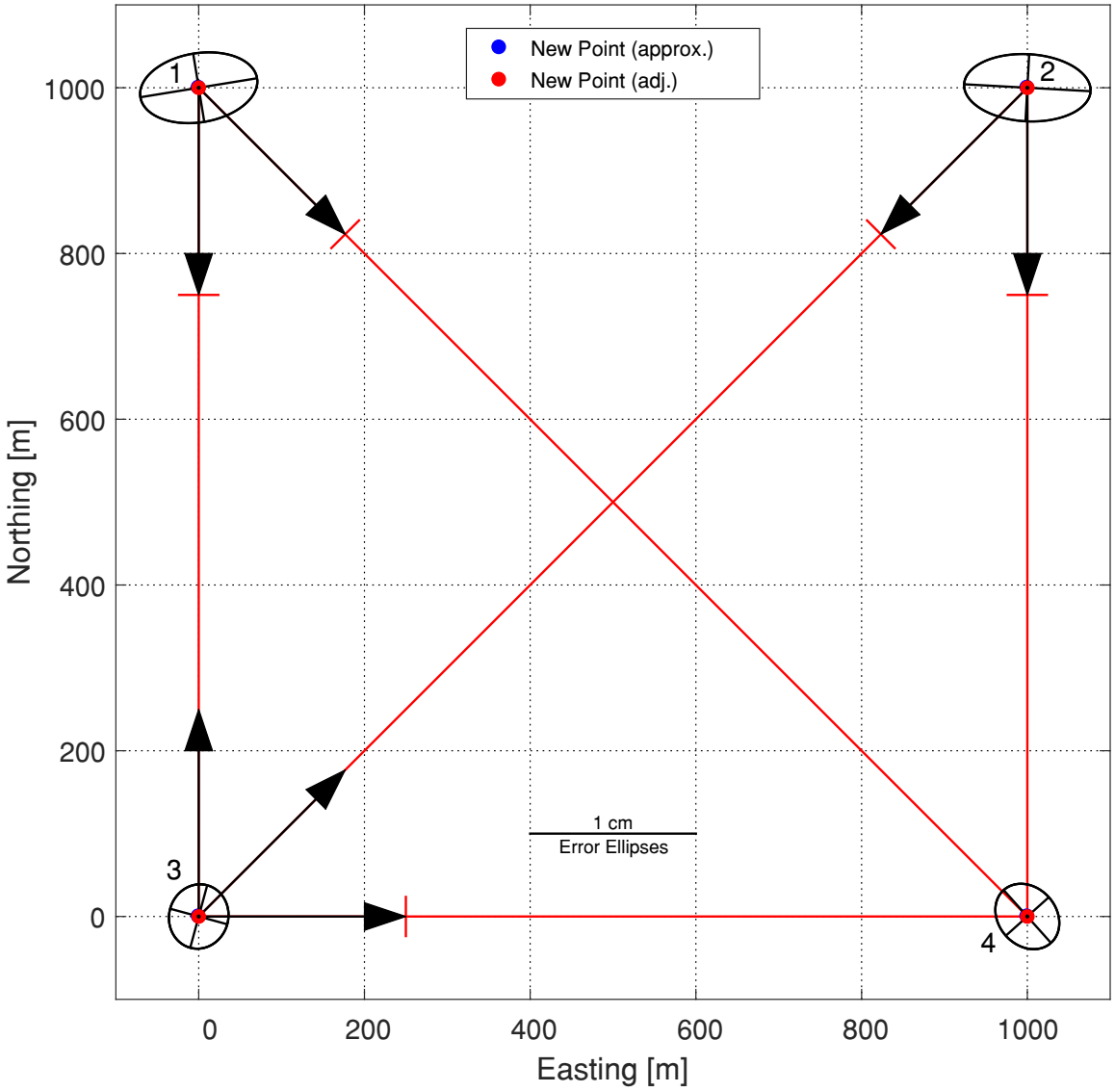
Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
1	1.33	0.78	89.732 495
2	1.42	0.75	103.498 066
3	0.73	0.67	16.900 809
4	0.80	0.65	153.509 110

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
1	149.999 733	−0.27	0.34
2	200.001 738	1.74	0.35
3	0.000 831	0.83	0.25

Network graph



Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 0.493,\ r = 2.94,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.17,\ \alpha_G = 0.53\%,\ k^F_{\alpha_G;r,\infty} = 4.28$

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	4	0.000 137	149.999 870	−0.137	0.301	42.14	0.21	6.37	−0.325	1414	3.0	1.78	0.09	81.8	4.2	1.7	0.1	0.53
	3	50.000 863	200.000 596	0.137	0.301	42.14	0.21	6.37	+0.325	1000	2.2	1.78	0.09	57.9	3.0	14.5	0.7	0.53
2	4	399.999 608	200.001 346	0.392	0.306	40.22	0.62	6.52	+0.974	1000	6.2	2.04	0.30	61.2	9.1	20.2	3.0	1.56
	3	49.998 392	250.000 130	−0.392	0.306	40.22	0.62	6.52	−0.974	1414	8.7	2.04	0.30	86.5	12.9	0.2	0.0	1.56
3	1	399.999 765	0.000 596	0.235	0.322	34.07	0.40	7.08	+0.690	1000	3.7	4.04	0.39	73.3	7.1	40.1	3.9	1.02
	2	49.999 298	50.000 130	−0.298	0.297	43.88	0.45	6.24	−0.680	1414	6.6	2.98	0.32	77.8	8.5	24.9	2.7	1.14
	4	99.996 937	99.997 768	0.063	0.277	51.08	0.09	5.78	+0.124	1000	1.0	2.28	0.05	44.4	0.9	15.4	0.3	0.22

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 0.135,\ r = 1.06,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.13,\ \alpha_G = 0.11\%,\ k^F_{\alpha_G;r,\infty} = 10.24$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	3	1000.0215	−0.150	0.368	13.8	0.4	11.1	−1.084	10.3	1.0	95.9	9.3	95.9	9.3	1.02
	4	1414.1973	0.273	0.336	28.2	0.5	7.8	+0.971	6.6	0.8	55.9	7.0	55.9	7.0	1.30
2	3	1414.2404	−0.042	0.365	15.0	0.1	10.7	−0.279	9.8	0.3	90.6	2.4	90.6	2.4	0.27
	4	999.9819	−0.190	0.359	17.8	0.5	9.8	−1.068	8.9	1.0	80.6	8.8	80.6	8.8	1.14
3	4	999.9999	0.012	0.328	31.5	0.0	7.4	+0.039	6.1	0.0	50.5	0.3	50.5	0.3	0.06

Supplementary information

Observed directions	:	7
Observed distances	:	5
Orientation unknowns	:	3
Coordinate unknowns	:	8
Datum defect	:	3
Datum definition	:	free
Number of datum constraints	:	3
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.9
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.0
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$7.3 \cdot 10^{-13}$
Stop criterion (target)	:	$1 \cdot 10^{-10}$
Redundancy r	:	4
Redundancy directions	:	2.94
Redundancy distances	:	1.06
Redundancy (Check)	:	4.00
Weighted square sum of residuals Ω [m^2]	:	$6.27657 \cdot 10^{-5}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$3.96124 \cdot 10^{-3}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.3961
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.1569 ($k^F_{\alpha_G;r,\infty}$ =3.38)
Number of outliers (Data snooping)	:	0
Number of outliers (τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	3.047
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.50375
Trace coordinate cofactor matrix, tr$Q_{\widehat{X}}$	:	3.21033

3.4.3 Carosio A (1983)

Carosio A (1983): Verfahren der multivariaten Statistik zur Beurteilung der Resultate und der Zuverlässigkeit geodätischer Messsysteme. Institut für Geodäsie und Photogrammetrie. ETH Zürich, Mitteilungen Nr. 35, pp. 65-71

Available data files: [2D] Carosio_DistanceDirection_fix∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
A	−1000.0000 (D)	100.0000 (D)
B	100.0000	1000.0000
C	1000.0000 (D)	100.0000 (D)
P	100.0000 (D)	0.0000 (D)

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [−]
B	A	1421.2730	10	1
	C	1272.7993	10	1
	P	1000.0098	10	1

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [m²/rad²]	ω [gon]
A	B	56.344 760	5	16 211.389 38	0.000 171
	P	105.771 590	5	16 211.389 38	
B	C	150.000 340	5	16 211.389 38	0.000 008
	P	199.999 980	5	16 211.389 38	
	A	256.344 760	5	16 211.389 38	
C	P	292.955 340	5	16 211.389 38	399.999 831
	B	350.000 340	5	16 211.389 38	
P	C	92.955 340	5	16 211.389 38	0.000 007
	A	305.771 590	5	16 211.389 38	
	B	399.999 980	5	16 211.389 38	

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_P	y_P	ω_A	ω_B	ω_C	ω_P	Δy
$r_{A,B}$	−28.3642	34.6674	28.3642	−34.6674	0.0000	0.0000	0.0000	0.0000	−1	0	0	0	−0.17
$r_{A,P}$	5.2182	57.4001	0.0000	0.0000	0.0000	0.0000	−5.2182	−57.4001	−1	0	0	0	0.17
$r_{B,C}$	0.0000	0.0000	35.3678	35.3678	−35.3678	−35.3678	0.0000	0.0000	0	−1	0	0	0.35
$r_{B,P}$	0.0000	0.0000	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0	−1	0	0	−0.01
$r_{B,A}$	−28.3642	34.6674	28.3642	−34.6674	0.0000	0.0000	0.0000	0.0000	0	−1	0	0	−0.34
$r_{C,P}$	0.0000	0.0000	0.0000	0.0000	7.7637	−69.8729	−7.7637	69.8729	0	0	−1	0	−0.17
$r_{C,B}$	0.0000	0.0000	35.3678	35.3678	−35.3678	−35.3678	0.0000	0.0000	0	0	−1	0	0.17
$r_{P,C}$	0.0000	0.0000	0.0000	0.0000	7.7637	−69.8729	−7.7637	69.8729	0	0	0	−1	0.00
$r_{P,A}$	5.2182	57.4001	0.0000	0.0000	0.0000	0.0000	−5.2182	−57.4001	0	0	0	−1	0.01
$r_{P,B}$	0.0000	0.0000	63.6620	0.0000	0.0000	0.0000	−63.6620	0.0000	0	0	0	−1	−0.01

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{mm}]}$ distances (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_P	y_P	Δy
$s_{B,A}$	−0.7740	−0.6332	0.7740	0.6332	0.0000	0.0000	0.0000	0.0000	5.96
$s_{B,C}$	0.0000	0.0000	−0.7071	0.7071	0.7071	−0.7071	0.0000	0.0000	7.09
$s_{B,P}$	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	−1.0000	9.80

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_A	y_A	x_B	y_B	x_C	y_C	x_P	y_P	ω_A	ω_B	ω_C	ω_P
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{m,mgon}]}$ (1. iteration)

	x_A	y_A	x_B	y_B	x_C	y_C	x_P	y_P	ω_A	ω_B	ω_C	ω_P
$\widehat{\Delta x}^{\mathsf{T}}$	0.0000	0.0000	−0.0003	0.0098	0.0000	0.0000	0.0000	0.0000	−0.173	−0.010	0.168	−0.006

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
B	99.9997	−0.028	0.001	1000.0098	0.978	0.001	0.002

Absolute error ellipses

in	A [mm]	B [mm]	ϕ [gon]
B	0.01	0.01	99.329 716

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [mm]	B [mm]	ϕ [gon]
B	0.04	0.03	99.329 716

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
A	399.999 997	−0.17	$5{\cdot}10^{-3}$
B	399.999 998	−0.01	$4{\cdot}10^{-3}$
C	399.999 999	0.17	$5{\cdot}10^{-3}$
P	0.000 001	−0.01	$4{\cdot}10^{-3}$

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 0.000,\ r = 5.98,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.00,\ \alpha_G = 1.76\%,\ k^F_{\alpha_G;r,\infty} = 2.56$

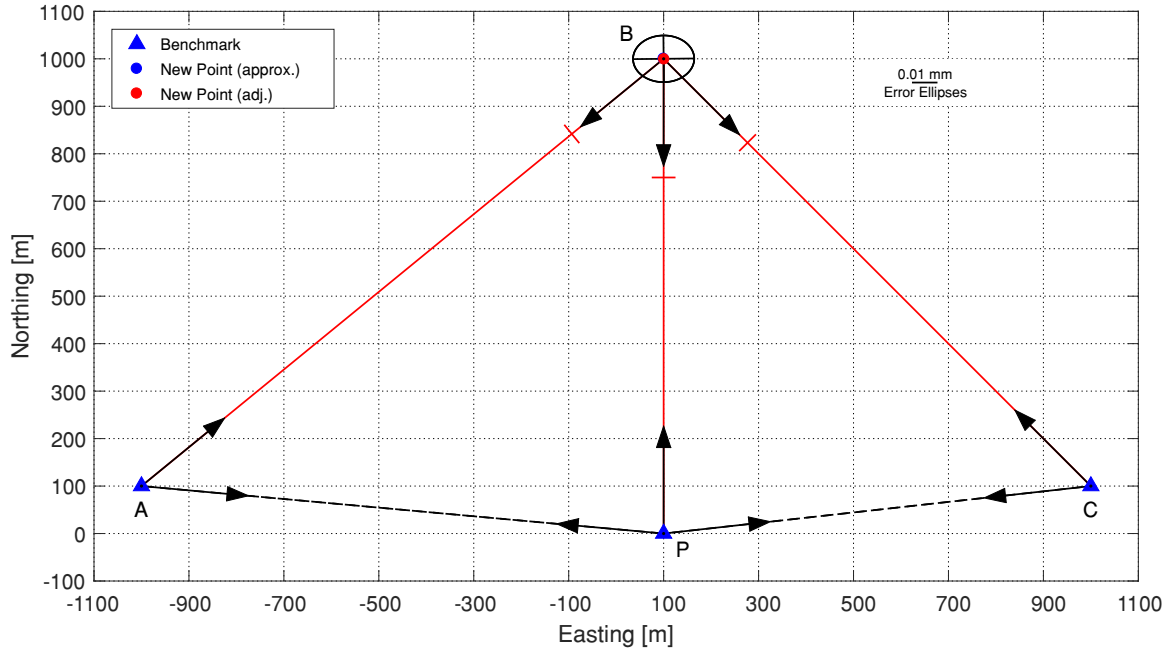
in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
B	A	256.344 758	256.344 757	0.002	0.004	66.34	0.00	25.37	+0.002	1421	0.0	0.29	0.00	190.6	0.0	0.6	0.0	0.30
	P	199.999 984	199.999 982	−0.004	0.004	66.51	0.00	25.33	−0.006	1000	0.1	0.20	0.00	133.3	0.0	1.9	0.0	0.71
	C	150.000 338	150.000 336	0.002	0.004	66.39	0.00	25.36	+0.003	1273	0.0	0.27	0.00	170.4	0.0	0.9	0.0	0.42
P	B	399.999 981	399.999 982	−0.001	0.004	66.02	0.00	25.43	−0.002	1000	0.0	0.41	0.00	135.7	0.0	3.9	0.0	0.26
	C	92.955 342	92.955 343	−0.002	0.004	66.51	0.00	25.33	−0.003	906	0.0	0.20	0.00	120.7	0.0	0.0	0.0	0.31
	A	305.771 587	305.771 588	0.003	0.004	66.51	0.00	25.33	+0.005	1105	0.1	0.20	0.00	147.2	0.0	0.0	0.0	0.56
A	B	56.344 759	56.344 757	0.001	0.005	49.87	0.00	29.26	+0.001	1421	0.0	0.21	0.00	327.5	0.0	1.8	0.0	0.11
	P	105.771 591	105.771 588	−0.001	0.005	49.87	0.00	29.26	−0.001	1105	0.0	0.21	0.00	254.5	0.0	0.0	0.0	0.11
C	P	292.955 343	292.955 343	−0.003	0.005	49.82	0.00	29.27	−0.007	906	0.0	0.25	0.00	208.9	0.0	0.0	0.0	0.68
	B	350.000 337	350.000 336	0.003	0.005	49.82	0.00	29.27	+0.007	1273	0.1	0.25	0.00	293.7	0.1	2.1	0.0	0.68

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 0.000,\ r = 1.02,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.00,\ \alpha_G = 0.10\%,\ k^F_{\alpha_G;r,\infty} = 10.60$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
B	A	1421.2730	−0.002	0.001	25.0	0.0	8.3	−0.006	7.1	0.0	61.9	0.0	61.9	0.0	2.32
	C	1272.7993	−0.002	0.001	29.5	0.0	7.6	−0.006	6.4	0.0	53.6	0.0	53.6	0.0	2.40
	P	1000.0098	0.002	0.001	47.8	0.0	6.0	+0.005	4.3	0.0	31.2	0.0	31.2	0.0	2.35

Network graph



Supplementary information (Adjustment Program Version: V21)

Observed directions	:	10
Observed distances	:	3
Orientation unknowns	:	4
Coordinate unknowns	:	2
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	6
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	2.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.4
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$1.8 \cdot 10^{-13}$
Redundancy r	:	7
Redundancy directions	:	5.98
Redundancy distances	:	1.02
Weighted square sum of residuals Ω [m^2]	:	$1.29735 \cdot 10^{-9}$
(a priori) standard deviation σ_0 [m]	:	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$1.36138 \cdot 10^{-5}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.0014
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	$2 \cdot 10^{-6}$ ($k^F_{\alpha_G;r,\infty}$ =2.32)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.978
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	$2.63 \cdot 10^{-6}$

3.5 Fix Distance-Direction network

Jäger R et al (2005): Klassische und robuste Ausgleichungsverfahren. Wichmann, Heidelberg, pp. 241

Coordinates

Point name	Easting x [m]	Northing y [m]
A	410.7800 (D)	380.1300 (D)
B	1183.4600 (D)	1762.6700 (D)
C	2077.0300 (D)	433.3800 (D)
D	1207.5700 (D)	124.6300 (D)
N	1175.1500	997.7200

Datum: fix, (D)...Datum coordinate

Approximate orientations

Point name	ω [gon]
N	63.561 000 0

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [rad²/m²]
N	A	982.6900	1	$6.168\,50 \cdot 10^{-5}$
	B	765.0000	1	
	C	1063.8900	1	

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
N	A	193.174 900	0.500	1	63.561 000
	B	337.130 400	0.500		
	C	72.034 400	0.500		
	D	134.075 800	0.500		

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_N	y_N	ω_N	Δy
$r_{N,A}$	−40.7144	50.3908	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	40.7144	−50.3908	−1	−0.48
$r_{N,B}$	0.0000	0.0000	83.2139	−0.9040	0.0000	0.0000	0.0000	0.0000	−83.2139	0.9040	−1	−0.16
$r_{N,C}$	0.0000	0.0000	0.0000	0.0000	−31.7414	−50.7263	0.0000	0.0000	31.7414	50.7263	−1	0.12
$r_{N,D}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−72.8153	−2.7038	72.8153	2.7038	−1	−0.36

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[mm]}$ horizontal distances (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_N	y_N	Δy
$s_{N,A}$	−0.7778	−0.6285	0.0000	0.0000	0.0000	0.0000	0	0	0.7778	0.6285	0.37
$s_{N,B}$	0.0000	0.0000	0.0109	0.9999	0.0000	0.0000	0	0	−0.0109	−0.9999	4.86
$s_{N,C}$	0.0000	0.0000	0.0000	0.0000	0.8477	−0.5304	0	0	−0.8477	0.5304	−2.46

Matrix $D^T{}_{[-]}$ of datum constraints (1. iteration)

	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_N	y_N	ω_N
D^T	1	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0

Least squares solution $\widehat{\Delta x}_{[mm,mgon]}$ (datum parameters removed, 1. iteration)

	x_N	y_N	ω_N
$\widehat{\Delta x}^T$	1.764	−3.478	0.2435

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	$\hat{y}$ [m]	$\hat{y} - y$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	$\lvert\hat{\sigma}_{2D}\rvert$ [mm]
N	1175.1518	1.764	1.081	997.7165	−3.478	0.975	1.456

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
N	0.108	0.097	109.448 377

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
N	0.404	0.362	109.448 377

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
N	63.561 243	0.24	0.32

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 2.290$, $r = 2.95$, $\hat{\sigma}_0^2/\sigma_0^2 = 0.78$, $\alpha_G = 0.53\%$, $k^F_{\alpha_G;r,\infty} = 4.27$

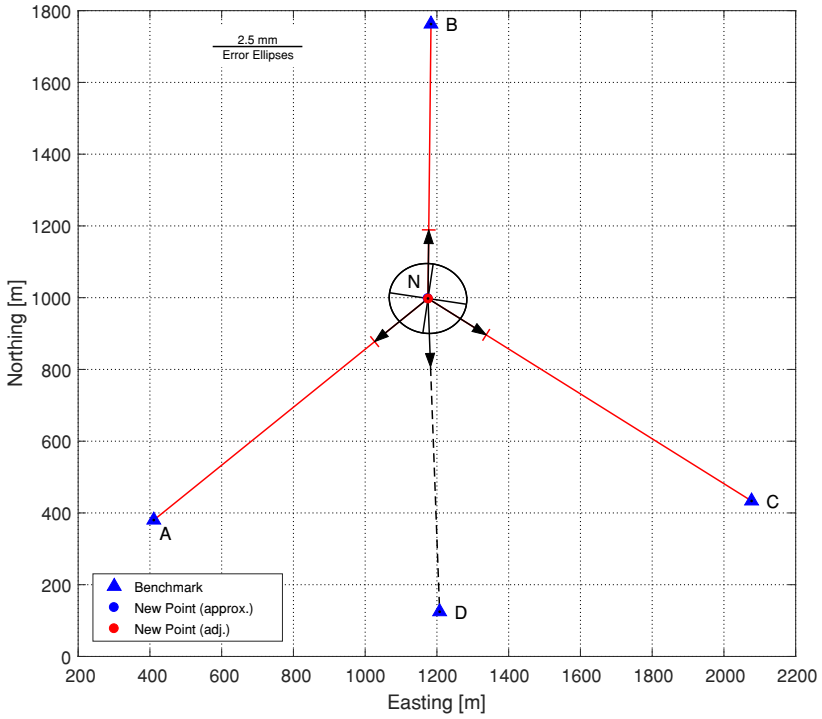
in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
N	A	193.175 381	256.736 624	−0.481	0.322	74.17	1.12	2.40	−0.648	983	7.4	0.44	0.12	9.6	2.6	0.3	0.1	0.88
	B	337.130 168	0.691 412	0.232	0.335	72.17	0.55	2.43	+0.321	765	2.8	0.82	0.11	8.1	1.1	0.7	0.1	0.43
	C	72.033 917	135.595 161	0.483	0.321	74.35	1.12	2.40	+0.649	1064	8.1	0.39	0.10	10.3	2.8	0.3	0.1	0.88
	D	134.076 034	197.637 277	−0.234	0.323	74.05	0.54	2.40	−0.316	874	3.2	0.47	0.06	8.6	1.1	0.4	0.1	0.43

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 4.152$, $r = 1.05$, $\hat{\sigma}_0^2/\sigma_0^2 = 3.94$, $\alpha_G = 0.11\%$, $k^F_{\alpha_G;r,\infty} = 10.33$

in	to	$\hat{s}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mm]	$\widehat{\nabla}$ [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
N	A	982.6888	1.186	1.025	34.8	2.0	7.0	+3.405	5.7	2.7	4.6	2.2	4.6	2.2	1.58
	B	764.9986	1.405	0.974	41.0	2.2	6.4	+3.422	5.0	2.6	3.8	2.0	3.8	2.0	1.73
	C	1063.8891	0.879	1.066	29.4	1.6	7.6	+2.990	6.4	2.5	5.4	2.1	5.4	2.1	1.28

Network graph



Supplementary information

Observed directions	:	4
Observed distances	:	3
Orientation unknowns	:	1
Coordinate unknowns	:	2
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	8
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.9
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.0
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$7.2 \cdot 10^{-14}$
Redundancy r	:	4
Redundancy directions	:	2.95
Redundancy distances	:	1.05
Weighted square sum of residuals Ω [mgon^2]	:	1.6104
(a priori) standard deviation σ_0 [mgon]	:	0.5
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	0.63452
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.2690
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	1.6104 ($k^F_{\alpha_G;r,\infty}$ =3.38)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.390
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.021187

3.5.1 Niemeier W (2008)

Niemeier W (2008): Ausgleichungsrechnung, 2. Auflage. Walter de Gruyter, pp. 156-162/278-281

Available data files: [2D] Niemeier_DistanceDirection_fix∗.∗

Coordinates

Point name	ID	Easting x [m]	Northing y [m]
104	10	40 686.7920 (D)	26 816.1430 (D)
106	01	41 932.8380 (D)	28 872.5520 (D)
113	11	42 242.2310 (D)	27 492.0070 (D)
280	28	40 350.8460 (D)	28 835.9790 (D)
Z108	Z1	40 759.4000	27 816.1000
Z110	02	41 373.0000	27 904.0000

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
Z1	10	1002.5980	5	40 000
	11	1517.8620	5	40 000
	28	1098.6430	5	40 000
02	10	1286.2150	5	40 000
	01	1118.6890	5	40 000
	11	961.9110	5	40 000
	Z1	619.9050	5	40 000

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]	ω [gon]
Z1	11	108.599 400	0.5	$1.621\,14{\cdot}10^{10}$	5.100 049
	10	199.513 100	0.5	$1.621\,14{\cdot}10^{10}$	
	28	370.644 400	0.5	$1.621\,14{\cdot}10^{10}$	
02	01	35.414 600	0.5	$1.621\,14{\cdot}10^{10}$	397.949 322
	11	130.227 800	0.5	$1.621\,14{\cdot}10^{10}$	
	10	237.876 300	0.5	$1.621\,14{\cdot}10^{10}$	
	Z1	292.994 300	0.5	$1.621\,14{\cdot}10^{10}$	

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	x_{10}	y_{10}	x_{01}	y_{01}	x_{11}	y_{11}	x_{28}	y_{28}	x_{Z1}	y_{Z1}	x_{02}	y_{02}	ω_{Z1}	ω_{02}	Δy
$r_{Z1,11}$	0.0000	0.0000	0.0000	0.0000	−8.9557	−40.9753	0.0000	0.0000	8.9557	40.9753	0.0000	0.0000	−1	0	0.67
$r_{Z1,10}$	−63.3308	4.5985	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	63.3308	−4.5985	0.0000	0.0000	−1	0	−1.32
$r_{Z1,28}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	53.7894	21.5475	−53.7894	−21.5475	0.0000	0.0000	−1	0	0.65
$r_{02,01}$	0.0000	0.0000	49.2684	−28.4779	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−49.2684	28.4779	0	−1	−1.16
$r_{02,11}$	0.0000	0.0000	0.0000	0.0000	−28.3457	−59.8043	0.0000	0.0000	0.0000	0.0000	28.3457	59.8043	0	−1	−0.37
$r_{02,10}$	−41.8633	26.4069	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	41.8633	−26.4069	0	−1	−0.23
$r_{02,Z1}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−14.5638	101.6653	14.5638	−101.6653	0	−1	1.76

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[cm]}$ distances (1. iteration)

A	x_{10}	y_{10}	x_{01}	y_{01}	x_{11}	y_{11}	x_{28}	y_{28}	x_{Z1}	y_{Z1}	x_{02}	y_{02}	Δy
$s_{Z1,10}$	−0.0724	−0.9974	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0724	0.9974	0.0000	0.0000	0.839
$s_{Z1,11}$	0.0000	0.0000	0.0000	0.0000	0.9769	−0.2135	0.0000	0.0000	−0.9769	0.2135	0.0000	0.0000	2.668
$s_{Z1,28}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.3719	0.9283	0.3719	−0.9283	0.0000	0.0000	−2.417
$s_{02,10}$	−0.5335	−0.8458	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5335	0.8458	1.351
$s_{02,01}$	0.0000	0.0000	0.5004	0.8658	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.5004	−0.8658	−2.078
$s_{02,11}$	0.0000	0.0000	0.0000	0.0000	0.9036	−0.4283	0.0000	0.0000	0.0000	0.0000	−0.9036	0.4283	−1.455
$s_{02,Z1}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.9899	−0.1418	0.9899	0.1418	4.101

Matrix $D^{T}_{[-]}$ of datum constraints

	x_{10}	y_{10}	x_{01}	y_{01}	x_{11}	y_{11}	x_{28}	y_{28}	x_{Z1}	y_{Z1}	x_{02}	y_{02}	ω_{Z1}	ω_{02}
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[m,mgon]}$ (1. iteration)

	x_{Z1}	y_{Z1}	x_{02}	y_{02}	ω_{Z1}	ω_{02}
$\widehat{\Delta x}^{T}$	−0.0231	0.0166	0.0193	0.0042	−0.060	0.636

Adjusted coordinates

ID	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
Z1	40 759.3769	−2.307	0.313	27 816.1166	1.664	0.301	0.434
02	41 373.0193	1.927	0.312	27 904.0042	0.421	0.289	0.425

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
Z1	0.33	0.29	59.231 558
02	0.32	0.28	134.379 098

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
Z1	0.98	0.85	59.231 558
02	0.97	0.82	134.379 098

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
Z1	5.099 989	−0.06	0.28
02	397.949 958	0.64	0.25

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 3.426,\ r = 3.78,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.91,\ \alpha_G = 0.81\%,\ k^F_{\alpha_G;r,\infty} = 3.53$

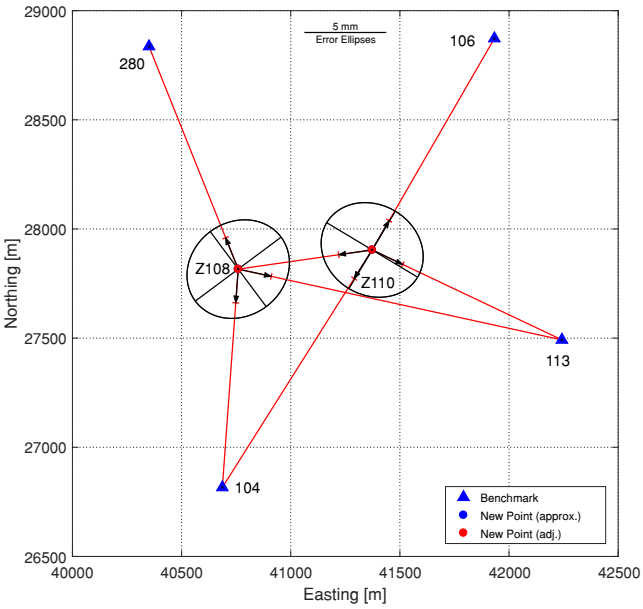
in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
Z1	28	370.644 695	375.744 685	−0.295	0.351	47.25	0.86	3.01	−0.625	1099	5.1	2.65	0.55	27.4	5.7	8.9	1.8	0.89
	10	199.512 942	204.612 932	0.158	0.331	53.19	0.43	2.83	+0.297	1003	2.5	2.08	0.22	20.9	2.2	6.6	0.7	0.45
	11	108.599 262	113.699 252	0.138	0.300	61.49	0.35	2.63	+0.224	1518	3.3	1.20	0.10	24.2	2.1	3.8	0.3	0.36
02	01	35.414 295	33.364 254	0.305	0.330	53.32	0.83	2.83	+0.571	1119	5.4	2.63	0.53	23.2	4.7	8.3	1.7	0.86
	Z1	292.993 783	290.943 742	0.517	0.380	38.29	1.67	3.34	+1.350	620	5.0	4.05	1.64	20.1	8.1	14.4	5.8	1.73
	10	237.876 592	235.826 550	−0.292	0.285	65.31	0.72	2.56	−0.447	1286	5.9	1.59	0.28	17.9	3.1	4.7	0.8	0.75
	11	130.228 329	128.178 288	−0.529	0.309	59.04	1.38	2.69	−0.897	962	8.0	2.15	0.72	16.6	5.5	5.6	1.9	1.43

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 4.046,\ r = 4.22,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.96,\ \alpha_G = 0.98\%,\ k^F_{\alpha_G;r,\infty} = 3.25$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
Z1	10	1002.6045	−0.653	0.304	60.4	1.7	2.7	−1.081	3.3	1.4	10.5	4.3	10.5	4.3	1.74
	11	1517.8614	0.059	0.304	60.4	0.2	2.7	+0.098	3.3	0.1	10.5	0.4	10.5	0.4	0.16
	28	1098.6431	−0.014	0.289	64.3	0.0	2.6	−0.022	3.1	0.0	9.2	0.1	9.2	0.1	0.04
02	10	1286.2153	−0.033	0.275	67.5	0.1	2.5	−0.049	2.9	0.1	8.2	0.2	8.2	0.2	0.08
	01	1118.6965	−0.749	0.275	67.5	1.8	2.5	−1.110	2.9	1.3	8.2	3.6	8.2	3.6	1.89
	11	961.9099	0.106	0.323	55.3	0.3	2.8	+0.191	3.7	0.3	12.4	0.9	12.4	0.9	0.29
	Z1	619.9041	0.086	0.353	46.7	0.3	3.0	+0.185	4.4	0.3	16.1	1.0	16.1	1.0	0.26

Network graph



Supplementary information

Observed directions	:	7
Observed distances	:	7
Orientation unknowns	:	2
Coordinate unknowns	:	4
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	8
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	2.8
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.5
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$3.4 \cdot 10^{-12}$
Redundancy r	:	8
Redundancy directions	:	3.78
Redundancy distances	:	4.22
Weighted square sum of residuals Ω [-]	:	7.47148
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	$9.66403 \cdot 10^{-1}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.9664
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.9339 ($k^F_{\alpha_G;r,\infty}$ =2.15)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	3.461
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.36896

3.5.2 Wolf (1979), Ex. II3.4-2, with one non-linear external restriction

Wolf W (1979): Ausgleichungsrechnung II. Dümmlers, pp. 273

Available data files: [2D] Wolf_Direction_fix_with_cond∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
A	182 581.4700 (D)	923 429.2900 (D)
B	184 292.4900 (D)	923 313.5300 (D)
C	185 487.6800 (D)	921 829.1200 (D)
D	184 601.0600 (D)	920 342.3100 (D)
E	182 710.8300 (D)	920 499.6900 (D)
F	180 714.5000 (D)	921 400.7000 (D)
G	182 275.7100	921 801.5700
H	183 716.6300	921 800.0400

Datum: fix, (D)...Datum coordinate

External restriction(s)

$$\sqrt{(x_G - x_H)^2 + (y_G - y_H)^2} = 1440.60\,\text{m}$$

Correlated horizontal distances with covariance matrix Σ_s [cm²]

in	to	s [m]	E-G	C-H
E	G	1372.4500	+9.000 000 0	+7.200 000 0
C	H	1771.4200	+7.200 000 0	+9.000 000 0

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
A	H	0.000 000	1.400	1	161.259 577
	G	50.562 000	1.400	1	
	F	86.100 000	1.400	1	
B	C	0.000 000	1.400	1	156.844 984
	H	66.299 000	1.400	1	
	A	147.457 000	1.400	1	
C	D	0.000 000	1.400	1	234.231 796
	H	64.723 000	1.400	1	
	B	122.613 000	1.400	1	
D	E	0.000 000	1.400	1	305.290 468
	H	59.989 000	1.400	1	
	C	128.942 000	1.400	1	

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
E	F	0.000 000	1.400	1	326.989 718
	G	52.477 000	1.400	1	
	H	114.923 000	1.400	1	
	D	178.298 000	1.400	1	
F	A	0.000 000	1.400	1	47.360 274
	G	36.639 000	1.400	1	
G	H	0.000 000	1.400	1	100.068 847
	E	79.398 000	1.400	1	
	F	183.928 000	1.400	1	
H	B	0.000 000	1.400	1	23.145 669
	C	75.807 000	1.400	1	
	G	276.924 000	1.400	1	

Design matrix $A_{[\text{mgon/m,-}]}$ **and reduced observation vector** $\Delta y_{[\text{mgon}]}$ **directions** (1. iteration)

Too large to be displayed !

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[m]}$ horizontal distances (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	x_F	y_F	x_G	y_G	x_H	y_H	Δy
$s_{E,G}$	0	0	0	0	0.0000	0.0000	0	0	0.3170	−0.9484	0	0	−0.3170	0.9484	0.0000	0.0000	−0.2193
$s_{C,H}$	0	0	0	0	0.9999	0.0164	0	0	0.0000	0.0000	0	0	0.0000	0.0000	−0.9999	−0.0164	0.1313

Matrix B^T of external restrictions and inhomogeneity c (datum parameters and orientation unknowns removed, 1. iteration)

	x_G	y_G	x_H	y_H	c
B^T	−2881.840 00	3.060 00	2881.840 00	−3.060 00	924.427 30

Least squares solution $\widehat{\Delta x}_{[cm,mgon]}$ (datum parameters removed, 1. iteration)

	x_G	y_G	x_H	y_H	ω_A	ω_B	ω_C	ω_D	ω_E	ω_F	ω_G	ω_H
$\widehat{\Delta x}^T$	15.779	−8.128	−16.303	−12.482	0.0127	1.4151	−1.4631	−2.5405	0.9273	2.3297	4.1146	0.5982

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
G	182 275.8678	15.781	4.561	921 801.4887	−8.128	4.576	6.461
H	183 716.4670	−16.305	4.562	921 799.9152	−12.484	5.630	7.246

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
G	5.34	3.64	150.287 708
H	5.79	4.35	176.835 350

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
G	14.49	9.87	150.287 708
H	15.72	11.82	176.835 350

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
A	161.259 590	0.01	2.58
B	156.846 399	1.42	2.46
C	234.230 333	−1.46	2.47
D	305.287 927	−2.54	2.44
E	326.990 646	0.93	2.25
F	47.362 603	2.33	3.07
G	100.072 962	4.12	2.63
H	23.146 268	0.60	2.61

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 115.068$, $r = 13.87$, $\hat{\sigma}_0^2/\sigma_0^2 = 8.30$, $\alpha_G = 6.55\%$, $k^F_{\alpha_G;r,\infty} = 1.62$

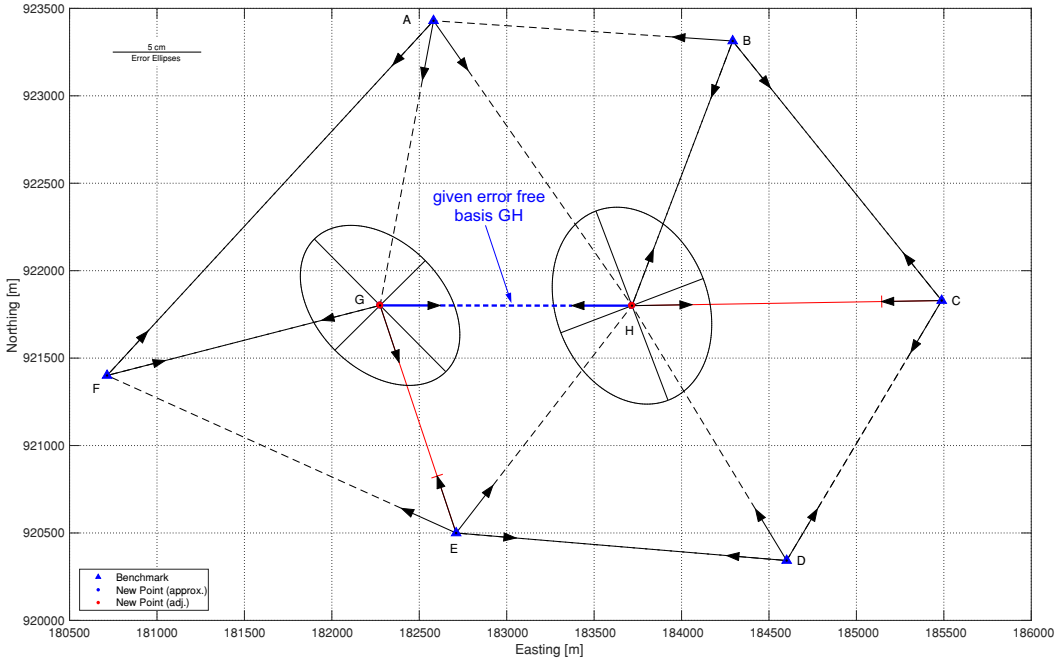
in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
A	H	0.006 561	161.266 151	−6.561	2.482	63.56	5.88	7.26	−10.322*	1986	204.6	0.91	1.30	82.5	117.3	10.5	14.9	2.00
	G	50.554 740	211.814 330	7.260	2.578	60.71	6.66	7.42	+11.959#	1656	188.9	1.29	2.08	75.9	122.2	20.0	32.1	2.27
	F	86.100 699	247.360 289	−0.699	2.576	60.77	0.64	7.42	−1.150	2757	30.3	1.29	0.20	126.1	19.5	0.0	0.0	0.22
B	C	399.998 396	156.844 795	1.604	2.464	64.09	1.43	7.23	+2.502	1906	48.0	0.83	0.29	77.7	26.9	0.0	0.0	0.49
	H	66.303 470	223.149 869	−4.470	2.717	56.35	4.25	7.71	−7.933*	1620	113.7	1.77	1.82	85.6	88.1	30.3	31.2	1.45
	A	147.454 134	304.300 533	2.866	2.464	64.09	2.56	7.23	+4.473	1715	77.2	0.83	0.51	69.9	43.3	0.0	0.0	0.87
C	D	0.001 472	234.231 805	−1.472	2.469	63.96	1.31	7.23	−2.302	1731	40.0	0.85	0.27	70.9	22.6	0.0	0.0	0.45
	H	64.720 065	298.950 398	2.935	2.732	55.86	2.80	7.74	+5.254	1771	81.7	1.82	1.23	95.1	64.5	34.9	23.7	0.95
	B	122.614 463	356.844 795	−1.463	2.469	63.96	1.31	7.23	−2.287	1906	43.8	0.85	0.27	78.0	24.7	0.0	0.0	0.44
D	E	0.000 353	305.288 280	−0.353	2.437	64.89	0.31	7.18	−0.544	1897	10.5	0.68	0.05	75.1	5.7	0.0	0.0	0.11
	H	59.986 769	365.274 697	2.231	2.616	59.55	2.06	7.50	+3.746	1705	59.7	1.43	0.71	81.2	40.6	21.4	10.7	0.70
	C	128.943 878	34.231 805	−1.878	2.437	64.89	1.66	7.18	−2.894	1731	51.1	0.68	0.28	68.6	27.6	0.0	0.0	0.57
E	F	399.999 617	326.990 263	0.383	2.246	70.17	0.33	6.91	+0.546	2190	13.2	1.08	0.09	70.9	5.6	0.0	0.0	0.11
	G	52.480 787	379.471 433	−3.787	2.346	67.46	3.29	7.04	−5.613*	1373	81.6	1.38	1.10	49.4	39.4	17.7	14.1	1.12
	H	114.919 962	41.910 608	3.038	2.445	64.66	2.70	7.19	+4.699	1644	78.4	1.65	1.08	65.6	42.9	29.5	19.3	0.92
	D	178.297 634	105.288 280	0.366	2.246	70.17	0.31	6.91	+0.521	1897	10.9	1.08	0.08	61.4	4.6	0.0	0.0	0.11
F	A	399.997 686	47.360 289	2.314	3.068	44.33	2.48	8.69	+5.220	2757	100.2	1.48	0.89	209.5	125.9	0.0	0.0	0.85
	G	36.641 314	84.003 917	−2.314	3.068	44.33	2.48	8.69	−5.220	1612	58.6	1.48	0.89	122.5	73.6	25.0	15.0	0.85
G	H	399.996 575	100.069 537	3.425	3.161	40.90	3.83	9.05	+8.373*	1441	77.5	3.28	3.04	121.0	112.0	54.0	50.0	1.30
	E	79.398 470	179.471 433	−0.470	2.728	56.00	0.45	7.73	−0.840	1373	10.1	1.80	0.20	73.3	8.0	19.7	2.1	0.15
	F	183.930 955	284.003 917	−2.955	2.963	48.08	3.04	8.34	−6.145	1612	74.8	2.57	1.89	109.7	80.8	35.5	26.2	1.04
H	B	0.003 601	23.149 869	−3.601	2.668	57.92	3.38	7.60	−6.217*	1620	91.6	1.61	1.31	81.4	66.6	24.3	19.9	1.15
	C	75.804 130	98.950 398	2.870	2.966	47.99	2.96	8.35	+5.981	1771	79.9	2.58	1.85	120.9	86.6	42.0	30.1	1.01
	G	276.923 269	300.069 537	0.731	3.382	32.38	0.92	10.17	+2.257	1441	16.5	4.25	0.94	155.6	34.5	71.5	15.9	0.31

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 14.360$, $r = 1.13$, $\hat{\sigma}_0^2/\sigma_0^2 = 12.71$, $\alpha_G = 0.12\%$, $k^F_{\alpha_G;r,\infty} = 9.69$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
E	G	1372.5422	−9.217	5.052	47.8	4.4	17.9	−19.296*	7.7	8.3	93.7	100.8	93.7	100.8	1.51
C	H	1771.4538	−3.381	4.544	65.2	1.4	15.4	−5.186	6.3	2.1	53.5	18.1	53.5	18.1	0.48

Network graph



Supplementary information

Observed directions	:	24
Observed distances	:	2
Orientation unknowns	:	8
Coordinate unknowns	:	4
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	12
Number of external restrictions	:	1
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	7.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.9
Number of iterations (Max=20)	:	4
Stop criterion (actual)	:	$7.9 \cdot 10^{-11}$
Redundancy r	:	15
Redundancy directions	:	13.87
Redundancy distances	:	1.13
Weighted square sum of residuals Ω [mgon^2]	:	253.68
(a priori) standard deviation σ_0 [mgon]	:	1.4
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	4.1124
Ratio $\hat{\sigma}_0/\sigma_0$	:	2.9374
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	8.6286 ($k^F_{\alpha_G;r,\infty}$ =1.57)
Number of outliers (Data snooping)	:	7 (Remove outliers or scale standard deviations by the factor 2.94)
Number of outliers (τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	27.144
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	94.246

3.5.3 Lösler (2010), Deformation Analysis Epoch 1

http://diegeodaeten.de/deformationsanalyse.html

Coordinates

Point name	ID	Easting x [m]	Northing y [m]
100	A	100.0000 (D)	200.0000 (D)
101	B	100.6757 (D)	215.0000 (D)
102	C	101.2010 (D)	230.0000 (D)
103	D	101.5761 (D)	245.0000 (D)
104	E	101.8012 (D)	260.0000 (D)
105	F	101.8762 (D)	275.0000 (D)
106	G	101.8012 (D)	290.0000 (D)
107	H	101.5761 (D)	305.0000 (D)
108	I	101.2010 (D)	320.0000 (D)
109	J	100.6757 (D)	335.0000 (D)
110	K	100.0000 (D)	350.0000 (D)
1001	L	159.1474 (D)	222.2700 (D)
1002	M	156.8485 (D)	331.4654 (D)
1003	N	238.1704 (D)	369.9712 (D)
1004	O	346.2164 (D)	316.2355 (D)
1005	P	276.3888 (D)	197.2700 (D)
1006	Q	215.7566 (D)	278.3045 (D)

Datum: free, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma_c\rvert$ [mm]	$\lvert\sigma_d\rvert$ [–]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
L	A	63.2013	2	$2.0{\cdot}10^{-6}$	2.004	249 005.37
	B	58.9214	2	$2.0{\cdot}10^{-6}$	2.003	249 135.07
	C	58.4595	2	$2.0{\cdot}10^{-6}$	2.003	249 148.53
	D	61.8952	2	$2.0{\cdot}10^{-6}$	2.004	249 045.90
	E	68.6409	2	$2.0{\cdot}10^{-6}$	2.005	248 827.63
	F	77.8486	2	$2.0{\cdot}10^{-6}$	2.006	248 494.03
	G	88.7493	2	$2.0{\cdot}10^{-6}$	2.008	248 046.28
	H	100.7917	2	$2.0{\cdot}10^{-6}$	2.010	247 485.80
	I	113.6167	2	$2.0{\cdot}10^{-6}$	2.013	246 813.94
	J	126.9908	2	$2.0{\cdot}10^{-6}$	2.016	246 032.32
	K	140.7605	2	$2.0{\cdot}10^{-6}$	2.020	245 142.86
	M	109.2188	2	$2.0{\cdot}10^{-6}$	2.012	247 052.97
	N	167.5110	2	$2.0{\cdot}10^{-6}$	2.028	243 176.48
	O	209.3428	2	$2.0{\cdot}10^{-6}$	2.043	239 503.88
	P	119.8798	2	$2.0{\cdot}10^{-6}$	2.014	246 458.11
	Q	79.6491	2	$2.0{\cdot}10^{-6}$	2.006	248 424.00

Horizontal distances (continued)

in	to	s [m]	$\lvert\sigma_c\rvert$ [mm]	$\lvert\sigma_d\rvert$ [–]	$\lvert\sigma\rvert$ [mm]	p [1/m²]
M	A	143.2314	2	$2.0{\cdot}10^{-6}$	2.020	244 974.30
	B	129.3022	2	$2.0{\cdot}10^{-6}$	2.017	245 888.97
	C	115.7251	2	$2.0{\cdot}10^{-6}$	2.013	246 696.17
	D	102.6172	2	$2.0{\cdot}10^{-6}$	2.011	247 394.86
	E	90.2077	2	$2.0{\cdot}10^{-6}$	2.008	247 982.06
	F	78.8056	2	$2.0{\cdot}10^{-6}$	2.006	248 457.00
	G	68.9164	2	$2.0{\cdot}10^{-6}$	2.005	248 818.25
	H	61.2814	2	$2.0{\cdot}10^{-6}$	2.004	249 064.66
	I	56.8134	2	$2.0{\cdot}10^{-6}$	2.003	249 195.66
	J	56.2817	2	$2.0{\cdot}10^{-6}$	2.003	249 210.59
	K	59.7921	2	$2.0{\cdot}10^{-6}$	2.004	249 109.41
	L	109.2203	2	$2.0{\cdot}10^{-6}$	2.012	247 052.89
	N	89.9763	2	$2.0{\cdot}10^{-6}$	2.008	247 992.32
	O	189.9824	2	$2.0{\cdot}10^{-6}$	2.036	241 291.01
	P	179.7175	2	$2.0{\cdot}10^{-6}$	2.032	242 178.04
	Q	79.3451	2	$2.0{\cdot}10^{-6}$	2.006	248 435.94
N	L	167.5118	2	$2.0{\cdot}10^{-6}$	2.028	243 176.42
	M	89.9779	2	$2.0{\cdot}10^{-6}$	2.008	247 992.25
	O	120.6707	2	$2.0{\cdot}10^{-6}$	2.015	246 411.89
	P	176.8830	2	$2.0{\cdot}10^{-6}$	2.031	242 415.40
	Q	94.3705	2	$2.0{\cdot}10^{-6}$	2.009	247 793.21
O	L	209.3439	2	$2.0{\cdot}10^{-6}$	2.043	239 503.78
	M	189.9795	2	$2.0{\cdot}10^{-6}$	2.036	241 291.26
	N	120.6693	2	$2.0{\cdot}10^{-6}$	2.015	246 411.98
	P	137.9435	2	$2.0{\cdot}10^{-6}$	2.019	245 331.73
	Q	135.8594	2	$2.0{\cdot}10^{-6}$	2.018	245 469.18
P	L	119.8765	2	$2.0{\cdot}10^{-6}$	2.014	246 458.30
	M	179.7162	2	$2.0{\cdot}10^{-6}$	2.032	242 178.15
	N	176.8830	2	$2.0{\cdot}10^{-6}$	2.031	242 415.40
	O	137.9460	2	$2.0{\cdot}10^{-6}$	2.019	245 331.56
	Q	101.2071	2	$2.0{\cdot}10^{-6}$	2.010	247 465.24
Q	A	139.7496	2	$2.0{\cdot}10^{-6}$	2.019	245 211.04
	B	131.3432	2	$2.0{\cdot}10^{-6}$	2.017	245 760.38
	C	124.3213	2	$2.0{\cdot}10^{-6}$	2.015	246 194.86
	D	118.9397	2	$2.0{\cdot}10^{-6}$	2.014	246 512.67
	E	115.4172	2	$2.0{\cdot}10^{-6}$	2.013	246 713.50
	F	113.9317	2	$2.0{\cdot}10^{-6}$	2.013	246 796.48
	G	114.5552	2	$2.0{\cdot}10^{-6}$	2.013	246 761.77
	H	117.2583	2	$2.0{\cdot}10^{-6}$	2.014	246 609.24
	I	121.9085	2	$2.0{\cdot}10^{-6}$	2.015	246 338.99
	J	128.2867	2	$2.0{\cdot}10^{-6}$	2.016	245 952.25
	K	136.1610	2	$2.0{\cdot}10^{-6}$	2.018	245 449.41
	L	79.6521	2	$2.0{\cdot}10^{-6}$	2.006	248 423.89
	M	79.3488	2	$2.0{\cdot}10^{-6}$	2.006	248 435.79
	N	94.3634	2	$2.0{\cdot}10^{-6}$	2.009	247 793.53
	O	135.8615	2	$2.0{\cdot}10^{-6}$	2.018	245 469.05
	P	101.2092	2	$2.0{\cdot}10^{-6}$	2.010	247 465.14

Directions

in	to	r [gon]	$\lvert\sigma_c\rvert$ [mgon]	$\lvert\sigma_d\rvert$ [mm]	s_{2D} [m]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]	ω [gon]
L	A	139.560 860	0.300	0.500	63	0.586	11 793 176 097	137.513 851
	B	154.611 270	0.300	0.500	59	0.618	10 613 956 777	
	C	170.928 520	0.300	0.500	58	0.622	10 486 708 807	
	D	186.425 120	0.300	0.500	62	0.595	11 433 534 020	
	E	199.533 320	0.300	0.500	69	0.552	13 287 084 135	
	F	209.860 030	0.300	0.500	78	0.507	15 758 526 206	
	G	217.759 490	0.300	0.500	89	0.468	18 536 076 622	
	H	223.781 790	0.300	0.500	101	0.436	21 360 193 475	
	I	228.414 450	0.300	0.500	114	0.410	24 054 049 266	
	J	232.024 540	0.300	0.500	127	0.391	26 519 170 335	
	K	234.878 150	0.300	0.500	141	0.376	28 715 535 943	
	M	261.146 130	0.300	0.500	109	0.418	23 167 408 652	
	N	293.761 350	0.300	0.500	168	0.355	32 137 808 339	
	O	332.852 130	0.300	0.500	209	0.336	35 827 909 428	
	P	375.861 150	0.300	0.500	120	0.401	25 250 424 816	
	Q	312.810 420	0.300	0.500	80	0.500	16 230 859 426	
M	A	76.980 240	0.300	0.500	143	0.373	29 075 791 411	149.003 092
	B	79.606 520	0.300	0.500	129	0.388	26 911 276 430	
	C	82.932 490	0.300	0.500	116	0.407	24 465 039 628	
	D	87.206 050	0.300	0.500	103	0.432	21 764 979 052	
	E	92.781 260	0.300	0.500	90	0.463	18 893 403 814	
	F	100.143 620	0.300	0.500	79	0.503	16 009 654 402	
	G	109.897 640	0.300	0.500	69	0.551	13 361 352 157	
	H	122.568 600	0.300	0.500	61	0.600	11 264 248 849	
	I	138.061 060	0.300	0.500	57	0.636	10 034 970 621	
	J	154.997 740	0.300	0.500	56	0.640	9 888 881 072.0	
	K	171.061 480	0.300	0.500	60	0.611	10 854 105 872	
	L	49.656 370	0.300	0.500	109	0.418	23 167 408 652	
	N	322.843 660	0.300	0.500	90	0.464	18 837 257 797	
	O	356.106 060	0.300	0.500	190	0.344	34 324 950 405	
	P	4.669 630	0.300	0.500	180	0.348	33 392 366 892	
	Q	397.735 240	0.300	0.500	79	0.501	16 151 728 428	
N	L	277.236 490	0.300	0.500	168	0.355	32 137 808 339	354.038 795
	M	317.808 310	0.300	0.500	90	0.464	18 837 257 797	
	O	175.342 270	0.300	0.500	121	0.399	25 396 688 341	
	P	232.096 610	0.300	0.500	177	0.350	33 115 524 017	
	Q	261.227 730	0.300	0.500	94	0.451	19 888 544 810	
O	L	311.358 320	0.300	0.500	209	0.336	35 827 909 428	359.007 932
	M	346.100 970	0.300	0.500	190	0.344	34 324 950 405	
	N	370.373 130	0.300	0.500	121	0.399	25 396 688 341	
	P	274.782 260	0.300	0.500	138	0.378	28 292 805 910	
	Q	322.978 890	0.300	0.500	136	0.381	27 971 652 078	

Directions (continued)

in	to	r [gon]	$\lvert\sigma_c\rvert$ [mgon]	$\lvert\sigma_d\rvert$ [mm]	s_{2D} [m]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]	ω [gon]
P	L	232.310 220	0.300	0.500	120	0.401	25 250 424 816	81.064 176
	M	272.608 370	0.300	0.500	180	0.348	33 392 366 892	
	N	305.071 020	0.300	0.500	177	0.350	33 115 524 017	
	O	352.726 300	0.300	0.500	138	0.378	28 292 805 910	
	Q	278.041 660	0.300	0.500	101	0.435	21 452 839 501	
Q	A	67.960 300	0.300	0.500	140	0.377	28 565 993 288	194.176 987
	B	73.806 770	0.300	0.500	131	0.386	27 249 093 343	
	C	80.418 940	0.300	0.500	124	0.394	26 054 389 280	
	D	87.754 900	0.300	0.500	119	0.402	25 075 852 052	
	E	95.683 770	0.300	0.500	115	0.408	24 405 642 459	
	F	103.975 820	0.300	0.500	114	0.410	24 115 264 734	
	G	112.334 200	0.300	0.500	115	0.409	24 237 905 974	
	H	120.445 080	0.300	0.500	117	0.405	24 759 420 142	
	I	128.045 360	0.300	0.500	122	0.398	25 622 226 821	
	J	134.964 960	0.300	0.500	128	0.389	26 740 254 055	
	K	141.126 270	0.300	0.500	136	0.380	28 018 194 411	
	L	56.147 310	0.300	0.500	80	0.500	16 230 859 426	
	M	152.560 700	0.300	0.500	79	0.501	16 151 728 428	
	N	221.089 920	0.300	0.500	94	0.451	19 888 544 810	
	O	287.810 140	0.300	0.500	136	0.381	27 971 652 078	
	P	364.928 570	0.300	0.500	101	0.435	21 452 839 501	

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

Too large to be displayed !

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{mm}]}$ horizontal distances (1. iteration)

Too large to be displayed !

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints (1. iteration)

Too large to be displayed !

Least squares solution $\widehat{\Delta x}_{[\text{mm}]}$ (1. iteration)

	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
$\widehat{\Delta x}^{\mathsf{T}}$	−0.323	−0.066	0.105	0.251	−0.072	−0.145	0.572	0.128	0.160	0.114

Least squares solution $\widehat{\Delta x}_{[\text{mm}]}$ (continued)

	x_F	y_F	x_G	y_G	x_H	y_H	x_I	y_I	x_J	y_J
$\widehat{\Delta x}^{\mathsf{T}}$	0.363	−0.240	−0.163	0.418	−0.248	0.406	0.462	−0.464	0.324	−0.318

Least squares solution $\widehat{\Delta x}_{[\mathrm{mm}]}$ (continued)

	x_K	y_K	x_L	y_L	x_M	y_M	x_N	y_N	x_O	y_O
$\widehat{\Delta x}^\mathsf{T}$	0.213	−0.058	−0.115	0.086	−0.528	−0.299	−0.354	0.286	0.195	0.052

Least squares solution $\widehat{\Delta x}_{[\mathrm{mm,mgon}]}$ (continued)

	x_P	y_P	x_Q	y_Q	ω_L	ω_M	ω_N	ω_O	ω_P	ω_Q
$\widehat{\Delta x}^\mathsf{T}$	−0.156	−0.233	−0.436	0.081	0.0210	−0.1162	−0.1168	0.0005	0.0200	−0.0596

Adjusted coordinates

ID	$\hat{x}$ [m]	$\hat{x} - x$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	$\hat{y}$ [m]	$\hat{y} - y$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	$\lvert\hat{\sigma}_{2D}\rvert$ [mm]
A	99.9997	−0.323	0.659	199.9999	−0.066	0.544	0.855
B	100.6758	0.105	0.633	215.0003	0.251	0.455	0.779
C	101.2009	−0.072	0.589	229.9999	−0.145	0.401	0.713
D	101.5767	0.572	0.545	245.0001	0.128	0.387	0.668
E	101.8013	0.160	0.514	260.0001	0.114	0.389	0.645
F	101.8765	0.363	0.502	274.9998	−0.240	0.391	0.637
G	101.8010	−0.163	0.510	290.0004	0.418	0.388	0.641
H	101.5759	−0.248	0.538	305.0004	0.406	0.383	0.661
I	101.2015	0.462	0.583	319.9995	−0.464	0.390	0.702
J	100.6760	0.324	0.633	334.9997	−0.318	0.434	0.767
K	100.0002	0.213	0.665	349.9999	−0.058	0.521	0.845
L	159.1473	−0.115	0.271	222.2701	0.086	0.198	0.336
M	156.8480	−0.528	0.264	331.4651	−0.299	0.193	0.328
N	238.1700	−0.354	0.385	369.9715	0.286	0.327	0.505
O	346.2166	0.195	0.467	316.2356	0.052	0.300	0.555
P	276.3887	−0.156	0.418	197.2698	−0.233	0.329	0.532
Q	215.7561	−0.436	0.249	278.3046	0.081	0.211	0.326

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
L	137.513 872	0.02	0.14
M	149.002 976	−0.12	0.14
N	354.038 678	−0.12	0.20
O	359.007 932	0.00	0.19
P	81.064 196	0.02	0.20
Q	194.176 928	−0.06	0.12

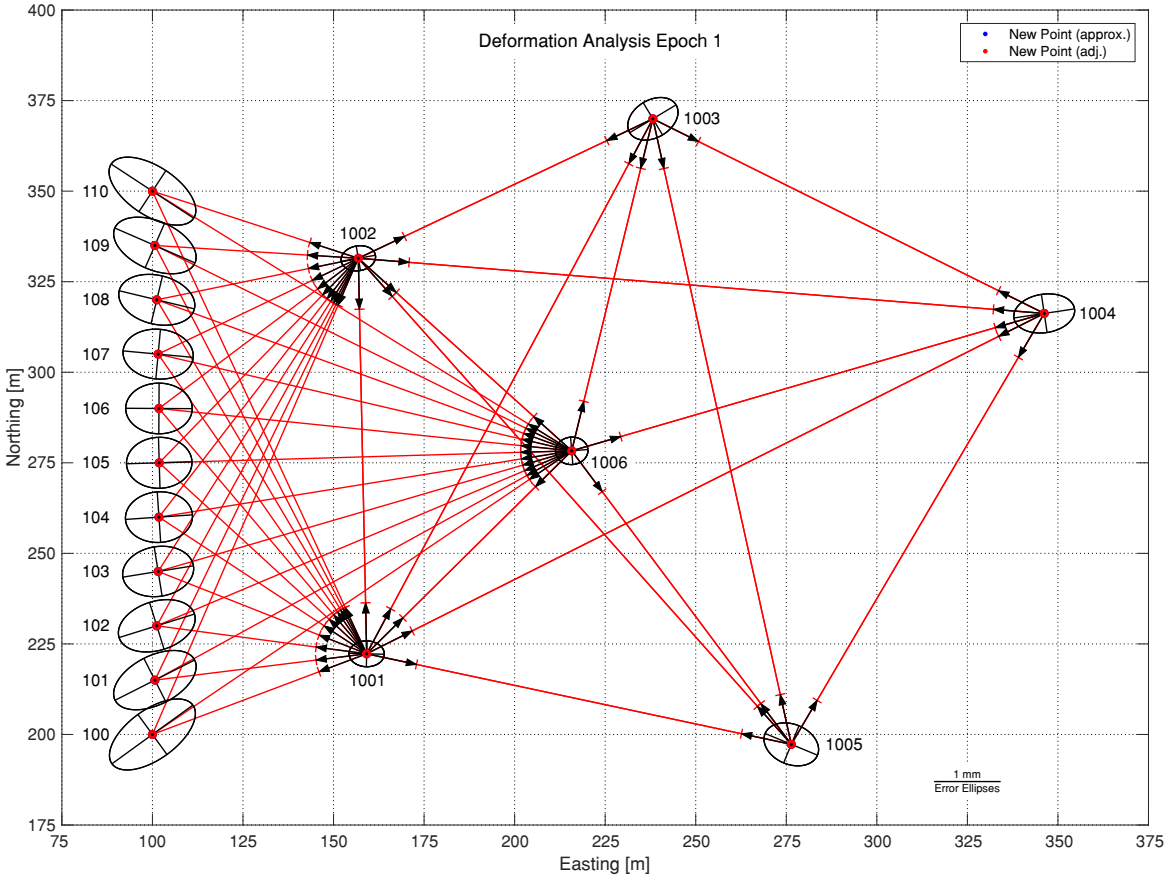
Absolute error ellipses

in	$A_{[mm]}$	$B_{[mm]}$	$\phi_{[gon]}$
A	0.77	0.38	60.068 766
B	0.68	0.37	70.201 036
C	0.61	0.38	80.781 269
D	0.55	0.38	89.857 344
E	0.51	0.39	95.641 228
F	0.50	0.39	98.111 958
G	0.51	0.39	100.371 226
H	0.54	0.38	105.880 420
I	0.59	0.37	115.043 785
J	0.67	0.37	126.142 094
K	0.76	0.38	136.990 855
L	0.27	0.20	101.576 793
M	0.27	0.19	88.917 884
N	0.41	0.29	65.568 605
O	0.47	0.30	90.525 525
P	0.43	0.31	124.987 114
Q	0.25	0.21	94.744 747

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	$A_{[mm]}$	$B_{[mm]}$	$\phi_{[gon]}$
A	1.91	0.94	60.068 766
B	1.70	0.93	70.201 036
C	1.51	0.94	80.781 269
D	1.37	0.95	89.857 344
E	1.28	0.97	95.641 228
F	1.25	0.97	98.111 958
G	1.27	0.97	100.371 226
H	1.34	0.95	105.880 420
I	1.48	0.93	115.043 785
J	1.67	0.93	126.142 094
K	1.88	0.94	136.990 855
L	0.67	0.49	101.576 793
M	0.66	0.47	88.917 884
N	1.03	0.72	65.568 605
O	1.17	0.74	90.525 525
P	1.08	0.77	124.987 114
Q	0.62	0.52	94.744 747

Network graph



Adjusted directions

Variance component: $\Omega = 14.776,\ r = 32.60, \hat{\sigma}^2 = 0.45,\ \alpha_G = 18.09\%,\ k^F_{\alpha_G;r,\infty} = 1.22$

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
L	A	139.561 382	277.075 254	−0.522	0.429	31.43	1.59	4.32	−1.660	63	0.5	5.94	2.28	2.9	1.1	2.8	1.1	1.80
	B	154.611 371	292.125 243	−0.101	0.461	28.56	0.30	4.78	−0.352	59	0.1	6.39	0.47	3.2	0.2	3.0	0.2	0.34
	C	170.928 512	308.442 384	0.008	0.472	26.20	0.03	5.02	+0.031	58	0.0	6.78	0.04	3.4	0.0	3.3	0.0	0.03
	D	186.425 174	323.939 046	−0.054	0.454	25.45	0.18	4.88	−0.213	62	0.1	6.90	0.30	3.5	0.2	3.5	0.2	0.20
	E	199.533 262	337.047 134	0.058	0.419	26.27	0.21	4.45	+0.222	69	0.1	6.73	0.34	3.5	0.2	3.5	0.2	0.23
	F	209.859 526	347.373 397	0.504	0.380	27.97	1.88	3.96	+1.804	78	0.6	6.40	2.92	3.5	1.6	3.4	1.6	2.13
	G	217.759 372	355.273 244	0.118	0.346	29.75	0.46	3.54	+0.397	89	0.2	6.09	0.68	3.5	0.4	3.4	0.4	0.52
	H	223.781 970	361.295 842	−0.180	0.319	31.21	0.74	3.22	−0.576	101	0.3	5.83	1.04	3.5	0.6	3.3	0.6	0.84
	I	228.414 396	365.928 268	0.054	0.297	32.73	0.23	2.96	+0.166	114	0.1	5.59	0.31	3.6	0.2	3.3	0.2	0.26
	J	232.024 877	369.538 748	−0.337	0.277	35.62	1.44	2.71	−0.945	127	0.7	5.19	1.81	3.5	1.2	3.1	1.1	1.63
	K	234.878 202	372.392 074	−0.052	0.256	40.75	0.22	2.43	−0.127	141	0.1	4.60	0.24	3.2	0.2	2.7	0.1	0.24
	M	261.145 841	398.659 713	0.289	0.189	73.94	0.80	2.01	+0.391	109	0.5	2.09	0.41	0.9	0.2	0.7	0.1	0.91
	N	293.761 171	31.275 043	0.179	0.171	70.14	0.60	1.75	+0.255	168	0.5	2.21	0.32	1.4	0.2	0.7	0.1	0.68
	O	332.852 215	70.366 087	−0.085	0.176	64.82	0.31	1.73	−0.131	209	0.3	2.52	0.19	2.0	0.2	0.8	0.1	0.35
	P	375.860 929	113.374 801	0.221	0.239	54.47	0.75	2.24	+0.406	120	0.4	3.44	0.62	1.9	0.3	1.2	0.2	0.85
	Q	312.810 741	50.324 613	−0.321	0.228	73.36	0.75	2.41	−0.438	80	0.4	2.24	0.41	0.8	0.1	0.6	0.1	0.85

Adjusted directions (continued)

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
M	A	76.980 008	225.982 984	0.232	0.252	41.48	0.97	2.40	+0.559	143	0.5	4.51	1.05	3.2	0.7	2.7	0.6	1.09
	B	79.606 473	228.609 449	0.047	0.274	36.29	0.20	2.66	+0.130	129	0.1	5.10	0.25	3.4	0.2	3.1	0.1	0.23
	C	82.932 467	231.935 443	0.023	0.294	33.08	0.10	2.92	+0.071	116	0.0	5.53	0.13	3.6	0.1	3.3	0.1	0.11
	D	87.206 001	236.208 977	0.049	0.316	31.37	0.20	3.18	+0.156	103	0.1	5.80	0.28	3.5	0.2	3.3	0.2	0.23
	E	92.781 092	241.784 068	0.168	0.342	29.96	0.66	3.50	+0.562	90	0.2	6.04	0.97	3.5	0.6	3.3	0.5	0.75
	F	100.143 669	249.146 645	−0.049	0.377	28.25	0.18	3.91	−0.172	79	0.1	6.35	0.28	3.5	0.2	3.4	0.1	0.21
	G	109.897 887	258.900 863	−0.247	0.417	26.51	0.87	4.42	−0.932	69	0.3	6.68	1.41	3.5	0.7	3.5	0.7	0.99
	H	122.568 777	271.571 753	−0.177	0.458	25.46	0.58	4.91	−0.695	61	0.2	6.90	0.98	3.5	0.5	3.5	0.5	0.66
	I	138.060 989	287.063 966	0.071	0.483	25.90	0.22	5.16	+0.272	57	0.1	6.84	0.36	3.4	0.2	3.3	0.2	0.25
	J	154.997 655	304.000 631	0.085	0.480	28.12	0.25	4.99	+0.302	56	0.1	6.46	0.39	3.2	0.2	3.1	0.2	0.28
	K	171.061 720	320.064 696	−0.240	0.448	31.14	0.70	4.52	−0.771	60	0.2	5.99	1.02	2.9	0.5	2.8	0.5	0.80
	L	49.656 736	198.659 713	−0.366	0.193	72.75	1.03	2.03	−0.504	109	0.6	2.17	0.54	0.9	0.2	0.8	0.2	1.16
	N	322.843 902	71.846 878	−0.242	0.269	56.99	0.69	2.54	−0.425	90	0.3	3.33	0.56	1.5	0.3	1.1	0.2	0.78
	O	356.105 882	105.108 858	0.178	0.186	62.27	0.66	1.80	+0.286	190	0.5	2.72	0.43	2.0	0.3	0.9	0.1	0.74
	P	4.669 765	153.672 741	−0.135	0.174	68.15	0.47	1.74	−0.198	180	0.4	2.31	0.26	1.6	0.2	0.8	0.1	0.53
	Q	397.734 885	146.737 861	0.355	0.232	72.53	0.83	2.43	+0.489	79	0.4	2.30	0.46	0.8	0.2	0.7	0.1	0.94
N	L	277.236 365	231.275 043	0.125	0.183	65.96	0.44	1.81	+0.190	168	0.3	1.54	0.16	1.6	0.2	0.3	0.0	0.49
	M	317.808 200	271.846 878	0.110	0.259	59.99	0.31	2.47	+0.184	90	0.2	2.69	0.20	1.4	0.1	0.8	0.1	0.35
	O	175.342 440	129.381 118	−0.170	0.251	49.38	0.60	2.35	−0.344	121	0.3	3.27	0.48	2.3	0.3	1.3	0.2	0.68
	P	232.096 468	186.135 147	0.142	0.194	60.46	0.52	1.86	+0.234	177	0.4	1.98	0.25	2.0	0.3	0.8	0.1	0.59
	Q	261.228 056	215.266 735	−0.326	0.218	70.16	0.86	2.23	−0.465	94	0.5	1.88	0.39	1.0	0.2	0.7	0.1	0.98

Adjusted directions (continued)

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
O	L	311.358 155	270.366 087	0.165	0.162	70.27	0.59	1.66	+0.235	209	0.5	1.22	0.17	1.6	0.2	0.1	0.0	0.66
	M	346.100 926	305.108 858	0.044	0.165	70.30	0.15	1.69	+0.063	190	0.1	1.31	0.05	1.5	0.1	0.2	0.0	0.17
	N	370.373 186	329.381 118	−0.056	0.229	57.85	0.18	2.17	−0.096	121	0.1	2.74	0.12	1.7	0.1	1.2	0.1	0.21
	P	274.782 235	233.790 167	0.025	0.225	54.74	0.09	2.11	+0.046	138	0.1	2.88	0.06	2.1	0.0	1.3	0.0	0.10
	Q	322.979 131	281.987 063	−0.241	0.177	72.26	0.74	1.85	−0.333	136	0.5	1.48	0.27	1.1	0.2	0.4	0.1	0.84
P	L	232.310 605	313.374 801	−0.385	0.224	60.11	1.24	2.14	−0.640	120	0.7	2.50	0.75	1.6	0.5	0.7	0.2	1.40
	M	272.608 545	353.672 741	−0.175	0.172	68.75	0.61	1.74	−0.255	180	0.5	1.38	0.20	1.5	0.2	0.3	0.0	0.69
	N	305.070 951	386.135 147	0.069	0.189	62.56	0.25	1.83	+0.111	177	0.2	1.96	0.12	1.9	0.1	0.8	0.1	0.28
	O	352.725 971	33.790 167	0.329	0.236	50.35	1.23	2.20	+0.653	138	0.7	3.17	0.94	2.4	0.7	1.4	0.4	1.39
	Q	278.041 476	359.105 672	0.184	0.205	71.61	0.50	2.12	+0.257	101	0.3	1.78	0.22	1.0	0.1	0.6	0.1	0.57
Q	A	67.959 973	262.136 900	0.327	0.199	64.41	1.08	1.94	+0.508	140	0.7	2.73	0.72	1.5	0.4	1.2	0.3	1.22
	B	73.806 773	267.983 701	−0.003	0.206	63.48	0.01	2.00	−0.005	131	0.0	2.82	0.01	1.5	0.0	1.2	0.0	0.01
	C	80.418 858	274.595 786	0.082	0.217	61.28	0.27	2.08	+0.134	124	0.2	2.99	0.19	1.6	0.1	1.4	0.1	0.30
	D	87.755 092	281.932 020	−0.192	0.228	58.76	0.62	2.17	−0.327	119	0.4	3.18	0.48	1.7	0.3	1.5	0.2	0.71
	E	95.683 718	289.860 646	0.052	0.237	56.86	0.17	2.23	+0.091	115	0.1	3.33	0.14	1.7	0.1	1.6	0.1	0.19
	F	103.976 107	298.153 034	−0.287	0.240	56.07	0.93	2.26	−0.511	114	0.5	3.39	0.77	1.8	0.4	1.6	0.4	1.06
	G	112.334 256	306.511 183	−0.056	0.238	56.55	0.18	2.25	−0.099	115	0.1	3.35	0.15	1.8	0.1	1.6	0.1	0.21
	H	120.444 866	314.621 793	0.214	0.231	58.25	0.69	2.19	+0.368	117	0.4	3.22	0.54	1.7	0.3	1.5	0.3	0.79
	I	128.045 533	322.222 460	−0.173	0.220	60.78	0.56	2.11	−0.284	122	0.3	3.03	0.41	1.6	0.2	1.4	0.2	0.63
	J	134.964 763	329.141 690	0.197	0.209	63.22	0.64	2.02	+0.312	128	0.4	2.84	0.44	1.5	0.2	1.3	0.2	0.72
	K	141.126 073	335.303 000	0.197	0.201	64.39	0.65	1.96	+0.306	136	0.4	2.74	0.43	1.5	0.2	1.2	0.2	0.73
	L	56.147 686	250.324 613	−0.376	0.218	75.69	0.86	2.37	−0.496	80	0.5	2.13	0.45	0.7	0.2	0.6	0.1	0.98
	M	152.560 934	346.737 861	−0.234	0.218	75.73	0.54	2.38	−0.309	79	0.3	2.13	0.28	0.7	0.1	0.6	0.1	0.61
	N	221.089 807	15.266 735	0.113	0.269	54.47	0.34	2.53	+0.207	94	0.2	3.56	0.29	1.7	0.1	1.4	0.1	0.38
	O	287.810 135	81.987 063	0.005	0.236	50.80	0.02	2.21	+0.009	136	0.0	3.76	0.02	2.3	0.0	1.5	0.0	0.02
	P	364.928 744	159.105 672	−0.174	0.266	52.03	0.56	2.49	−0.335	101	0.3	3.73	0.50	1.9	0.3	1.4	0.2	0.63

Adjusted horizontal distances

Variance component: $\Omega = 54.706,\ r = 56.40, \hat{\sigma}^2 = 0.97,\ \alpha_G = 28.40\%,\ k^F_{\alpha_G;r,\infty} = 1.10$

in	to	$\hat{s}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mm]	$\widehat{\nabla}$ [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
L	A	63.2012	0.072	0.846	77.2	0.0	9.4	+0.094	2.2	0.0	2.2	0.0	2.2	0.0	0.05
	B	58.9217	−0.259	0.754	81.9	0.1	9.2	−0.317	1.9	0.1	1.7	0.1	1.7	0.1	0.16
	C	58.4596	−0.093	0.665	85.9	0.1	8.9	−0.108	1.7	0.0	1.3	0.0	1.3	0.0	0.06
	D	61.8953	−0.056	0.597	88.6	0.0	8.8	−0.063	1.5	0.0	1.0	0.0	1.0	0.0	0.03
	E	68.6448	−3.917	0.556	90.2	2.1	8.7	−4.344	1.4	0.7	0.9	0.4	0.9	0.4	2.33
	F	77.8483	0.317	0.533	90.9	0.2	8.7	+0.349	1.3	0.1	0.8	0.0	0.8	0.0	0.19
	G	88.7468	2.518	0.526	91.2	1.3	8.7	+2.760	1.3	0.4	0.8	0.2	0.8	0.2	1.49
	H	100.7907	0.979	0.536	90.9	0.5	8.7	+1.077	1.3	0.2	0.8	0.1	0.8	0.1	0.58
	I	113.6167	−0.024	0.574	89.6	0.0	8.8	−0.027	1.4	0.0	0.9	0.0	0.9	0.0	0.01
	J	126.9915	−0.736	0.648	86.8	0.4	8.9	−0.848	1.6	0.2	1.2	0.1	1.2	0.1	0.44
	K	140.7597	0.807	0.746	82.5	0.4	9.2	+0.978	1.9	0.2	1.6	0.2	1.6	0.2	0.50
	M	109.2192	−0.409	0.312	96.9	0.2	8.4	−0.422	0.7	0.0	0.3	0.0	0.3	0.0	0.23
	N	167.5121	−1.113	0.472	93.1	0.6	8.7	−1.196	1.1	0.2	0.6	0.1	0.6	0.1	0.64
	O	209.3429	−0.115	0.541	91.0	0.1	8.9	−0.126	1.3	0.0	0.8	0.0	0.8	0.0	0.07
	P	119.8773	2.525	0.495	92.3	1.3	8.7	+2.737	1.2	0.4	0.7	0.2	0.7	0.2	1.48
	Q	79.6519	−2.822	0.336	96.4	1.4	8.4	−2.927	0.8	0.3	0.3	0.1	0.3	0.1	1.62

Adjusted horizontal distances (continued)

in	to	$\hat{s}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mm]	$\widehat{\nabla}$ [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
M	A	143.2299	1.452	0.760	81.9	0.8	9.2	+1.773	1.9	0.4	1.7	0.3	1.7	0.3	0.90
	B	129.3034	−1.220	0.665	86.1	0.7	9.0	−1.418	1.7	0.3	1.3	0.2	1.3	0.2	0.74
	C	115.7229	2.208	0.588	89.1	1.2	8.8	+2.479	1.4	0.4	1.0	0.3	1.0	0.3	1.32
	D	102.6212	−3.971	0.545	90.6	2.1	8.7	−4.384	1.3	0.7	0.8	0.4	0.8	0.4	2.35
	E	90.2074	0.280	0.531	91.0	0.1	8.7	+0.308	1.3	0.0	0.8	0.0	0.8	0.0	0.17
	F	78.8048	0.804	0.534	90.9	0.4	8.7	+0.884	1.3	0.1	0.8	0.1	0.8	0.1	0.48
	G	68.9166	−0.151	0.552	90.3	0.1	8.7	−0.168	1.4	0.0	0.8	0.0	0.8	0.0	0.09
	H	61.2812	0.193	0.588	89.0	0.1	8.8	+0.217	1.5	0.0	1.0	0.0	1.0	0.0	0.12
	I	56.8154	−2.048	0.650	86.5	1.1	8.9	−2.367	1.6	0.4	1.2	0.3	1.2	0.3	1.24
	J	56.2831	−1.395	0.738	82.6	0.8	9.1	−1.688	1.9	0.4	1.6	0.3	1.6	0.3	0.87
	K	59.7931	−0.988	0.832	77.9	0.6	9.4	−1.268	2.2	0.3	2.1	0.3	2.1	0.3	0.63
	L	109.2192	1.091	0.312	96.9	0.6	8.4	+1.126	0.7	0.1	0.3	0.0	0.3	0.0	0.62
	N	89.9779	−1.552	0.474	92.9	0.8	8.6	−1.672	1.1	0.2	0.6	0.1	0.6	0.1	0.91
	O	189.9800	2.408	0.539	91.0	1.2	8.8	+2.646	1.3	0.4	0.8	0.2	0.8	0.2	1.40
	P	179.7174	0.121	0.491	92.5	0.1	8.7	+0.131	1.2	0.0	0.7	0.0	0.7	0.0	0.07
	Q	79.3486	−3.532	0.323	96.7	1.8	8.4	−3.654	0.8	0.3	0.3	0.1	0.3	0.1	2.03
N	L	167.5121	−0.313	0.472	93.1	0.2	8.7	−0.336	1.1	0.0	0.6	0.0	0.6	0.0	0.18
	M	89.9779	0.048	0.474	92.9	0.0	8.6	+0.051	1.1	0.0	0.6	0.0	0.6	0.0	0.03
	O	120.6715	−0.790	0.519	91.5	0.4	8.7	−0.863	1.3	0.1	0.7	0.1	0.7	0.1	0.46
	P	176.8800	3.000	0.511	91.9	1.5	8.8	+3.264	1.2	0.5	0.7	0.3	0.7	0.3	1.74
	Q	94.3674	3.089	0.420	94.4	1.6	8.5	+3.273	1.0	0.4	0.5	0.2	0.5	0.2	1.79

Adjusted horizontal distances (continued)

in	to	$\hat{s}$ [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mm]	$\widehat{\nabla}$ [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
O	L	209.3429	0.985	0.541	91.0	0.5	8.9	+1.082	1.3	0.2	0.8	0.1	0.8	0.1	0.57
	M	189.9800	−0.492	0.539	91.0	0.3	8.8	−0.540	1.3	0.1	0.8	0.0	0.8	0.0	0.29
	N	120.6715	−2.190	0.519	91.5	1.1	8.7	−2.394	1.3	0.3	0.7	0.2	0.7	0.2	1.29
	P	137.9449	−1.401	0.521	91.5	0.7	8.7	−1.531	1.3	0.2	0.7	0.1	0.7	0.1	0.82
	Q	135.8628	−3.386	0.488	92.5	1.7	8.7	−3.660	1.2	0.5	0.6	0.3	0.6	0.3	1.97
P	L	119.8773	−0.775	0.495	92.3	0.4	8.7	−0.840	1.2	0.1	0.7	0.1	0.7	0.1	0.45
	M	179.7174	−1.179	0.491	92.5	0.6	8.7	−1.274	1.2	0.2	0.7	0.1	0.7	0.1	0.68
	N	176.8800	3.000	0.511	91.9	1.5	8.8	+3.264	1.2	0.5	0.7	0.3	0.7	0.3	1.74
	O	137.9449	1.099	0.521	91.5	0.6	8.7	+1.202	1.3	0.2	0.7	0.1	0.7	0.1	0.64
	Q	101.2074	−0.293	0.449	93.6	0.2	8.6	−0.312	1.1	0.0	0.5	0.0	0.5	0.0	0.17
Q	A	139.7540	−4.367	0.849	77.4	2.5	9.5	−5.645	2.2	1.3	2.1	1.3	2.1	1.3	2.78
	B	131.3428	0.446	0.766	81.5	0.2	9.2	+0.548	2.0	0.1	1.7	0.1	1.7	0.1	0.28
	C	124.3231	−1.803	0.688	85.1	1.0	9.0	−2.119	1.7	0.4	1.3	0.3	1.3	0.3	1.10
	D	118.9375	2.220	0.630	87.5	1.2	8.9	+2.539	1.6	0.4	1.1	0.3	1.1	0.3	1.33
	E	115.4155	1.654	0.598	88.7	0.9	8.8	+1.865	1.5	0.3	1.0	0.2	1.0	0.2	0.99
	F	113.9275	4.166	0.587	89.1	2.2	8.8	+4.675	1.4	0.8	1.0	0.5	1.0	0.5	2.48
	G	114.5537	1.450	0.593	88.9	0.8	8.8	+1.632	1.5	0.3	1.0	0.2	1.0	0.2	0.86
	H	117.2595	−1.212	0.620	87.9	0.6	8.9	−1.379	1.5	0.2	1.1	0.2	1.1	0.2	0.73
	I	121.9067	1.830	0.673	85.7	1.0	9.0	+2.135	1.7	0.4	1.3	0.3	1.3	0.3	1.11
	J	128.2878	−1.130	0.751	82.2	0.6	9.2	−1.374	1.9	0.3	1.6	0.2	1.6	0.2	0.70
	K	136.1604	0.590	0.837	78.0	0.3	9.4	+0.757	2.2	0.2	2.1	0.2	2.1	0.2	0.37
	L	79.6519	0.178	0.336	96.4	0.1	8.4	+0.185	0.8	0.0	0.3	0.0	0.3	0.0	0.10
	M	79.3486	0.168	0.323	96.7	0.1	8.4	+0.174	0.8	0.0	0.3	0.0	0.3	0.0	0.10
	N	94.3674	−4.011	0.420	94.4	2.1	8.5	−4.249	1.0	0.5	0.5	0.2	0.5	0.2	2.33
	O	135.8628	−1.286	0.488	92.5	0.7	8.7	−1.390	1.2	0.2	0.6	0.1	0.6	0.1	0.75
	P	101.2074	1.807	0.449	93.6	0.9	8.6	+1.931	1.1	0.2	0.5	0.1	0.5	0.1	1.05

Supplementary information

Observed directions	:	63
Observed distances	:	63
Orientation unknowns	:	6
Coordinate unknowns	:	34
Datum defect	:	3
Datum definition	:	free
Number of datum constraints	:	3
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	37.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	3.2
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$2.6 \cdot 10^{-13}$
Redundancy r	:	89
Redundancy directions	:	32.60
Redundancy distances	:	56.40
Weighted square sum of residuals Ω [-]	:	69.48172
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	0.883569
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.8836
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	0.7807 ($k^F_{\alpha_G;r,\infty}$ =1.04)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.167
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.069291

3.6 Trilateration-Triangulation networks

3.6.1 Ghilani (2010), Ex. 16.1

Ghilani (2010): Adjustment Computations. Spatial Data Analysis. 5th Edition. Ex. 16.1, pp. 300

Available data files: [2D] Ghilani16_1_Traverse∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
Q	1000.0000 (D)	800.0000 (D)
R	1000.0000 (D)	1000.0000 (D)
U	1173.2000	1100.0000
S	1223.0000 (D)	1186.5000 (D)
T	1400.0000 (D)	1186.5000 (D)

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
R	U	200	50	400.00
U	S	100	80	156.25

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
R	Q	U	266.666 666 7	9.2593	$4.727\,24{\cdot}10^7$
S	U	T	266.685 185 2	9.2593	$4.727\,24{\cdot}10^7$
U	R	S	166.666 666 7	9.2593	$4.727\,24{\cdot}10^7$

Design matrix A[-] and reduced observation vector Δy[m] distances (1. iteration)

A	x_Q	y_Q	x_R	y_R	x_U	y_U	x_S	y_S	x_T	y_T	Δy
$s_{R,U}$	0	0	−0.8660	−0.5000	0.8660	0.5000	0.0000	0.0000	0	0	0.0044
$s_{U,S}$	0	0	0.0000	0.0000	−0.4989	−0.8666	0.4989	0.8666	0	0	0.1887

Design matrix A[mgon/m] and reduced observation vector Δy[mgon] angles (1. iteration)

A	x_Q	y_Q	x_R	y_R	x_U	y_U	x_S	y_S	x_T	y_T	Δy
$\alpha_{R,Q,U}$	318.3099	0.0000	−477.4718	275.6685	159.1619	−275.6685	0.0000	0.0000	0.0000	0.0000	0.8086
$\alpha_{S,U,T}$	0.0000	0.0000	0.0000	0.0000	552.7606	−318.2367	−552.7606	677.9089	0.0000	−359.6722	−59.2544
$\alpha_{U,R,S}$	0.0000	0.0000	159.1619	−275.6685	−711.9225	593.9052	552.7606	−318.2367	0.0000	0.0000	76.9643

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_{Q}	y_{Q}	x_{R}	y_{R}	x_{U}	y_{U}	x_{S}	y_{S}	x_{T}	y_{T}
D^{T}	1	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	1

Least squares solution $\widehat{\Delta x}_{[\mathrm{cm}]}$ (1. iteration)

	x_{Q}	y_{Q}	x_{R}	y_{R}	x_{U}	y_{U}	x_{S}	y_{S}	x_{T}	y_{T}
$\widehat{\Delta x}^{\mathsf{T}}$	0	0	0	0	−11.14	−1.28	0	0	0	0

Adjusted coordinates

Point name	$\hat{x}_{[\mathrm{m}]}$	$\hat{x} - x_{[\mathrm{cm}]}$	$\lvert\hat{\sigma}\rvert_{[\mathrm{cm}]}$	$\hat{y}_{[\mathrm{m}]}$	$\hat{y} - y_{[\mathrm{cm}]}$	$\lvert\hat{\sigma}\rvert_{[\mathrm{cm}]}$	$\lvert\hat{\sigma}_{2D}\rvert_{[\mathrm{cm}]}$
U	1173.0886	−11.136	4.194	1099.9872	−1.277	5.264	6.730

Absolute error ellipses

in	$A_{[\mathrm{cm}]}$	$B_{[\mathrm{cm}]}$	$\phi_{[\mathrm{gon}]}$
U	6.57	1.45	42.080 159

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	$A_{[\mathrm{cm}]}$	$B_{[\mathrm{cm}]}$	$\phi_{[\mathrm{gon}]}$
U	28.73	6.34	42.080 159

Adjusted horizontal distances

Variance component: $\Omega = 6.926,\ r = 1.35, \hat{\sigma}_0^2 = 5.14,\ \alpha_G = 0.15\%,\ k^F_{\alpha_G;r,\infty} = 8.27$

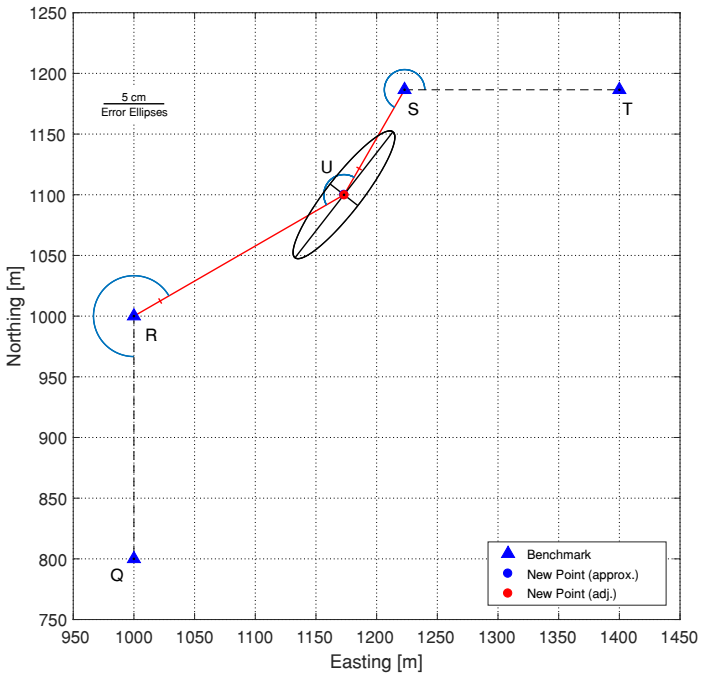
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
R	U	199.8928	10.722	6.113	54.8	2.9	27.9	+19.562	3.8	2.6	126.1	88.4	126.1	88.4	1.59
U	S	99.8779	12.206	6.513	80.0	1.7	37.0	+15.265	2.1	0.9	74.1	30.6	74.1	30.6	0.94

Adjusted horizontal angles

Variance component: $\Omega = 2.997,\ r = 1.65, \hat{\sigma}_0^2 = 1.81,\ \alpha_G = 0.21\%,\ k^F_{\alpha_G;r,\infty} = 6.91$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
R	Q	U	266.651 645	15.022	8.966	71.65	1.92	45.20	+20.96	2.60	1.21	12.8	5.9	1.05
S	U	T	266.686 983	−1.798	10.810	58.79	0.25	49.90	−3.06	3.46	0.21	20.6	1.3	0.14
U	R	S	166.661 372	5.295	13.599	34.78	0.97	64.87	+15.22	5.66	1.33	42.3	9.9	0.53

Network graph



Supplementary information

Observed distances	:	2
Observed angles	:	3
Coordinate unknowns	:	2
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	8
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.6
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.7
Number of iterations (Max=20)	:	4
Stop criterion (actual)	:	$3.1 \cdot 10^{-12}$
Redundancy r	:	3
Redundancy distances	:	1.35
Redundancy angles	:	1.65
Weighted square sum of residuals Ω [-]	:	9.92316
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	1.81871
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.8187
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	3.3077 ($k^F_{\alpha_G;r,\infty}$ =4.21)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	11.209
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	45.294

3.6.2 Ghilani (2010), Ex. 16.2

Ghilani Charles D. (2010): Adjustment Computations. Spatial Data Analysis. Fifth Edition, John Wiley & Sons, Inc., ISBN 978-0-470-46491-5, Ex. 16.2, pp. 307/528

Available data files: [2D] Ghilani16_2_DistanceAngleAzimuth_fix∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
Q	1000.0000 (D)	1000.0000 (D)
R	1003.0600	2640.0100
S	2323.0700	2638.4700
T	2661.7500	1096.0700

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
Q	R	1640.0160	26	1479.29
	S	2105.9620	29	1189.06
R	S	1320.0010	24	1736.11
	T	2266.0350	30	1111.11
S	T	1579.1230	25	1600.00
T	Q	1664.5240	26	1479.29

Grid bearings

in	to	T [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
Q	R	0.118 672 8	0.000 31	$4.254\,52\cdot10^{16}$

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
Q	R	S	43.126 759 3	1.2346	$2.659\,07\cdot10^{9}$
	S	T	53.077 901 2	1.2346	$2.659\,07\cdot10^{9}$
	T	R	303.795 216 0	1.3580	$2.197\,58\cdot10^{9}$
R	Q	S	299.954 753 1	1.4506	$1.925\,99\cdot10^{9}$
	S	T	47.645 370 4	1.3272	$2.300\,98\cdot10^{9}$
	S	Q	100.045 277 8	1.3889	$2.101\,00\cdot10^{9}$
S	R	T	286.165 679 0	1.4506	$1.925\,99\cdot10^{9}$
	Q	R	56.828 703 7	1.3272	$2.300\,98\cdot10^{9}$
	T	Q	57.005 000 0	1.2346	$2.659\,07\cdot10^{9}$
T	S	Q	310.083 703 7	1.3889	$2.101\,00\cdot10^{9}$
	R	S	38.520 277 8	1.2346	$2.659\,07\cdot10^{9}$

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ grid bearings (1. iteration)

A	x_Q	y_Q	x_R	y_R	x_S	y_S	x_T	y_T	Δy
$T_{\text{Q,R}}$	−38.8179	0.0724	38.8179	−0.0724	0	0	0	0	−0.11

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ distances (1. iteration)

A	x_Q	y_Q	x_R	y_R	x_S	y_S	x_T	y_T	Δy
$s_{\text{Q,R}}$	−0.0019	−1.0000	0.0019	1.0000	0.0000	0.0000	0.0000	0.0000	0.315
$s_{\text{Q,S}}$	−0.6282	−0.7780	0.0000	0.0000	0.6282	0.7780	0.0000	0.0000	−0.528
$s_{\text{R,S}}$	0.0000	0.0000	−1.0000	0.0012	1.0000	−0.0012	0.0000	0.0000	−0.990
$s_{\text{R,T}}$	0.0000	0.0000	−0.7320	0.6813	0.0000	0.0000	0.7320	−0.6813	−1.955
$s_{\text{S,T}}$	0.0000	0.0000	0.0000	0.0000	−0.2145	0.9767	0.2145	−0.9767	−2.294
$s_{\text{T,Q}}$	−0.9983	−0.0577	0.0000	0.0000	0.0000	0.0000	0.9983	0.0577	−0.071

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ angles (1. iteration)

A	x_Q	y_Q	x_R	y_R	x_S	y_S	x_T	y_T	Δy
$\alpha_{\text{Q,R,S}}$	15.2991	18.9191	−38.8179	0.0724	23.5188	−18.9915	0.0000	0.0000	−0.0028
$\alpha_{\text{Q,S,T}}$	21.3114	19.1911	0.0000	0.0000	−23.5188	18.9915	2.2074	−38.1826	−0.1848
$\alpha_{\text{Q,T,R}}$	−36.6105	−38.1102	38.8179	−0.0724	0.0000	0.0000	−2.2074	38.1826	0.0641
$\alpha_{\text{R,Q,S}}$	38.8179	−0.0724	−38.7616	48.3008	−0.0563	−48.2283	0.0000	0.0000	−0.7356
$\alpha_{\text{R,S,T}}$	0.0000	0.0000	19.0850	−27.6645	0.0563	48.2283	−19.1412	−20.5639	−0.3293
$\alpha_{\text{R,S,Q}}$	−38.8179	0.0724	38.7616	−48.3008	0.0563	48.2283	0.0000	0.0000	0.7664
$\alpha_{\text{S,R,T}}$	0.0000	0.0000	−0.0563	−48.2283	39.4324	56.8745	−39.3761	−8.6462	0.4578
$\alpha_{\text{S,Q,R}}$	23.5188	−18.9915	0.0563	48.2283	−23.5751	−29.2368	0.0000	0.0000	−0.0229
$\alpha_{\text{S,T,Q}}$	−23.5188	18.9915	0.0000	0.0000	−15.8573	−27.6377	39.3761	8.6462	−1.0522
$\alpha_{\text{T,S,Q}}$	−2.2074	38.1826	0.0000	0.0000	−39.3761	−8.6462	41.5835	−29.5364	−0.4346
$\alpha_{\text{T,R,S}}$	0.0000	0.0000	−19.1412	−20.5639	39.3761	8.6462	−20.2349	11.9177	0.7562

Matrix $D^\text{T}{}_{[\text{-}]}$ of datum constraints

	x_Q	y_Q	x_R	y_R	x_S	y_S	x_T	y_T
D^T	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (1. iteration)

	x_Q	y_Q	x_R	y_R	x_S	y_S	x_T	y_T
$\widehat{\Delta x}^\text{T}$	0	0	−0.285	−0.492	−0.735	0.420	−1.139	1.671

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
R	1003.0572	−0.285	0.001	2640.0051	−0.492	0.597	0.597
S	2323.0626	−0.735	0.549	2638.4742	0.420	0.660	0.858
T	2661.7386	−1.139	0.590	1096.0867	1.671	0.727	0.936

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
R	0.60	0.00	0.118 673
S	0.68	0.52	173.648 263
T	0.77	0.54	29.094 063

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
R	1.66	0.00	0.118 673
S	1.91	1.45	173.648 263
T	2.13	1.50	29.094 063

Adjusted grid bearings

Variance component: $\Omega = 0.000,\ r = 0.00, \hat{\sigma}_0^2 = 0.00$

in	to	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\widehat{T}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
Q	R	0.118 672 8	0	0.007	0	∞	21 396.89	∞	∞	∞	551 209.3	∞	551 209.3	∞	∞

Adjusted horizontal distances

Variance component: $\Omega = 0.462,\ r = 3.72, \hat{\sigma}_0^2 = 0.12,\ \alpha_G = 0.79\%,\ k^F_{\alpha_G;r,\infty} = 3.57$

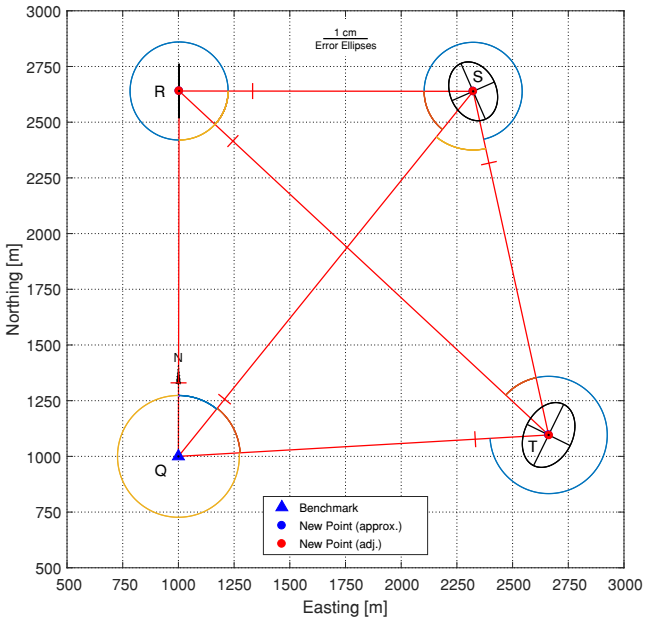
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
Q	R	1640.0079	0.807	0.597	57.6	0.4	14.2	+1.403	3.5	0.4	60.1	6.0	60.1	6.0	1.16
	S	2105.9659	−0.393	0.558	70.2	0.2	14.3	−0.559	2.7	0.1	42.6	1.7	42.6	1.7	0.46
R	S	1320.0064	−0.539	0.549	57.9	0.3	13.0	−0.930	3.5	0.3	54.9	3.9	54.9	3.9	0.84
	T	2266.0336	0.144	0.580	70.0	0.1	14.8	+0.205	2.7	0.0	44.5	0.6	44.5	0.6	0.16
S	T	1579.1329	−0.986	0.560	59.7	0.5	13.4	−1.652	3.4	0.4	53.9	6.7	53.9	6.7	1.45
T	Q	1664.5143	0.970	0.602	56.9	0.5	14.2	+1.705	3.6	0.4	61.4	7.3	61.4	7.3	1.40

Adjusted horizontal angles

Variance component: $\Omega = 1.030,\ r = 8.28, \hat{\sigma}_0^2 = 0.12,\ \alpha_G = 3.00\%,\ k^F_{\alpha_G;r,\infty} = 2.10$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
Q	R	S	43.126 620	0.140	0.197	79.49	0.13	5.72	+0.18	2.10	0.06	1.2	0.0	0.36
	S	T	53.077 676	0.226	0.214	75.74	0.21	5.86	+0.30	2.34	0.12	1.4	0.1	0.60
	T	R	303.795 705	−0.489	0.274	67.17	0.44	6.85	−0.73	2.89	0.31	2.2	0.2	1.25
R	Q	S	299.955 159	−0.406	0.247	76.70	0.32	6.84	−0.53	2.28	0.18	1.6	0.1	0.91
	S	T	47.645 858	−0.488	0.198	82.08	0.41	6.05	−0.59	1.93	0.19	1.1	0.1	1.15
	S	Q	100.044 841	0.437	0.247	74.59	0.36	6.65	+0.59	2.41	0.21	1.7	0.1	1.03
S	R	T	286.165 712	−0.033	0.272	71.64	0.03	7.08	−0.05	2.60	0.02	2.0	0.0	0.08
	Q	R	56.828 539	0.164	0.226	76.70	0.14	6.26	+0.21	2.28	0.08	1.5	0.0	0.40
	T	Q	57.005 749	−0.749	0.230	72.18	0.71	6.00	−1.04	2.57	0.44	1.7	0.3	2.02
T	S	Q	310.083 424	0.280	0.268	70.00	0.24	6.86	+0.40	2.70	0.16	2.1	0.1	0.68
	R	S	38.519 854	0.424	0.188	81.45	0.38	5.65	+0.52	1.97	0.18	1.0	0.1	1.08

Network graph



Supplementary information

Observed grid bearings	:	1
Observed distances	:	6
Observed angles	:	11
Coordinate unknowns	:	6
Datum defect	:	2
Datum definition	:	fix
Number of datum constraints	:	2
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	5.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.8
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$1.1 \cdot 10^{-12}$
Redundancy r	:	12
Redundancy grid bearings	:	0.00
Redundancy distances	:	3.72
Redundancy angles	:	8.28
Weighted square sum of residuals Ω [-]	:	1.49205
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	$3.52616 \cdot 10^{-1}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.3526
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	0.1243 ($k^F_{\alpha_G;r,\infty}$ =1.73)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	2.265
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	1.9704

3.6.3 Ghilani (2010), Ex. 21.1

Ghilani (2010): Adjustment Computations. Spatial Data Analysis. 5th Edition. Ex. 21.1, pp. 446

Available data files: [2D] Ghilani21_1_DistanceAngle_fix*.*

This network/observation scenario is a great example for the application of outlier detection, using both the DIA-approach by Baarda (DIA=Detection, Identification and Adaptation) and the Pope-test. The original list of observations contains a 50m distance blunder and a 2.2gon angle blunder. In two successive steps, first the corrupted distance is eliminated (thus reducing the test quantity of the global test – ratio of a posteriori and a priori variance – from 232982 to 938), then the erroneous angle is deleted. In the final set up, no further outlier is detected. As can be seen from the network graphs (in particular the scale bar of the error ellipses), the point precision is drastically improved.

Coordinates

Point name	ID	Easting x [m]	Northing y [m]
1	1	2 477 233.7200	420 353.5900
2	2	2 477 497.8900	419 951.9800
3	3	2 477 832.5500	420 210.2100
4	4	2 477 991.6400	420 400.5800
5	5	2 477 630.4300	420 567.4500
6	6	2 477 665.2200	420 323.3200
102	10	2 476 455.8900	419 741.3800
103	01	2 476 735.0500	419 912.4200
201	20	2 476 576.2300	419 589.2300
202	02	2 476 948.7400	419 331.2900
203	03	2 477 463.8400	419 819.5800
2000	04	2 476 334.6000 (D)	419 710.0900 (D)
2001	05	2 476 297.9800 (D)	419 266.8200 (D)

Datum: fix, (D)...Datum coordinate

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ distances (1. iteration)

Too large to be displayed !

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ angles (1. iteration)

Too large to be displayed !

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5	x_6	y_6	x_{10}	y_{10}	x_{01}	y_{01}	x_{20}	y_{20}	x_{02}	y_{02}	x_{03}	y_{03}	x_{04}	y_{04}	x_{05}	y_{05}
D^{T}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [–]
1	2	480.7100	22	1
	3	615.7400	22	1
3	1	615.7400	22	1
	2	422.7000	22	1
	4	298.1000*	22	1
	5	410.4400	22	1
	6	201.9800	22	1
5	1	450.6700	22	1
	2	629.5800	22	1
	3	410.4600	22	1
	4	397.8900	22	1
	6	246.6100	22	1
10	01	327.3700	22	1
01	1	665.7900	22	1
20	02	453.1000	22	1
02	03	709.7800	22	1
03	3	537.1800	22	1
04	10	125.2400	22	1
05	20	425.9000	22	1

* 50 m blunder

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [m²/rad²]
1	2	3	352.000 154 3	1.944	518 817.395 40
3	2	1	56.799 691 4	2.222	397 219.568 35
	2	4	186.156 790 1	4.475	97 939.892 62
	2	5	109.066 821 0	3.179	194 098.052 82
	2	6	79.682 561 7	4.660	90 311.225 05
	03	2	9.998 765 4	2.006	487 381.359 13
5	1	3	298.695 216 0	3.025	214 409.229 73
	2	3	353.720 524 7	2.160	420 242.090 27
	3	4	360.328 395 1	2.500	313 852.498 45
	6	3	376.234 104 9	3.302	179 857.301 28
10	04	01	181.079 012 3	8.920	24 654.712 50
01	10	1	191.142 901 2*	3.6420	147 887.549 72
20	05	02	293.227 993 8	2.994	218 852.826 27
02	20	03	113.146 604 9	2.500	313 852.498 45
03	02	3	196.466 049 4	2.593	291 834.784 91
04	10	05	121.312 963 0	7.870	31 667.608 49
05	04	20	40.082 160 5	2.284	376 038.393 41

* 2.22 gon blunder

3.6.3.1 Original observation scenario

Least squares solution $\widehat{\Delta x}_{[m]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5
$\widehat{\Delta x}^T$	3.1543	−1.9583	2.1790	−2.9197	3.1275	−4.0539	15.4756	9.4043	1.3767	−1.2991

	x_6	y_6	x_{10}	y_{10}	x_{01}	y_{01}	x_{20}	y_{20}	x_{02}	y_{02}
$\widehat{\Delta x}^T$	2.0778	−2.4505	−0.4768	0.9673	−3.7646	6.0327	0.3875	−0.2276	0.0277	−1.2278

	x_{03}	y_{03}	x_{04}	y_{04}	x_{05}	y_{05}
$\widehat{\Delta x}^T$	1.6635	−2.7979	0.0000	0.0000	0.0000	0.0000

Adjusted coordinates

ID	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
1	2 477 236.7770	305.705	2613.810	420 351.5745	−201.552	2758.232	3799.980
2	2 477 500.0185	212.850	1886.194	419 949.0581	−292.189	3288.278	3790.844
3	2 477 835.6134	306.336	2297.210	420 206.1767	−403.330	4172.628	4763.192
4	2 478 007.5936	1595.362	2883.889	420 410.1663	958.629	4726.375	5536.735
5	2 477 631.6261	119.606	3263.607	420 566.1543	−129.573	3631.968	4882.860
6	2 477 667.1989	197.887	2625.872	420 320.8860	−243.402	3762.002	4587.795
10	2 476 455.4193	−47.069	993.129	419 742.3499	96.993	602.276	1161.483
01	2 476 731.2541	−379.594	1444.469	419 918.4377	601.771	1330.976	1964.176
20	2 476 576.6086	37.863	832.853	419 588.9965	−23.348	907.677	1231.877
02	2 476 948.7591	1.913	1240.969	419 330.0639	−122.612	1649.433	2064.131
03	2 477 465.4687	162.871	1564.914	419 816.7925	−278.749	3028.710	3409.111

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
1	3501.30	1476.74	152.452 816
2	3475.18	1514.46	176.592 042
3	4555.58	1390.92	172.304 550
4	5323.46	1521.90	168.113 042
5	4654.00	1477.36	154.163 006
6	4351.53	1453.29	164.197 377

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
10	1013.90	566.62	84.385 885
01	1498.61	1269.70	133.440 665
20	1007.15	709.34	41.800 223
02	1685.85	1191.03	18.870 616
03	3136.80	1335.12	181.428 760

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 2259345.624$, $r = 8.52$, $\hat{\sigma}_0^2/\sigma_0^2 = 265227.41$, $\alpha_G = 3.14\%$, $k^F_{\alpha_G;r,\infty} = 2.07$

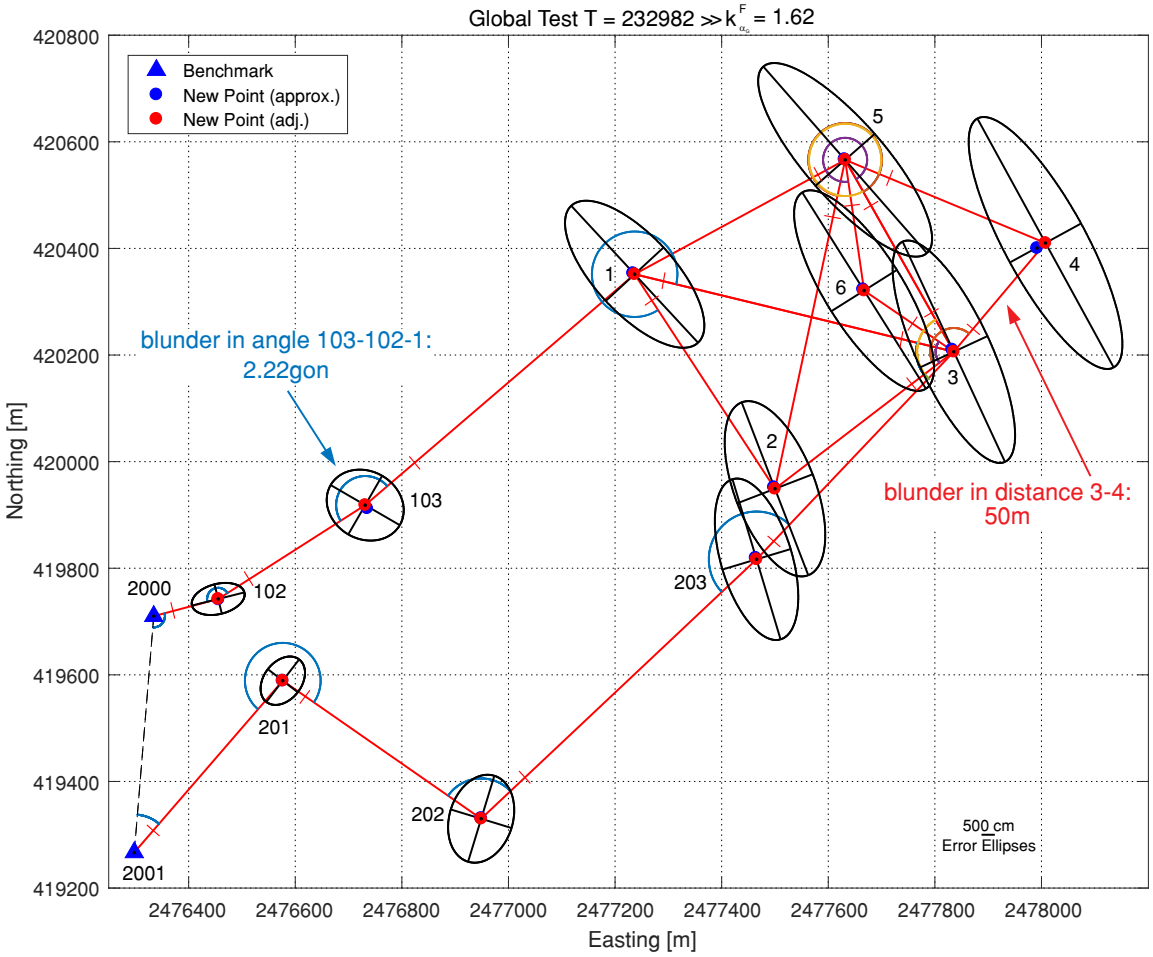
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	2	480.9527	−24.268	592.060	68.91	13	10.95	−35.2 *	2.78	9	34.0	109.5	34.0	109.5	0.03
	3	616.2349	−49.489	535.554	74.56	26	10.53	−66.4 *	2.41	15	26.8	168.8	26.8	168.8	0.05
3	1	616.2349	−49.489	535.554	74.56	26	10.53	−66.4 *	2.41	15	26.8	168.8	26.8	168.8	0.05
	2	422.7693	−6.930	545.872	73.57	4	10.60	−9.4 *	2.48	2	28.0	24.9	28.0	24.9	0.01
	4	266.8126	3128.740	652.490	62.24	1803	11.52	+5026.5 #	3.22	1404	43.5	18 978.0	43.5	18 978.0	3.73#
	5	413.7568	−331.678	505.888	77.30	171	10.34	−429.1 *	2.24	93	23.5	973.8	23.5	973.8	0.36
	6	203.7686	−178.865	688.888	57.91	107	11.95	−308.8 *	3.52	91	50.3	1299.8	50.3	1299.8	0.22
5	1	449.3887	128.127	719.233	54.13	79	12.36	+236.7 *	3.80	73	56.7	1086.0	56.7	1086.0	0.16
	2	630.9740	−139.403	601.224	67.94	77	11.03	−205.2 *	2.84	53	35.4	657.7	35.4	657.7	0.16
	3	413.7568	−329.678	505.888	77.30	170	10.34	−426.5 *	2.24	92	23.5	967.9	23.5	967.9	0.35
	4	407.0428	−915.282	781.854	45.79	615	13.43	−1998.9 *	4.50	669	72.8	10 836.1	72.8	10 836.1	1.27
	6	247.8345	−122.454	709.969	55.30	75	12.22	−221.4 *	3.72	67	54.6	989.8	54.6	989.8	0.16
10	01	327.2487	12.129	1005.568	10.33	17	28.29	+117.4 *	12.18	51	253.7	1053.1	253.7	1053.1	0.04
01	1	665.7034	8.665	1004.849	10.46	12	28.11	+82.9 *	12.09	36	251.7	742.0	251.7	742.0	0.03
20	02	453.3675	−26.752	1052.586	1.75	92	68.79	−1531.7 *	30.99	690	675.9	15 049.8	675.9	15 049.8	0.19
02	03	709.8546	−7.460	1005.071	10.42	11	28.17	−71.6 *	12.12	31	252.3	641.6	252.3	641.6	0.02
03	3	537.2403	−6.027	1005.627	10.32	9	28.30	−58.4 *	12.18	25	253.8	523.9	253.8	523.9	0.02
04	10	125.0520	18.798	1013.811	8.85	29	30.55	+212.4 *	13.26	92	278.5	1935.6	278.5	1935.6	0.06
05	20	425.9479	−4.791	1006.346	10.19	7	28.48	−47.0 *	12.27	20	255.8	422.3	255.8	422.3	0.01

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 1002398.564$, $r = 5.48$, $\hat{\sigma}_0^2/\sigma_0^2 = 182870.14$, $\alpha_G = 1.52\%$, $k^F_{\alpha_G;r,\infty} = 2.71$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
1	2	3	352.035 132	−34.977	720.482	41.07	28	12.54	−85.16*	4.95	34	7.4	50.2	0.06
3	2	1	56.783 516	16.175	760.126	49.78	10	13.01	+32.49*	4.15	10	6.5	16.3	0.02
	2	4	186.212 771	−55.981	1779.045	32.17	22	32.60	−174.00*	6.00	32	22.1	118.0	0.05
	2	5	108.798 916	267.905	862.287	68.42	102	15.88	+391.55*	2.81	69	5.0	123.6	0.21
	2	6	79.685 655	−3.094	1746.968	39.69	1	30.57	−7.79	5.09	1	18.4	4.7	0.00
	03	2	9.992 482	6.284	941.354	5.50	13	35.36	+114.32*	17.13	55	33.4	108.0	0.03
5	1	3	298.869 983	−174.766	1002.129	52.88	79	17.19	−330.47*	3.90	75	8.1	155.7	0.16
	2	3	353.802 400	−81.875	577.672	69.31	46	10.72	−118.12*	2.75	30	3.3	36.2	0.09
	3	4	357.858 023	2470.372	994.688	32.05	1745	18.25	+7707.21#	6.02	2541	12.4	5236.8	3.62*
	6	3	376.348 490	−114.385	1330.096	30.37	63	24.76	−376.58*	6.26	95	17.2	262.2	0.13
10	04	01	180.440 404	638.608	3279.278	41.99	110	56.88	+1520.98*	4.86	130	33.0	882.4	0.23
01	10	1	191.070 145	72.756	1729.960	3.16	112	84.72	+2305.90*	22.89	623	82.0	2233.1	0.23
20	05	02	293.305 431	−77.437	1400.083	6.13	104	49.97	−1263.48*	16.17	409	46.9	1186.0	0.22
02	20	03	113.202 437	−55.832	1113.748	14.81	58	26.84	−376.90*	9.91	139	22.9	321.1	0.12
03	02	3	196.486 191	−20.141	1178.740	11.28	23	31.90	−178.64*	11.59	65	28.3	158.5	0.05
04	10	05	121.858 264	−545.301	2885.343	42.31	107	50.00	−1288.75*	4.82	124	28.8	743.4	0.22
05	04	20	40.146 262	−64.102	1061.888	7.22	104	35.13	−888.04*	14.81	375	32.6	823.9	0.22

Network graph



3.6.3.2 Distance observation, blunder removed

Least squares solution $\widehat{\Delta x}_{[m]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5
$\widehat{\Delta x}^T$	3.3575	−2.0170	2.0361	−2.8157	2.8395	−3.8945	3.4518	−4.4099	3.9926	−3.2601

	x_6	y_6	x_{10}	y_{10}	x_{01}	y_{01}	x_{20}	y_{20}	x_{02}	y_{02}
$\widehat{\Delta x}^T$	3.2102	−3.3653	−0.4364	0.9743	−3.6640	6.0247	0.3464	−0.2479	−0.0021	−1.2047

	x_{03}	y_{03}	x_{04}	y_{04}	x_{05}	y_{05}
$\widehat{\Delta x}^T$	1.5111	−2.7079	0	0	0	0

Adjusted coordinates

ID	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
1	2 477 236.9744	325.436	165.845	420 351.5440	−204.598	175.038	241.128
2	2 477 499.8614	197.143	119.681	419 949.1657	−281.426	208.625	240.516
3	2 477 835.3008	275.085	145.783	420 206.3463	−386.374	264.711	302.200
4	2 477 994.9850	334.495	181.470	420 396.2147	−436.531	298.271	349.137
5	2 477 634.3011	387.107	206.759	420 564.2001	−324.993	230.921	309.958
6	2 477 668.3317	311.169	166.448	420 319.9697	−335.025	238.875	291.147
10	2 476 455.4632	−42.677	63.015	419 742.3601	98.011	38.225	73.702
01	2 476 731.3579	−369.209	91.655	419 918.4430	602.302	84.463	124.638
20	2 476 576.5638	33.381	52.841	419 588.9738	−25.622	57.592	78.160
02	2 476 948.7262	−1.378	78.738	419 330.0872	−120.275	104.656	130.968
03	2 477 465.3033	146.333	99.298	419 816.8872	−269.280	192.153	216.294

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
1	222.18	93.69	152.459 984
2	220.49	96.08	176.585 265
3	289.03	88.25	172.292 815
4	334.37	100.48	168.573 684
5	295.49	93.61	154.298 564
6	276.18	92.15	164.257 874
10	64.33	35.96	84.388 923
01	95.10	80.57	133.475 978
20	63.90	45.00	41.797 112
02	106.97	75.57	18.868 510
03	199.02	84.71	181.421 326

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
1	612.96	258.49	152.459 984
2	608.30	265.07	176.585 265
3	797.38	243.46	172.292 815
4	922.46	277.21	168.573 684
5	815.19	258.24	154.298 564
6	761.93	254.23	164.257 874
10	177.48	99.22	84.388 923
01	262.36	222.27	133.475 978
20	176.30	124.16	41.797 112
02	295.10	208.48	18.868 510
03	549.05	233.70	181.421 326

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 211.620$, $r = 7.87$, $\hat{\sigma}_0^2/\sigma_0^2 = 26.90$, $\alpha_G = 2.77\%$, $k^F_{\alpha_G;r,\infty} = 2.17$

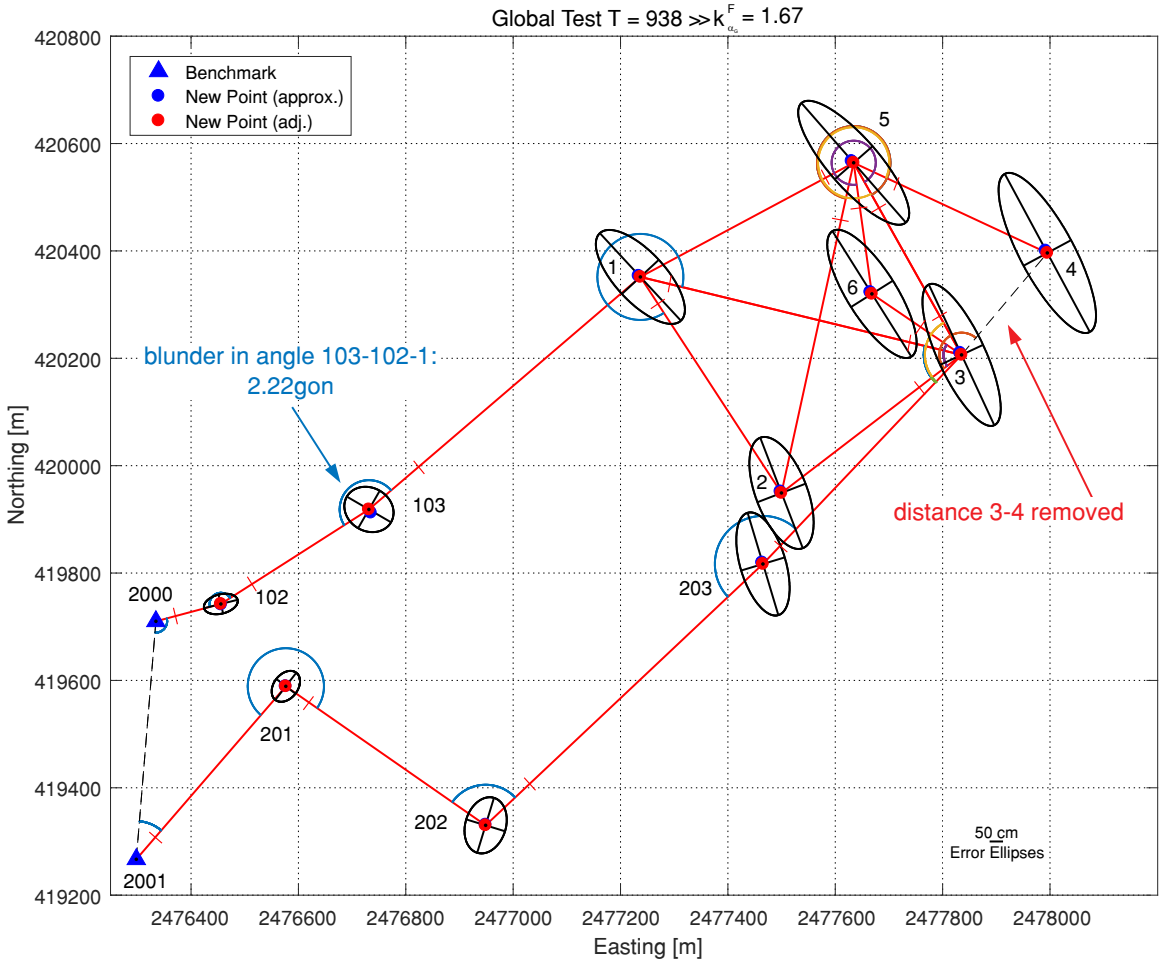
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	2	480.6432	6.680	37.583	68.89	3.66	10.95	+9.7 *	2.78	2.5	34.1	30.2	34.1	30.2	0.12
	3	615.6923	4.773	33.991	74.55	2.51	10.53	+6.4	2.41	1.5	26.8	16.3	26.8	16.3	0.08
3	1	615.6923	4.773	33.991	74.55	2.51	10.53	+6.4	2.41	1.5	26.8	16.3	26.8	16.3	0.08
	2	422.6836	1.640	34.622	73.60	0.87	10.60	+2.2	2.47	0.5	28.0	5.9	28.0	5.9	0.03
	5	410.4391	0.090	32.504	76.73	0.05	10.38	+0.1	2.28	0.0	24.2	0.3	24.2	0.3	0.00
	6	201.9629	1.714	43.651	58.03	1.02	11.93	+3.0	3.51	0.9	50.1	12.4	50.1	12.4	0.03
5	1	450.6563	1.369	45.606	54.19	0.85	12.35	+2.5	3.80	0.8	56.6	11.6	56.6	11.6	0.03
	2	629.5564	2.362	38.205	67.85	1.30	11.04	+3.5	2.84	0.9	35.5	11.2	35.5	11.2	0.04
	3	410.4391	2.090	32.504	76.73	1.08	10.38	+2.7	2.28	0.6	24.2	6.3	24.2	6.3	0.04
	4	397.8843	0.566	50.607	43.59	0.39	13.77	+1.3	4.70	0.4	77.7	7.3	77.7	7.3	0.01
	6	246.5898	2.019	44.915	55.56	1.23	12.20	+3.6	3.70	1.1	54.2	16.1	54.2	16.1	0.04
10	01	327.2966	7.340	63.803	10.33	10.38	28.28	+71.0 *	12.17	30.6	253.6	637.1	253.6	637.1	0.34
01	1	665.7511	3.894	63.758	10.46	5.47	28.11	+37.2 *	12.09	16.0	251.7	333.4	251.7	333.4	0.18
20	02	453.3510	−25.096	66.787	1.75	86.29	68.76	−1436.0 #	30.98	647.0	675.6	14 108.6	675.6	14 108.6	2.82*
02	03	709.8071	−2.709	63.772	10.42	3.81	28.16	−26.0 *	12.12	11.2	252.3	232.9	252.3	232.9	0.12
03	3	537.1932	−1.319	63.808	10.32	1.87	28.30	−12.8	12.18	5.5	253.8	114.6	253.8	114.6	0.06
04	10	125.0971	14.291	64.326	8.86	21.83	30.55	+161.4 *	13.26	70.0	278.4	1471.0	278.4	1471.0	0.71
05	20	425.9014	−0.140	63.853	10.19	0.20	28.48	−1.4	12.27	0.6	255.7	12.3	255.7	12.3	0.01

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 11982.322$, $r = 5.13$, $\hat{\sigma}_0^2/\sigma_0^2 = 2333.82$, $\alpha_G = 1.36\%$, $k^F_{\alpha_G;r,\infty} = 2.84$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
1	2	3	351.998 199	1.955	45.731	41.03	1.6	12.54	+4.77	4.95	1.9	7.4	2.8	0.05
3	2	1	56.797 500	2.192	48.268	49.70	1.4	13.02	+4.41	4.16	1.4	6.6	2.2	0.05
	2	4	186.157 736	−0.946	115.180	29.38	0.4	34.12	−3.22	6.41	0.6	24.1	2.3	0.01
	2	5	109.061 313	5.508	55.100	67.97	2.1	15.93	+8.10	2.84	1.4	5.1	2.6	0.07
	2	6	79.680 894	1.668	111.267	39.23	0.6	30.75	+4.25	5.14	0.7	18.7	2.6	0.02
	03	2	9.989 590	9.175	59.733	5.49	19.5	35.39	+167.23*	17.15	81.0	33.4	158.1	0.64
5	1	3	298.704 829	−9.613	63.641	52.80	4.4	17.20	−18.20*	3.91	4.1	8.1	8.6	0.14
	2	3	353.719 577	0.948	36.766	69.13	0.5	10.74	+1.37	2.76	0.4	3.3	0.4	0.02
	3	4	360.328 272	0.123	75.952	1.60	0.4	81.70	+7.71	32.42	3.1	80.4	7.6	0.01
	6	3	376.233 682	0.423	84.501	30.20	0.2	24.83	+1.40	6.28	0.4	17.3	1.0	0.01
10	04	01	180.446 712	632.300	208.076	41.99	109.4	56.88	+1506.00*	4.86	128.6	33.0	873.7	3.57*
01	10	1	191.071 486	71.415	109.768	3.16	110.4	84.73	+2263.55#	22.89	611.6	82.1	2192.1	3.60#
20	05	02	293.302 004	−74.010	88.837	6.13	99.9	49.97	−1207.46*	16.17	390.8	46.9	1133.5	3.26*
02	20	03	113.195 905	−49.300	70.668	14.81	51.2	26.84	−332.80*	9.91	122.9	22.9	283.5	1.67
03	02	3	196.480 242	−14.192	74.796	11.27	16.3	31.92	−125.97*	11.60	45.8	28.3	111.8	0.53
04	10	05	121.857 501	−544.538	183.075	42.31	106.4	49.99	−1286.88*	4.82	124.2	28.8	742.3	3.47*
05	04	20	40.143 419	−61.259	67.378	7.22	99.8	35.13	−848.58*	14.81	357.9	32.6	787.3	3.26*

Network graph



3.6.3.3 Distance and angle observations, blunders removed

Least squares solution $\widehat{\Delta x}_{[m]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5
$\widehat{\Delta x}^T$	0.0045	−0.0015	0.0043	−0.0046	−0.0034	0.0011	−0.0004	−0.0001	0.0046	−0.0013

	x_6	y_6	x_{10}	y_{10}	x_{01}	y_{01}	x_{20}	y_{20}	x_{02}	y_{02}
$\widehat{\Delta x}^T$	0.0024	−0.0016	0.0005	−0.0040	0.0016	−0.0030	0.0041	−0.0033	0.0016	−0.0027

	x_{03}	y_{03}	x_{04}	y_{04}	x_{05}	y_{05}
$\widehat{\Delta x}^T$	−0.0046	0.0037	0	0	0	0

Adjusted coordinates

ID	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
1	2 477 233.7245	0.447	7.136	420 353.5885	−0.146	6.864	9.901
2	2 477 497.8943	0.429	4.967	419 951.9754	−0.455	8.331	9.700
3	2 477 832.5466	−0.337	6.229	420 210.2111	0.112	10.695	12.377
4	2 477 991.6396	−0.037	7.710	420 400.5799	−0.007	12.058	14.313
5	2 477 630.4346	0.458	8.804	420 567.4487	−0.127	9.267	12.782
6	2 477 665.2224	0.241	7.089	420 323.3184	−0.161	9.589	11.925
10	2 476 455.8905	0.048	2.407	419 741.3760	−0.404	1.752	2.977
01	2 476 735.0516	0.155	5.115	419 912.4170	−0.296	7.018	8.684
20	2 476 576.2341	0.405	2.010	419 589.2267	−0.332	2.172	2.959
02	2 476 948.7416	0.164	2.948	419 331.2873	−0.272	4.113	5.061
03	2 477 463.8354	−0.457	4.048	419 819.5837	0.369	7.722	8.719

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
1	9.21	3.64	148.304 669
2	8.97	3.69	173.347 104
3	11.90	3.41	169.774 650
4	13.78	3.86	166.340 003
5	12.27	3.59	151.937 379
6	11.38	3.55	161.601 786
10	2.42	1.73	89.840 139
01	8.10	3.14	163.610 651
20	2.39	1.74	41.922 617
02	4.18	2.85	15.824 460
03	8.10	3.23	178.713 034

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
1	25.66	10.16	148.304 669
2	25.00	10.29	173.347 104
3	33.16	9.52	169.774 650
4	38.42	10.77	166.340 003
5	34.20	10.00	151.937 379
6	31.73	9.90	161.601 786
10	6.75	4.83	89.840 139
01	22.57	8.76	163.610 651
20	6.67	4.85	41.922 617
02	11.66	7.94	15.824 460
03	22.57	9.01	178.713 034

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 8.145$, $r = 7.84$, $\hat{\sigma}_0^2/\sigma_0^2 = 1.04$, $\alpha_G = 2.75\%$, $k^F_{\alpha_G;r,\infty} = 2.17$

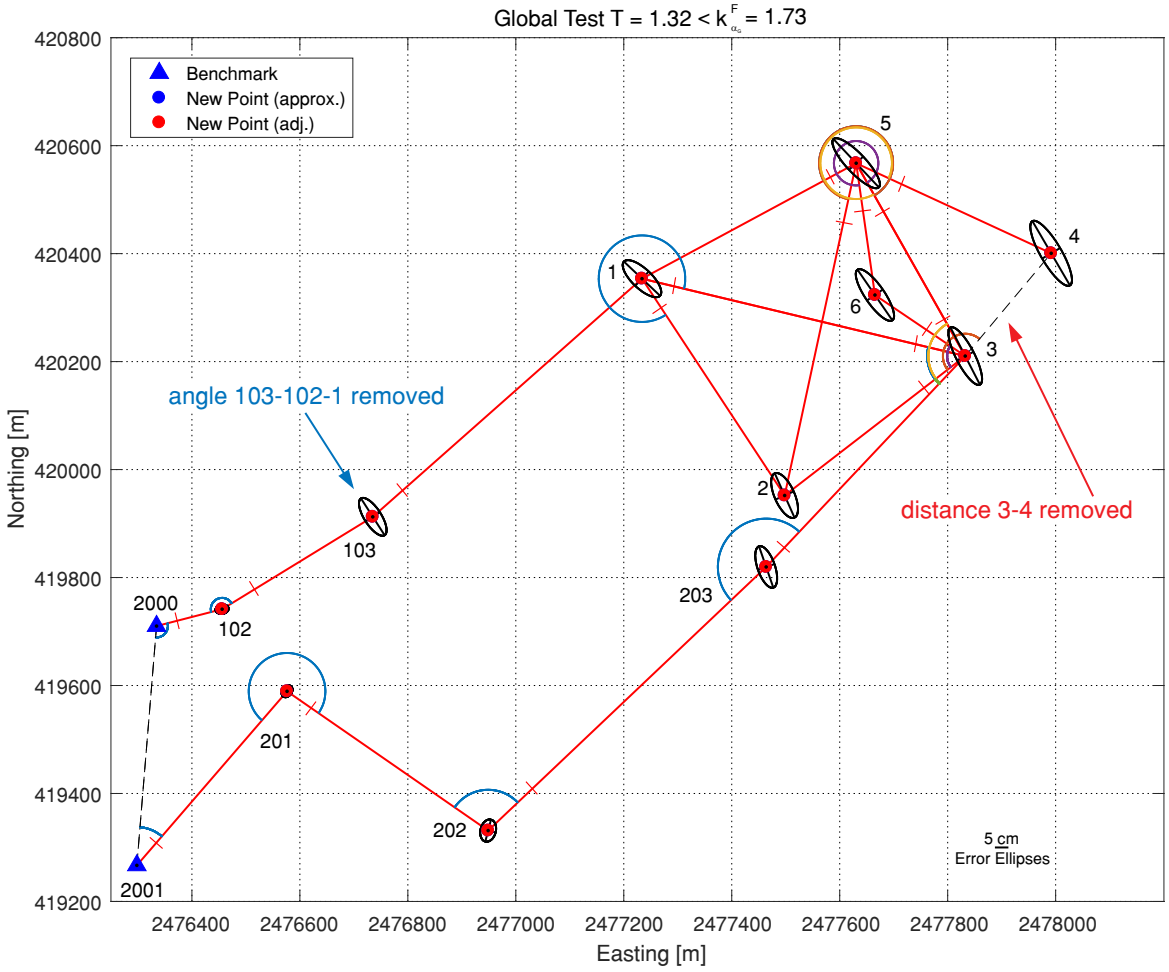
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	2	480.7065	0.347	1.409	68.82	0.19	10.96	+0.504	2.78	0.13	34.2	1.6	34.2	1.6	0.17
	3	615.7476	−0.756	1.274	74.49	0.40	10.53	−1.015	2.42	0.23	26.9	2.6	26.9	2.6	0.35
3	1	615.7476	−0.756	1.274	74.49	0.40	10.53	−1.015	2.42	0.23	26.9	2.6	26.9	2.6	0.35
	2	422.7030	−0.303	1.297	73.59	0.16	10.60	−0.412	2.48	0.10	28.0	1.1	28.0	1.1	0.14
	5	410.4485	−0.853	1.217	76.73	0.44	10.38	−1.111	2.28	0.24	24.2	2.6	24.2	2.6	0.39
	6	201.9670	1.305	1.635	58.03	0.78	11.93	+2.249	3.51	0.66	50.1	9.4	50.1	9.4	0.68
5	1	450.6829	−1.292	1.708	54.17	0.80	12.35	−2.384	3.80	0.73	56.6	10.9	56.6	10.9	0.70
	2	629.5826	−0.263	1.431	67.84	0.15	11.04	−0.388	2.85	0.10	35.5	1.2	35.5	1.2	0.13
	3	410.4485	1.147	1.217	76.73	0.60	10.38	+1.496	2.28	0.33	24.2	3.5	24.2	3.5	0.52
	4	397.8873	0.273	1.895	43.59	0.19	13.77	+0.625	4.70	0.21	77.7	3.5	77.7	3.5	0.16
	6	246.5965	1.354	1.682	55.56	0.83	12.20	+2.437	3.70	0.74	54.2	10.8	54.2	10.8	0.72
10	01	327.3927	−2.266	2.393	10.07	3.25	28.65	−22.506	12.35	9.70	257.6	202.4	257.6	202.4	2.83*
01	1	665.8130	−2.301	2.389	10.38	3.25	28.21	−22.164	12.14	9.54	252.8	198.6	252.8	198.6	2.83#
20	02	453.0945	0.549	2.516	0.59	3.25	118.27	+92.911	53.60	42.11	1175.7	923.6	1175.7	923.6	2.83*
02	03	709.7570	2.300	2.389	10.37	3.25	28.23	+22.177	12.15	9.54	253.0	198.8	253.0	198.8	2.83*
03	3	537.1571	2.292	2.390	10.30	3.25	28.33	+22.254	12.20	9.58	254.1	199.6	254.1	199.6	2.83*
04	10	125.2605	−2.050	2.417	8.24	3.25	31.67	−24.883	13.79	10.83	290.6	228.3	290.6	228.3	2.83*
05	20	425.8772	2.280	2.391	10.20	3.25	28.47	+22.366	12.26	9.63	255.7	200.9	255.7	200.9	2.83*

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 7.643$, $r = 4.16$, $\hat{\sigma}_0^2/\sigma_0^2 = 1.84$, $\alpha_G = 0.95\%$, $k^F_{\alpha_G;r,\infty} = 3.29$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
1	2	3	352.000 861	−0.707	1.713	41.02	0.57	12.54	−1.72	4.95	0.68	7.4	1.0	0.49
3	2	1	56.800 756	−1.065	1.808	49.70	0.68	13.03	−2.14	4.16	0.68	6.6	1.1	0.59
	2	4	186.157 245	−0.455	4.314	29.38	0.19	34.11	−1.55	6.41	0.29	24.1	1.1	0.16
	2	5	109.062 331	4.490	2.063	67.98	1.71	15.93	+6.60	2.84	1.18	5.1	2.1	1.49
	2	6	79.681 644	0.918	4.167	39.23	0.31	30.75	+2.34	5.14	0.39	18.7	1.4	0.27
	03	2	9.997 250	1.515	2.238	5.41	3.25	35.62	+27.99	17.27	13.57	33.7	26.5	2.83*
5	1	3	298.698 674	−3.458	2.384	52.77	1.57	17.21	−6.55	3.91	1.49	8.1	3.1	1.37
	2	3	353.719 464	1.060	1.377	69.13	0.59	10.74	+1.53	2.76	0.39	3.3	0.5	0.51
	3	4	360.328 336	0.059	2.845	1.60	0.19	81.71	+3.71	32.42	1.47	80.4	3.7	0.16
	6	3	376.233 639	0.466	3.165	30.20	0.26	24.83	+1.54	6.28	0.39	17.3	1.1	0.22
10	04	01	181.075 633	3.379	10.161	1.36	3.25	315.83	+248.11	35.17	27.63	311.5	244.7	2.83*
20	05	02	293.227 078	0.916	3.419	0.89	3.25	131.27	+103.12	43.65	34.29	130.1	102.2	2.83*
02	20	03	113.143 912	2.693	2.705	11.01	3.25	31.14	+24.46	11.75	9.23	27.7	21.8	2.83*
03	02	3	196.463 276	2.773	2.808	10.86	3.25	32.51	+25.54	11.84	9.30	29.0	22.8	2.83*
04	10	05	121.318 228	−5.265	8.834	4.25	3.25	157.82	−123.98	19.62	15.41	151.1	118.7	2.83*
05	04	20	40.081 405	0.755	2.606	1.04	3.25	92.62	+72.76	40.34	31.69	91.7	72.0	2.83#

Network graph



Supplementary information

Observed distances	:	19	18	18
Observed angles	:	17	17	16
Number of observations	:	36	35	34
Coordinate unknowns	:	22	22	22
Datum defect	:	3	3	3
Datum definition	:	fix	fix	fix
Number of datum constraints	:	4	4	4
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	6.6	6.0	5.3
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.8	2.8	2.8
Number of iterations (Max=20)	:	20	20	20
Stop criterion (actual)	:	$3.8 \cdot 10^{-10}$	$4.2 \cdot 10^{-10}$	$3.6 \cdot 10^{-10}$
Redundancy r	:	14	13	12
Redundancy distances	:	8.52	7.87	7.84
Redundancy angles	:	5.48	5.13	4.16
Weighted square sum of residuals Ω [m^2]	:	$1.58 \cdot 10^3$	5.90	$7.64 \cdot 10^{-3}$
(a priori) standard deviation σ_0 [m]	:	$2.20 \cdot 10^{-2}$	$2.20 \cdot 10^{-2}$	$2.20 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	10.7	0.674	$2.52 \cdot 10^{-2}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	482.6818	30.6267	1.1470
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	232982	938	1.3156
Critical value $k^F_{\alpha_G;r,\infty}$	:	1.62	1.67	1.73
Number of outliers (Data snooping)	:	35	15	0
Number of outliers (τ-criterion)	:	2	5	14
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	2180.369	1370.566	1.431
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	$1.497 \cdot 10^8$	$6.011 \cdot 10^5$	1050.4

3.6.4 Ghilani (2010), Ex. 21.10

Ghilani (2010): Adjustment Computations. Spatial Data Analysis. 5th Edition. Ex. 21.10, pp. 459

Available data files: [2D] Ghilani21_10_DistanceAngle_fix*.*

Coordinates

Point name	Easting x [m]	Northing y [m]
A	5600.5440 (D)	4966.2360 (D)
B	6061.6240 (D)	8043.1730 (D)
C	9787.8230	8038.5290
D	9260.8860	4843.9110

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
A	B	3111.2910	10	10 000.00
	C	5193.4710	16	3906.25
B	C	3726.2200	12	6944.44
	D	4524.4710	14	5102.04
C	D	3237.7830	10	10 000.00
D	A	3662.3720	12	6944.44

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
A	B	C	50.232 716 0	0.6481	$9.647\,43 \cdot 10^9$
	C	D	42.424 074 1	0.6481	$9.647\,43 \cdot 10^9$
B	C	D	49.920 679 0	0.6481	$9.647\,43 \cdot 10^9$
	D	A	59.470 061 7	0.6481	$9.647\,43 \cdot 10^9$
C	D	A	49.295 987 7	0.6481	$9.647\,43 \cdot 10^9$
	A	B	40.378 395 1	0.6481	$9.647\,43 \cdot 10^9$
D	A	B	47.892 284 0	0.6481	$9.647\,43 \cdot 10^9$
	B	C	60.407 407 4	0.6481	$9.647\,43 \cdot 10^9$

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[cm]}$ distances (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	Δy
$s_{A,B}$	−0.1482	−0.9890	0.1482	0.9890	0.0000	0.0000	0.0000	0.0000	−0.070
$s_{A,C}$	−0.8063	−0.5916	0.0000	0.0000	0.8063	0.5916	0.0000	0.0000	−1.431
$s_{B,C}$	0.0000	0.0000	−1.0000	0.0012	1.0000	−0.0012	0.0000	0.0000	1.811
$s_{B,D}$	0.0000	0.0000	−0.7071	0.7071	0.0000	0.0000	0.7071	−0.7071	3.129
$s_{C,D}$	0.0000	0.0000	0.0000	0.0000	0.1627	0.9867	−0.1627	−0.9867	−0.124
$s_{D,A}$	−0.9994	0.0334	0.0000	0.0000	0.0000	0.0000	0.9994	−0.0334	−1.342

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ angles (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	Δy
$\alpha_{A,B,C}$	12.9842	6.8508	−20.2357	3.0323	7.2515	−9.8831	0.0000	0.0000	0.0979
$\alpha_{A,C,D}$	7.8320	7.4898	0.0000	0.0000	−7.2515	9.8831	−0.5806	−17.3730	−0.7273
$\alpha_{B,C,D}$	0.0000	0.0000	9.9282	−7.1355	0.0213	17.0849	−9.9495	−9.9495	0.0215
$\alpha_{B,D,A}$	−20.2357	3.0323	10.2862	−12.9818	0.0000	0.0000	9.9495	9.9495	0.7526
$\alpha_{C,D,A}$	−7.2515	9.8831	0.0000	0.0000	−12.1486	−6.6832	19.4001	−3.1999	1.0938
$\alpha_{C,A,B}$	7.2515	−9.8831	0.0213	17.0849	−7.2727	−7.2018	0.0000	0.0000	0.9798
$\alpha_{D,A,B}$	−0.5806	−17.3730	9.9495	9.9495	0.0000	0.0000	−9.3689	7.4235	19.0126
$\alpha_{D,B,C}$	0.0000	0.0000	−9.9495	−9.9495	19.4001	−3.1999	−9.4506	13.1494	0.3739

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D
D^{T}	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	1	0	0	0	0	0
	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (1. iteration)

	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D
$\widehat{\Delta x}^{\mathsf{T}}$	0	0	0.000	0.000	0.199	0.635	−2.557	2.311

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
C	9787.8250	0.199	9.523	8038.5354	0.635	16.778	19.292
D	9260.8604	−2.557	9.761	4843.9341	2.311	15.117	17.994

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
C	17.32	8.51	181.678 833
D	15.93	8.37	24.166 591

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
C	49.60	24.37	181.678 833
D	45.63	23.98	24.166 591

Adjusted horizontal distances

Variance component: $\Omega = 27.680$, $r = 2.68$, $\hat{\sigma}_0^2 = 10.35$, $\alpha_G = 0.45\%$, $k^F_{\alpha_G;r,\infty} = 4.61$

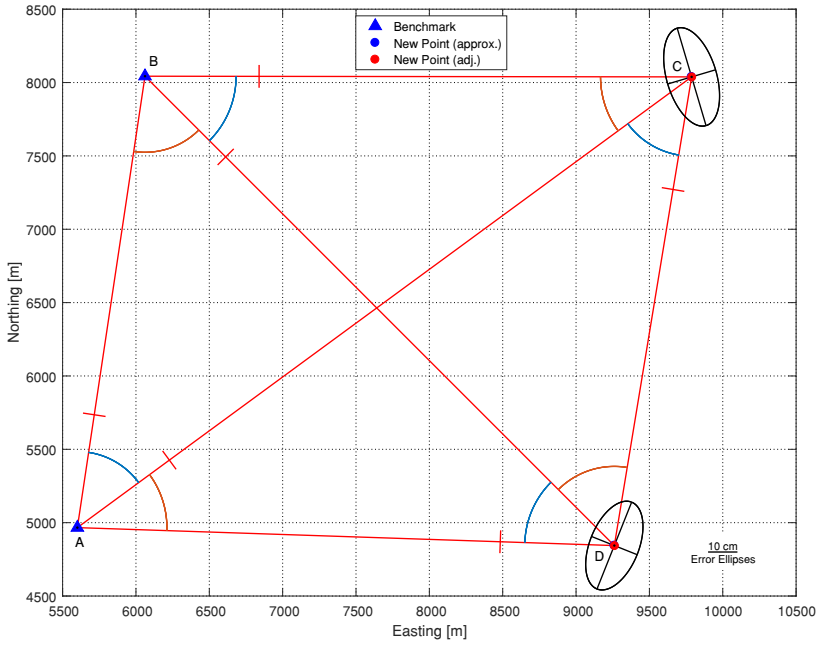
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
A	B	3111.2917	−0.070	0.000	100.00	0.07	4.13	−0.070	0.00	0.00	0.0	0.0	0.0	0.0	0.01
	C	5193.4907	−1.968	9.921	55.45	1.65	8.88	−3.548	3.70	1.48	39.6	15.8	39.6	15.8	0.18
B	C	3726.2039	1.612	9.531	26.90	2.59	9.56	+5.995	6.81	4.27	69.9	43.8	69.9	43.8	0.28
	D	4524.4053	6.571	9.934	41.66	7.27	8.96	+15.775*	4.89	8.61	52.3	92.0	52.3	92.0	0.78
C	D	3237.7722	1.081	8.466	16.94	2.63	10.04	+6.379	9.15	5.81	83.4	53.0	83.4	53.0	0.28
D	A	3662.3591	1.291	9.551	26.60	2.09	9.61	+4.853	6.86	3.47	70.6	35.6	70.6	35.6	0.22

Adjusted horizontal angles

Variance component: $\Omega = 835.324$, $r = 7.32$, $\hat{\sigma}_0^2 = 114.05$, $\alpha_G = 2.46\%$, $k^F_{\alpha_G;r,\infty} = 2.26$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
A	B	C	50.232 570	0.146	2.028	88.65	0.24	2.84	+0.16	1.48	0.09	0.3	0.0	0.03
	C	D	42.424 463	−0.389	1.590	93.02	0.62	2.78	−0.42	1.13	0.17	0.2	0.0	0.07
B	C	D	49.920 791	−0.112	1.688	92.14	0.18	2.79	−0.12	1.21	0.05	0.2	0.0	0.02
	D	A	59.469 285	0.777	2.111	87.71	1.28	2.86	+0.89	1.55	0.48	0.4	0.1	0.14
C	D	A	49.294 257	1.731	1.915	89.88	2.82	2.82	+1.93	1.39	0.94	0.3	0.2	0.30
	A	B	40.377 355	1.040	1.136	96.44	1.63	2.73	+1.08	0.79	0.31	0.1	0.0	0.18
D	A	B	47.873 682	18.601	1.103	96.64	29.19	2.72	+19.25*	0.77	5.44	0.1	0.6	3.14*
	B	C	60.407 597	−0.190	2.090	87.96	0.31	2.86	−0.22	1.53	0.12	0.3	0.0	0.03

Network graph



Supplementary information

Observed distances	:	6
Observed angles	:	8
Coordinate unknowns	:	4
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	4
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	4.0
Power of test γ [%] (Baarda)	:	80.0
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.7
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$2.5 \cdot 10^{-11}$
Redundancy r	:	10
Redundancy distances	:	2.68
Redundancy angles	:	7.32
Weighted square sum of residuals Ω [-]	:	$8.63004 \cdot 10^2$
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	9.28980
Ratio $\hat{\sigma}_0/\sigma_0$	:	9.2898
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	86.3004 ($k^F_{\alpha_G;r,\infty}$ =1.90)
Number of outliers (Data snooping)	:	2
Number of outliers (τ-criterion)	:	1
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	3.510
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	696

3.6.5 Polygon traverse: with 3 different datum definitions; one with non-linear external restriction

Available data files: [2D] Krumm_Traverse∗.∗

This example treats a polygon traverse using three different datum definitions (fixed, dynamic and free), and one solution is constrained by a non-linear external restriction: Point C is to be located at a fixed distance from the origin. Furthermore, two azimuths at the end points of the traverse are given.

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [−]
B	C	281.8320	16	1
C	D	271.3000	16	1
D	E	274.1000	16	1

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [m²/rad²]
C	B	D	205.967 284 0	3.0864	108 915.635 96
D	C	E	231.598 456 8	3.0864	108 915.635 96

Horizontal angles and azimuths

in	from	to	α [gon]	A [gon]	$\lvert\sigma\rvert$ [mgon]	p [m²/rad²]
B	*A	C	192.103 086 4	75.839 722 2	3.0864	108 915.635 96
E	D	*F	228.034 259 3	333.546 450 6	3.0864	108 915.635 96

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ distances (1. iteration)

A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	Δy
$s_{B,C}$	0.8759	0.4825	−0.8759	−0.4825	0.0000	0.0000	0.0000	0.0000	0.000
$s_{C,D}$	0.0000	0.0000	0.9172	0.3984	−0.9172	−0.3984	0.0000	0.0000	0.000
$s_{D,E}$	0.0000	0.0000	0.0000	0.0000	0.9963	−0.0864	−0.9963	0.0864	−4.386

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ angles (1. iteration)

A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	Δy
$\alpha_{B,A,C}$	108.9993	−197.8478	−108.9993	197.8478	0.0000	0.0000	0.0000	0.0000	0.0000
$\alpha_{C,B,D}$	−108.9993	197.8478	202.4962	−413.0719	−93.4969	215.2241	0.0000	0.0000	0.0000
$\alpha_{D,C,E}$	0.0000	0.0000	−93.4969	215.2241	73.4397	−446.5774	20.0572	231.3532	3.1212
$\alpha_{E,D,F}$	0.0000	0.0000	0.0000	0.0000	20.0572	231.3532	−20.0572	−231.3532	−6.7632

3.6.5.1 Datum definition 1: Points B and E fixed

Coordinates

Point name	Easting x [m]	Northing y [m]
B	8478.1390 (D)	2483.8260 (D)
C	8231.2898	2347.8306
D	7982.4554	2239.7328
E	7709.3360 (D)	2263.4110 (D)

Datum: fix, (D)...Datum coordinate

Matrix D^{T}[-] of datum constraints

	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
D^{T}	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	1

Least squares solution $\widehat{\Delta x}$[cm] (1. iteration)

	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
$\widehat{\Delta x}^{\mathsf{T}}$	0	0	−1.535	−0.880	−3.165	−1.505	0	0

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
C	8231.2745	−1.535	1.403	2347.8218	−0.880	0.999	1.722
D	7982.4237	−3.165	1.503	2239.7178	−1.504	0.860	1.731

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
C	1.49	0.87	73.252 025
D	1.50	0.86	96.072 779

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
C	6.50	3.80	73.252 025
D	6.58	3.74	96.072 779

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 3.136,\ r = 1.03,\ \hat{\sigma}_0^2/\sigma_0^2 = 3.03,\ \alpha_G = 0.10\%,\ k^F_{\alpha_G;r,\infty} = 10.50$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
B	C	281.8497	−1.769	1.483	34.7	1.9	11.2	−5.097	5.7	2.6	73.3	33.3	73.3	33.3	1.64
C	D	271.3174	−1.744	1.482	34.8	1.8	11.2	−5.005	5.7	2.5	73.0	32.6	73.0	32.6	1.61
D	E	274.1136	−1.363	1.493	33.8	1.5	11.4	−4.031	5.8	2.0	75.2	26.7	75.2	26.7	1.28

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 0.812,\ r = 1.97,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.41,\ \alpha_G = 0.28\%,\ k^F_{\alpha_G;r,\infty} = 5.95$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
B	A	C	192.103 018	0.068	1.978	68.80	0.03	15.38	+0.10	2.78	0.02	4.8	0.0	0.02
C	B	D	205.967 532	−0.248	2.970	29.65	0.15	23.42	−0.84	6.37	0.23	16.5	0.6	0.13
D	C	E	231.599 271	−0.814	2.938	31.15	0.47	22.85	−2.61	6.14	0.70	15.7	1.8	0.41
E	D	F	228.036 907	−2.647	2.033	67.03	1.05	15.58	−3.95	2.90	0.73	5.1	1.3	0.91

3.6.5.2 Datum definition 2: Points B and E dynamic

Coordinates

Point name	Easting x [m]	Northing y [m]
B	8478.1390 (D)	2483.8260 (D)
C	8231.2898	2347.8306
D	7982.4554	2239.7328
E	7709.3360 (D)	2263.4110 (D)

Datum: dynamic, (D)...Datum coordinate

Dynamic coordinates

Point name	Easting x [m]	$\lvert\sigma\rvert$ [cm]	p [–]	Northing y [m]	$\lvert\sigma\rvert$ [cm]	p [–]
B	8478.1390	1.00	2.56	2483.8260	1.00	2.56
E	7709.3360	1.00	2.56	2263.4110	1.00	2.56

Design matrix $A_{[\text{-}]}$ **and reduced observation vector** $\Delta y_{[\text{mm}]}$ **dynamic coordinates** (1. iteration)

A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	Δy
x_B	1	0	0	0	0	0	0	0	0
y_B	0	1	0	0	0	0	0	0	0
x_E	0	0	0	0	0	0	1	0	0
y_E	0	0	0	0	0	0	0	1	0

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (1. iteration)

	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
$\widehat{\Delta x}^T$	−0.445	−0.319	−1.688	−0.949	−3.029	−1.305	0.445	0.319

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
B	8478.1345	−0.445	0.975	2483.8228	−0.319	0.981	1.383
C	8231.2729	−1.688	1.473	2347.8211	−0.949	1.181	1.888
D	7982.4251	−3.029	1.549	2239.7198	−1.305	1.102	1.901
E	7709.3405	0.445	0.975	2263.4142	0.319	0.981	1.383

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
B	0.98	0.97	178.110 144
C	1.53	1.10	73.702 651
D	1.55	1.10	95.298 194
E	0.98	0.97	178.110 144

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
B	4.29	4.26	178.110 144
C	6.71	4.81	73.702 651
D	6.78	4.80	95.298 194
E	4.29	4.26	178.110 144

Adjusted dynamic coordinates

Variance component: $\Omega/\sigma_0^2 = 0.600,\ r = 0.40,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.51,\ \alpha_G = 0.03\%,\ k^F_{\alpha_G;r,\infty} = 25.84$

Point name	$\hat{x}$ [m]	$\hat{e}_x$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	$\hat{y}$ [m]	$\hat{e}_y$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
B	8478.1390	4.45	9.75				10.53	1.37	12.73	+4.23	12.04	4.00	11.39	3.78	11.39	3.78	1.33
B				2483.8260	3.19	9.81	9.37	1.04	13.50	+3.41	12.85	3.24	12.24	3.09	12.24	3.09	1.01
E	7709.3360	−4.45	9.75				10.53	1.37	12.73	−4.23	12.04	4.00	11.39	3.78	11.39	3.78	1.33
E				2263.4110	−3.19	9.81	9.37	1.04	13.50	−3.41	12.85	3.24	12.24	3.09	12.24	3.09	1.01

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 1.936,\ r = 0.81,\ \hat{\sigma}_0^2/\sigma_0^2 = 2.38,\ \alpha_G = 0.08\%,\ k^F_{\alpha_G;r,\infty} = 13.09$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
B	C	281.8459	−1.393	1.406	27.3	1.7	12.7	−5.101	6.7	2.7	92.0	37.1	92.0	37.1	1.62
C	D	271.3137	−1.371	1.405	27.4	1.6	12.6	−5.007	6.7	2.7	91.7	36.4	91.7	36.4	1.59
D	E	274.1107	−1.065	1.412	26.7	1.3	12.8	−3.986	6.8	2.1	93.7	29.2	93.7	29.2	1.25

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 0.651,\ r = 1.79,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.36,\ \alpha_G = 0.24\%,\ k^F_{\alpha_G;r,\infty} = 6.45$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
B	A	C	192.103 194	−0.108	1.993	60.74	0.04	16.36	−0.18	3.32	0.04	6.4	0.1	0.04
C	B	D	205.967 663	−0.379	2.685	28.79	0.23	23.77	−1.32	6.50	0.36	16.9	0.9	0.22
D	C	E	231.599 303	−0.847	2.662	30.00	0.50	23.29	−2.82	6.31	0.77	16.3	2.0	0.49
E	D	F	228.036 567	−2.308	2.030	59.26	0.97	16.57	−3.89	3.43	0.81	6.7	1.6	0.94

3.6.5.3 Datum definition 3: Free - Total trace minimization

Coordinates

Point name	Easting x [m]	Northing y [m]
B	8478.1390 (D)	2483.8260 (D)
C	8231.2898 (D)	2347.8306 (D)
D	7982.4554 (D)	2239.7328 (D)
E	7709.3360 (D)	2263.4110 (D)

Datum: free, (D)...Datum coordinate

Matrix D^{T}[-] of datum constraints

	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
D^{T}	1	0	1	0	1	0	1	0
	0	1	0	1	0	1	0	1

Least squares solution $\widehat{\Delta x}$[cm] (1. iteration)

	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
$\widehat{\Delta x}^{\mathsf{T}}$	−0.846	−1.154	−1.041	−0.801	−1.350	−0.089	3.238	2.043

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
B	8478.1305	−0.846	0.869	2483.8145	−1.154	0.910	1.258
C	8231.2794	−1.041	0.548	2347.8226	−0.801	0.466	0.719
D	7982.4419	−1.350	0.579	2239.7319	−0.089	0.451	0.734
E	7709.3684	3.238	0.875	2263.4314	2.043	0.885	1.244

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
B	0.91	0.87	2.743 509
C	0.57	0.43	69.868 489
D	0.58	0.45	90.667 104
E	0.92	0.84	154.325 425

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
B	18.17	17.36	2.743 509
C	11.46	8.64	69.868 489
D	11.61	8.94	90.667 104
E	18.28	16.84	154.325 425

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 0.000,\ r = 0.00,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.00$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
B	C	281.8320	0.000	0.944	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
C	D	271.3000	0.000	0.944	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
D	E	274.1000	0.000	0.944	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 0.348,\ r = 1.00,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.35,\ \alpha_G = 0.10\%,\ k^F_{\alpha_G;r,\infty} = 10.83$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
B	A	C	192.103 997	−0.910	1.577	25.00	0.59	25.51	−3.64	7.16	1.02	19.1	2.7	1.00
C	B	D	205.968 194	−0.910	1.577	25.00	0.59	25.51	−3.64	7.16	1.02	19.1	2.7	1.00
D	C	E	231.599 367	−0.910	1.577	25.00	0.59	25.51	−3.64	7.16	1.02	19.1	2.7	1.00
E	D	F	228.035 170	−0.910	1.577	25.00	0.59	25.51	−3.64	7.16	1.02	19.1	2.7	1.00

3.6.5.4 Datum definition 4: Points B and E fixed, 1 non-linear restriction for point C: $x_C^2 + y_C^2 = 8559.5^2$

Coordinates

Point name	Easting x [m]	Northing y [m]
B	8478.1390 (D)	2483.8260 (D)
C	8231.2898	2347.8306
D	7982.4554	2239.7328
E	7709.3360 (D)	2263.4110 (D)

Datum: fix, (D)...Datum coordinate

External restriction(s)

$$x_C^2 + y_C^2 - 8559.5^2 = 0$$

Matrix B^T of external restrictions and inhomogeneity c (1. iteration)

	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	c
B^T	0	0	16 462.580	4695.661	0	0	0	0	1400.121

Matrix D^T[-] of datum constraints

	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
D^T	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	1

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
C	8231.2140	−7.583	0.565	2347.7982	−3.233	1.982	2.061
D	7982.3916	−6.382	3.083	2239.7133	−1.954	2.008	3.679

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
C	2.06	0.00	182.311 336
D	3.08	2.01	98.805 008

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
C	7.68	0.00	182.311 336
D	11.49	7.48	98.805 008

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 28.560$, $r = 2.01$, $\hat{\sigma}_0^2/\sigma_0^2 = 14.24$, $\alpha_G = 0.28\%$, $k^F_{\alpha_G;r,\infty} = 5.85$

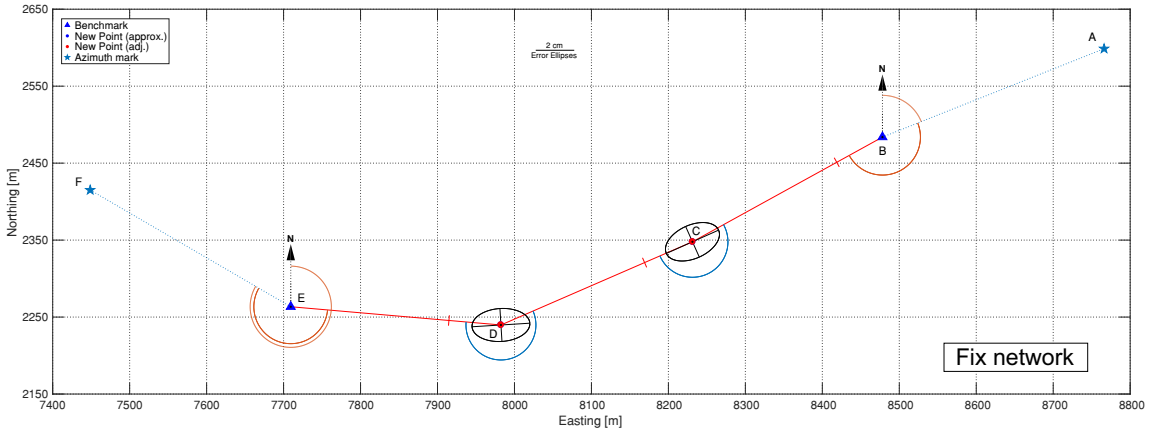
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
B	C	281.9140	−8.202	0.461	98.9	5.2	6.6	−8.296*	0.4	0.6	0.8	0.9	0.8	0.9	1.91
C	D	271.2839	1.611	2.983	52.3	1.4	9.1	+3.081	3.9	1.3	43.6	14.7	43.6	14.7	0.52
D	E	274.0820	1.803	3.073	49.4	1.6	9.4	+3.652	4.2	1.6	47.7	18.5	47.7	18.5	0.59

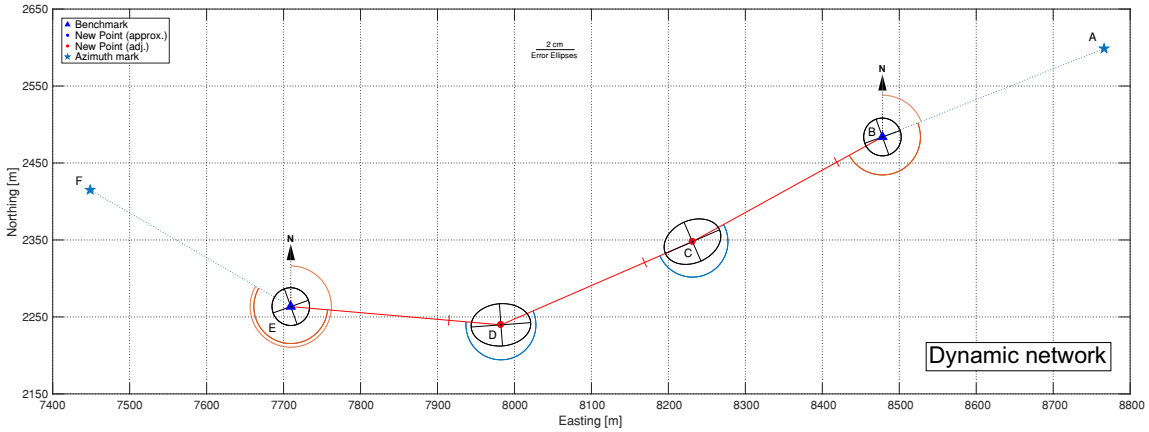
Adjusted horizontal angles

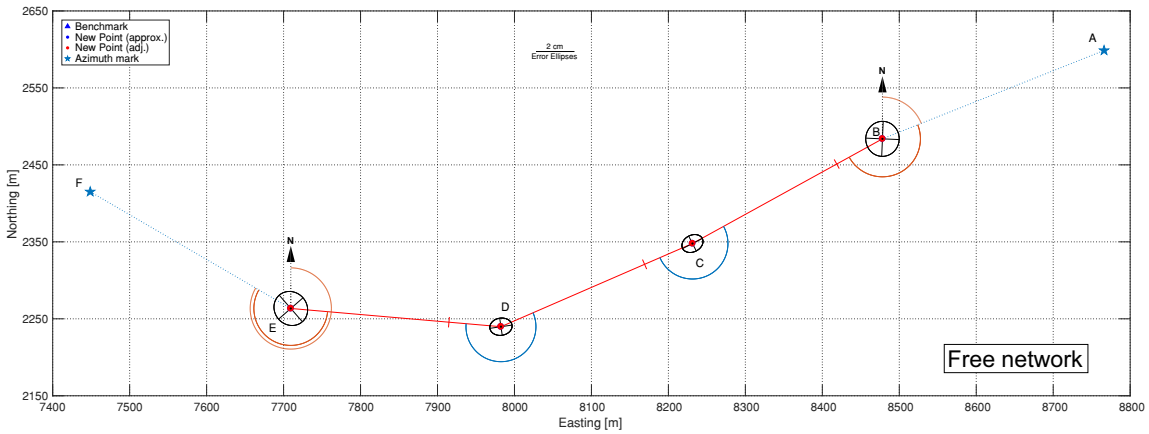
Variance component: $\Omega/\sigma_0^2 = 0.584$, $r = 1.99$, $\hat{\sigma}_0^2/\sigma_0^2 = 0.29$, $\alpha_G = 0.28\%$, $k^F_{\alpha_G;r,\infty} = 5.88$

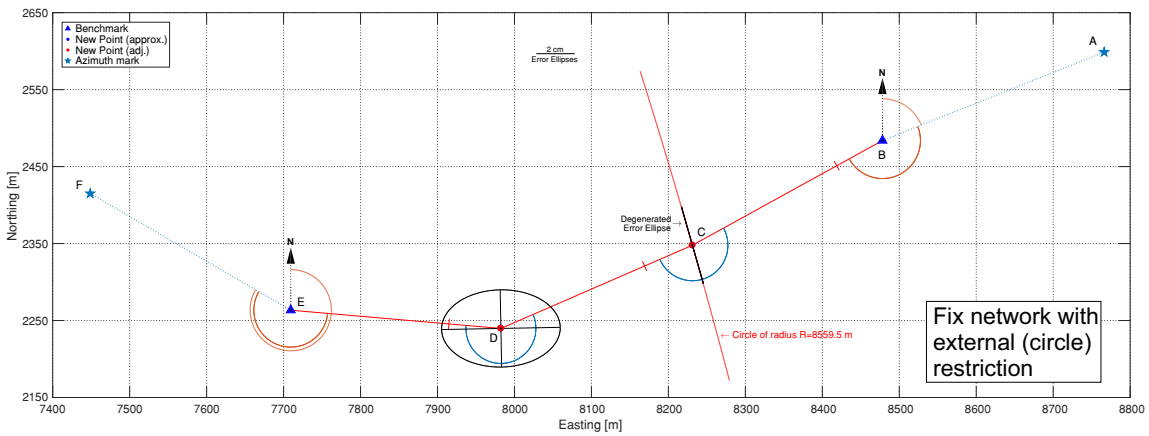
in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
B	A	C	192.104 954	−1.867	4.535	70.36	0.72	15.20	−2.65	2.68	0.47	4.5	0.8	0.27
C	B	D	205.967 048	0.236	6.983	29.74	0.14	23.38	+0.79	6.35	0.22	16.4	0.6	0.05
D	C	E	231.599 506	−1.049	6.912	31.17	0.61	22.84	−3.37	6.14	0.90	15.7	2.3	0.23
E	D	F	228.035 221	−0.961	4.697	68.21	0.38	15.44	−1.41	2.82	0.26	4.9	0.4	0.14

Network graphs









Supplementary information

Datum definition	:	1	2	3	4
Dynamic coordinates	:	0	4	0	0
Observed distances	:	3	3	3	3
Observed angles	:	4	4	4	4
Given azimuths	:	2	2	2	2
Coordinate unknowns	:	4	8	8	4
Datum defect	:	2	0	2	2
Datum definition	:	fix	dynamic	free	fix
Number of datum constraints	:	4	0	2	4
Number of external restrictions	:	0	0	0	1
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.6	0.6	0.1	0.9
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.7	1.7	∞	2.0
Number of iterations (Max=20)	:	3	3	3	4
Stop criterion (actual)	:	$5.8 \cdot 10^{-11}$	$5.6 \cdot 10^{-11}$	$9.2 \cdot 10^{-13}$	$4.6 \cdot 10^{-13}$
Redundancy r	:	3	3	1	4
Redundancy dynamic coordinates	:	0.00	0.40	0.00	0.00
Redundancy distances	:	1.03	0.81	0.00	2.01
Redundancy angles	:	1.97	1.79	1.00	1.99
Weighted square sum of residuals Ω [m^2]	:	$1.01 \cdot 10^{-3}$	$8.16 \cdot 10^{-4}$	$8.91 \cdot 10^{-5}$	$7.46 \cdot 10^{-3}$
(a priori) standard deviation σ_0 [m]	:	$1.60 \cdot 10^{-2}$	$1.60 \cdot 10^{-2}$	$1.60 \cdot 10^{-2}$	$1.60 \cdot 10^{-2}$
(a posteriori) standard deviation $\hat{\sigma}_0$ [m]	:	$1.84 \cdot 10^{-2}$	$1.65 \cdot 10^{-2}$	$9.44 \cdot 10^{-3}$	$4.32 \cdot 10^{-2}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.1473	1.0307	0.5900	2.6993
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	1.3162	1.0623	0.3481	7.2861
Critical value $k^F_{\alpha_G;r,\infty}$	:	4.21	4.21	10.83	3.38
Number of outliers (Data snooping)	:	0	0	0	1
Number of outliers (τ-criterion)	:	0	0	0	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	3.926	3.903	4.501	10.606
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	5.9631	11.007	4.186	17.782

3.6.6 Leick A (1995), Ch. 5.3, Ex. 2a & Ch. 5.4, Ex. 2c

Leick A (1995): GPS Satellite Surveying. Second Edition, Ch. 5.3 & 5.4, pp. 187

Reference ellipsoid, reference meridian and scale

Major semi axis [m]	E^2	L_0 [°]	m_0
6 378 137.000	0.006 694 380 020	291	0.999 60

Horizontal distances with stochastic model $\sigma = \sigma_c + s_{[km]} \cdot \sigma_l$

in	to	s [m]	σ_c [mm]	σ_l [mm/km]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
6	9	6806.1500	2	50	342.308	8.534 29
15	9	8751.9610	2	50	439.598	5.174 74
	1	6399.3120	2	50	321.966	9.646 75
1	6	4307.8140	2	50	217.391	21.160 12
	9	10 759.5850	2	50	539.979	3.429 62

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
6	1	9	233.102 746 9	0.9259	$4.727\,24 \cdot 10^9$
9	15	6	53.290 000 0	0.9259	$4.727\,24 \cdot 10^9$
15	1	9	98.996 018 5	0.9259	$4.727\,24 \cdot 10^9$
1	6	15	80.817 098 8	0.9259	$4.727\,24 \cdot 10^9$

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[m]}$ distances (1. iteration)

A	x_1	y_1	x_6	y_6	x_9	y_9	x_{15}	y_{15}	Δy
$s_{6,9}$	0.0000	0.0000	−0.9957	−0.0925	0.9957	0.0925	0.0000	0.0000	−0.0629
$s_{15,9}$	0.0000	0.0000	0.0000	0.0000	0.5982	0.8013	−0.5982	−0.8013	2.6364
$s_{15,1}$	−0.7921	0.6103	0.0000	0.0000	0.0000	0.0000	0.7921	−0.6103	1.9148
$s_{1,6}$	−0.8182	−0.5750	0.8182	0.5750	0.0000	0.0000	0.0000	0.0000	−0.2011
$s_{1,9}$	−0.9574	−0.2887	0.0000	0.0000	0.9574	0.2887	0.0000	0.0000	−0.2974

Design matrix $A_{[mgon/m]}$ and reduced observation vector $\Delta y_{[mgon]}$ angles (1. iteration)

A	x_1	y_1	x_6	y_6	x_9	y_9	x_{15}	y_{15}	Δy
$\alpha_{6,1,9}$	8.4971	−12.0903	−9.3622	21.4037	0.8651	−9.3134	0.0000	0.0000	−0.5516
$\alpha_{9,15,6}$	0.0000	0.0000	−0.8651	9.3134	−4.9656	−4.9607	5.8307	−4.3527	11.0605
$\alpha_{15,1,9}$	−6.0736	−7.8828	0.0000	0.0000	5.8307	−4.3527	0.2429	12.2356	−35.8687
$\alpha_{1,6,15}$	14.5707	−4.2075	−8.4971	12.0903	0.0000	0.0000	−6.0736	−7.8828	24.6270

3.6.6.1 Ex. 2a: Fix network with minimal datum constraints

Available data files: [2D] Leick53∗.∗

Coordinates

Point number	ID	Longitude L	Latitude B	Easting x [m]	Northing y [m]
Six Mile	1	291°10' 3.11" (D)	44°51'42.44" (D)	13 235.6760 (D)	4 967 610.7338 (D)
Trav-01	6	291°12'44.01"	44°53' 2.45"	16 760.2885	4 970 087.8491
Trav-09	9	291°17'53.04"	44°53'22.16"	23 537.3284	4 970 717.3439
Trav-14	15	291°13'53.52" (D)	44°49'35.50"	18 303.3487 (D)	4 963 706.2000

Datum: fix, (D)...Datum coordinate

Matrix D^{T}[-] of datum constraints

	x_1	y_1	x_6	y_6	x_9	y_9	x_{15}	y_{15}
D^{T}	1	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0

Least squares solution $\widehat{\Delta x}$[cm] (1. iteration)

	x_1	y_1	x_6	y_6	x_9	y_9	x_{15}	y_{15}
$\widehat{\Delta x}^{\mathsf{T}}$	0.000	0.000	−25.911	8.465	−16.804	49.378	0.000	−269.533

Adjusted coordinates

ID	$\widehat{L}$	$\hat{x}$ [m]	$\hat{x}-x$ [m]	$\lvert\hat{\sigma}\rvert$ [m]	$\widehat{B}$	$\hat{y}$ [m]	$\hat{y}-y$ [m]	$\lvert\hat{\sigma}\rvert$ [m]	$\lvert\hat{\sigma}_{2D}\rvert$ [m]
6	291°12'43.9982"	16 760.0292	−0.259	0.120	44°53' 2.4528"	4 970 087.9338	0.085	0.117	0.168
9	291°17'53.0324"	23 537.1598	−0.169	0.216	44°53'22.1760"	4 970 717.8380	0.494	0.269	0.345
15	291°13'53.5200"	18 303.3487	0.000	0.000	44°49'35.4126"	4 963 703.5044	−2.696	0.190	0.190

Absolute error ellipses

in	A[cm]	B[cm]	ϕ[gon]
6	12.35	11.38	61.251 509
9	32.67	11.03	158.626 735
15	19.02	0.00	0.000 000

Absolute confidence ellipses ($1-\alpha = 95\%$)

in	A[cm]	B[cm]	ϕ[gon]
6	46.03	42.39	61.251 509
9	121.76	41.09	158.626 735
15	70.87	0.00	0.000 000

Adjusted horizontal distances

Variance component: $\Omega = 1.476,\ r = 2.90, \hat{\sigma}_0^2 = 0.51,\ \alpha_G = 0.52\%,\ k^F_{\alpha_G;r,\infty} = 4.33$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
6	9	6806.341	−19.100	15.415	48.6	0.8	202.9	−39.297	4.2	0.8	1042.7	202.0	1042.7	202.0	1.27
15	9	8751.780	18.090	13.429	76.3	0.5	207.9	+23.694	2.3	0.3	491.7	56.0	491.7	56.0	0.75
	1	6399.043	26.917	11.611	67.0	1.0	162.5	+40.149	2.9	0.7	535.5	132.3	535.5	132.3	1.63
1	6	4307.852	−3.765	12.351	18.2	0.4	210.6	−20.688	8.8	0.9	1722.5	169.2	1722.5	169.2	0.65
	9	10 759.864	−27.862	15.397	79.4	0.6	250.4	−35.093	2.1	0.3	515.9	72.3	515.9	72.3	0.92

Adjusted horizontal angles

Variance component: $\Omega = 0.103,\ r = 1.10, \hat{\sigma}_0^2 = 0.09,\ \alpha_G = 0.12\%,\ k^F_{\alpha_G;r,\infty} = 9.89$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
6	1	9	233.102 792	−0.045	0.502	25.37	0.10	7.60	−0.18	7.09	0.17	5.7	0.1	0.15
9	15	6	53.290 069	−0.069	0.485	30.51	0.13	6.93	−0.23	6.24	0.20	4.8	0.2	0.21
15	1	9	98.995 779	0.239	0.498	26.66	0.50	7.41	+0.90	6.85	0.83	5.4	0.7	0.80
1	6	15	80.816 944	0.155	0.494	27.87	0.32	7.25	+0.56	6.65	0.51	5.2	0.4	0.50

3.6.6.2 Ex. 2c: Free network

Available data files: [2D] Leick54∗.∗

Coordinates

Point number	ID	Longitude L	Latitude B	Easting x [m]	Northing y [m]
Six Mile	1	291°10' 3.11" (D)	44°51'42.44" (D)	13 235.6760 (D)	4 967 610.7338 (D)
Trav-01	6	291°12'44.01" (D)	44°53' 2.45" (D)	16 760.2885 (D)	4 970 087.8491 (D)
Trav-09	9	291°17'53.04" (D)	44°53'22.16" (D)	23 537.3284 (D)	4 970 717.3439 (D)
Trav-14	15	291°13'53.52" (D)	44°49'35.50" (D)	18 303.3487 (D)	4 963 706.2000 (D)

Datum: free, (D)...Datum coordinate

Matrix D^{T}[-] of datum constraints (1. iteration)

	x_1	y_1	x_6	y_6	x_9	y_9	x_{15}	y_{15}
D^{T}	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	419.7989	−4723.4844	−2057.3174	−1198.8719	−2686.8122	5578.1680	4324.3317	344.1883

Least squares solution $\widehat{\Delta x}$[cm] (1. iteration)

	x_1	y_1	x_6	y_6	x_9	y_9	x_{15}	y_{15}
$\widehat{\Delta x}^{\mathsf{T}}$	9.346	67.926	−8.698	65.196	2.409	84.582	−3.057	−217.703

Adjusted coordinates

ID	$\widehat{L}$	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{B}$	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [m]
1	291°10' 3.1143"	13 235.7696	9.366	6.856	44°51'42.4620"	4 967 611.4132	0.679	4.441	0.082
6	291°12'44.0061"	16 760.2017	−8.685	8.470	44°53' 2.4711"	4 970 088.5012	0.652	4.070	0.094
9	291°17'53.0412"	23 537.3522	2.381	8.862	44°53'22.1874"	4 970 718.1898	0.846	4.761	0.101
15	291°13'53.5183"	18 303.3181	−3.061	5.433	44°49'35.4294"	4 963 704.0226	−2.177	6.538	0.085

Absolute error ellipses

in	A[cm]	B[cm]	ϕ[gon]
1	6.89	4.38	91.388 764
6	8.67	3.61	84.739 483
9	9.40	3.58	76.480 295
15	6.56	5.41	190.566 340

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A[cm]	B[cm]	ϕ[gon]
1	25.69	16.33	91.388 764
6	32.33	13.47	84.739 483
9	35.04	13.34	76.480 295
15	24.45	20.15	190.566 340

Adjusted horizontal distances

Variance component: $\Omega = 1.476,\ r = 2.90, \hat{\sigma}_0^2 = 0.51,\ \alpha_G = 0.52\%,\ k^F_{\alpha_G;r,\infty} = 4.33$

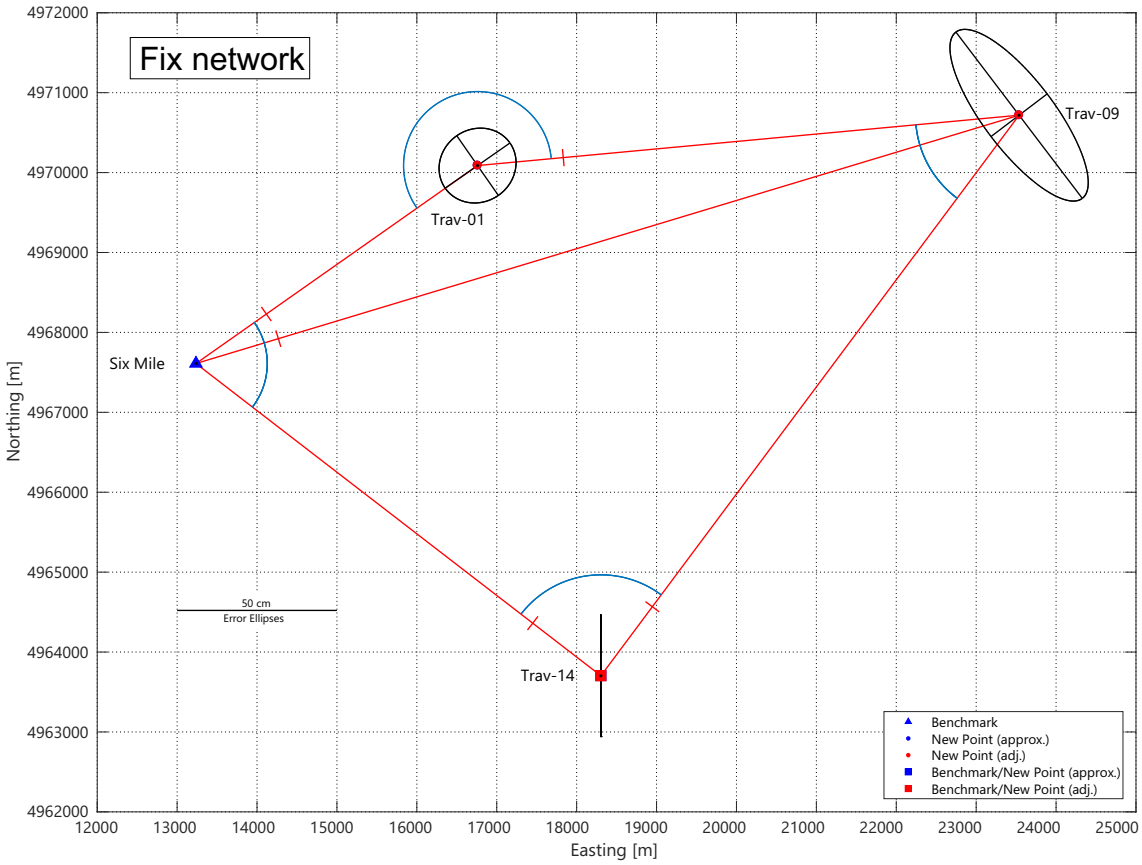
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
6	9	6806.341	−19.100	15.415	48.60	0.80	202.89	−39.297	4.25	0.82	104.27	20.20	104.27	20.20	1.27
15	9	8751.780	18.090	13.429	76.35	0.47	207.89	+23.694	2.30	0.26	49.17	5.60	49.17	5.60	0.75
	1	6399.043	26.917	11.611	67.04	1.02	162.48	+40.149	2.90	0.72	53.55	13.23	53.55	13.23	1.63
1	6	4307.852	−3.765	12.351	18.20	0.41	210.57	−20.688	8.76	0.86	172.25	16.92	172.25	16.92	0.65
	9	10 759.864	−27.862	15.397	79.40	0.58	250.41	−35.093	2.10	0.29	51.59	7.23	51.59	7.23	0.92

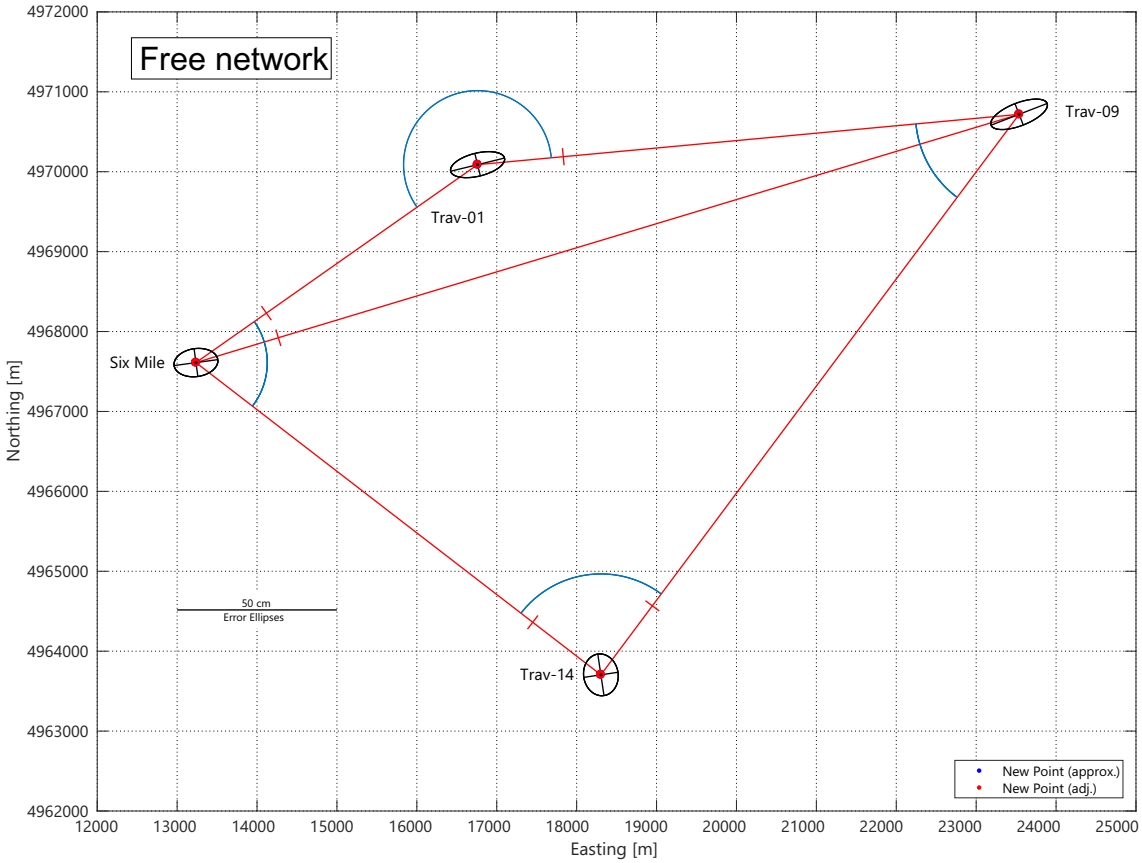
Adjusted horizontal angles

Variance component: $\Omega = 0.103,\ r = 1.10, \hat{\sigma}_0^2 = 0.09,\ \alpha_G = 0.12\%,\ k^F_{\alpha_G;r,\infty} = 9.89$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
6	1	9	233.102 792	−0.045	0.502	25.37	0.10	0.12	−0.18	7.09	0.17	5.7	0.1	0.15
9	15	6	53.290 069	−0.069	0.485	30.51	0.13	0.11	−0.23	6.24	0.20	4.8	0.2	0.21
15	1	9	98.995 779	0.239	0.498	26.66	0.50	0.12	+0.90	6.85	0.83	5.4	0.7	0.80
1	6	15	80.816 944	0.155	0.494	27.87	0.32	0.11	+0.56	6.65	0.51	5.2	0.4	0.50

Network graphs





Supplementary information

Observed distances	:	5	5
Observed angles	:	4	4
Coordinate unknowns	:	5	8
Datum defect	:	3	3
Datum definition	:	fix	free
Number of datum constraints	:	3	3
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.9	0.9
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.0	2.0
Number of iterations (Max=20)	:	20	20
Stop criterion (actual)	:	$6 \cdot 10^{-10}$	$4.6 \cdot 10^{-10}$
Redundancy r	:	4	4
Redundancy distances	:	2.90	2.90
Redundancy angles	:	1.10	1.10
Weighted square sum of residuals Ω [-]	:	1.57837	1.57837
(a priori) standard deviation σ_0 [-]	:	1	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	$6.28166 \cdot 10^{-1}$	$6.28166 \cdot 10^{-1}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.6282	0.6282
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	0.3946	0.3946
Critical value $k^F_{\alpha_G;r,\infty}$	:	3.38	3.38
Number of outliers (Data snooping & τ-criterion)	:	0	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	275.929	252.218
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	1832.6	328.5
Trace coordinate cofactor matrix, $\mathrm{tr}Q_{\widehat{X}}$	:	0.46442	0.08325

3.6.7 Leick A (1995), Ch. 5.5, Ex. 3 & Ch. 5.6, Ex. 4

Leick A (1995): GPS Satellite Surveying. Second Edition, Ch. 5.5 & Ch. 5.6, pp. 195

Reference ellipsoid, reference meridian and scale

Major semi axis [m]	E^2	L_0 [°]	m_0
6 378 137	0.006 694 380 020	291	0.999 60

Coordinates

Point number	ID	Longitude L	Latitude B	Easting x [m]	Northing y [m]
Six Mile	1	291°10' 3.11" (D)	44°51'42.44" (D)	13 235.6760 (D)	4 967 610.7338 (D)
Otter	3	291°22'42.61"	44°56'38.69"	29 860.8125	4 976 808.1650
Chemo	5	291°25'44.38"	44°48'20.70"	33 925.3768	4 961 461.5584
Trav-01	6	291°12'44.01"	44°53' 2.45" (D)	16 760.2885	4 970 087.8491 (D)
Vert-01	7	291°15' 0.01"	44°56' 0.01"	19 726.8786	4 975 575.3275
Trav-02	8	291°16'48.50"	44°55'54.03"	22 105.4551	4 975 398.5713
Trav-09	9	291°17'53.04"	44°53'22.16"	23 537.3284	4 970 717.3439
Verl-02	10	291°19' 0.00"	44°54' 0.00"	25 001.5535	4 971 890.5344
Trav-10	11	291°19'28.57"	44°51'19.69"	25 647.9020	4 966 946.3351
Trav-11	12	291°23'34.69"	44°48'20.05"	31 076.5746	4 961 427.1006
Trav-12	13	291°21'39.42"	44°48'58.63"	28 539.1463	4 962 605.8070
Trav-13	14	291°17'46.33"	44°47'42.91"	23 428.3050	4 960 248.6568
Trav-14	15	291°13'53.52"	44°49'35.50"	18 303.3487	4 963 706.2000

Datum: fix, (D)...Datum coordinate

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
5	12	11	38.025 463 0	0.9259	$4.727\,24 \cdot 10^9$
6	7	1	229.448 333 3	0.9259	$4.727\,24 \cdot 10^9$
	1	9	233.102 746 9	0.9259	$4.727\,24 \cdot 10^9$
8	9	7	123.610 401 2	0.9259	$4.727\,24 \cdot 10^9$
9	10	8	324.105 679 0	0.9259	$4.727\,24 \cdot 10^9$
	15	6	53.290 000 0	0.9259	$4.727\,24 \cdot 10^9$
11	5	10	254.471 512 3	0.9259	$4.727\,24 \cdot 10^9$
12	13	5	171.546 913 6	0.9259	$4.727\,24 \cdot 10^9$
13	14	12	255.192 993 8	0.9259	$4.727\,24 \cdot 10^9$
14	15	13	134.731 388 9	0.9259	$4.727\,24 \cdot 10^9$
15	1	14	195.942 561 7	0.9259	$4.727\,24 \cdot 10^9$
	1	9	98.996 018 5	0.9259	$4.727\,24 \cdot 10^9$
10	11	9	65.273 209 9	0.9259	$4.727\,24 \cdot 10^9$
7	8	6	126.838 518 5	0.9259	$4.727\,24 \cdot 10^9$
1	6	15	80.817 098 8	0.9259	$4.727\,24 \cdot 10^9$

Horizontal distances with stochastic model $\sigma = \sigma_c + s_{[km]} \cdot \sigma_l$

in	to	s [m]	σ_c [mm]	σ_l [mm/km]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]	Remark
5	11	9930.1120	2	50	498.506	4.024 02	
3	8	7882.2880	2	50	396.114	6.373 22	
	7	10 208.7990	2	50	512.440	3.808 15	
	5	15 876.7700	2	50	795.839	1.578 88	only Ex. 4
6	1	4307.8140	2	50	217.391	21.160 12	
8	6	7534.9780	2	50	378.749	6.971 03	
	7	2385.4200	2	50	121.271	67.996 43	
9	6	6806.1500	2	50	342.308	8.534 29	
	8	4895.4110	2	50	246.771	16.421 52	
	1	10 759.5850	2	50	539.979	3.429 62	
11	9	4321.5130	2	50	218.076	21.027 40	
	10	4986.5650	2	50	251.328	15.831 33	
12	5	2848.9430	2	50	144.447	47.927 20	
	11	7741.8770	2	50	389.094	6.605 28	
13	5	5506.1460	2	50	277.307	13.004 01	
	9	9530.2970	2	50	478.515	4.367 26	
	11	5215.1270	2	50	262.756	14.484 17	
	12	2797.9910	2	50	141.900	49.663 57	
	10	9936.2890	2	50	498.814	4.019 04	
14	6	11 885.8030	2	50	596.290	2.812 45	
	9	10 469.4370	2	50	525.472	3.621 61	
	13	5628.1810	2	50	283.409	12.450 09	
15	9	8751.9610	2	50	439.598	5.174 74	
	13	10 294.5640	2	50	516.728	3.745 21	
	14	6180.5730	2	50	311.029	10.337 11	
	1	6399.3120	2	50	321.966	9.646 75	
10	8	4548.9060	2	50	229.445	18.995 10	
	9	1876.2560	2	50	95.813	108.931 36	
	7	6434.4980	2	50	323.725	9.542 18	
7	6	6238.0000	2	50	313.900	10.148 86	

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[m]}$ distances (1. iteration)
Too large to be displayed !

Design matrix $A_{[mgon/m]}$ and reduced observation vector $\Delta y_{[mgon]}$ angles (1. iteration)
Too large to be displayed !

Matrix $D^T{}_{[-]}$ of datum constraints

	x_1	y_1	x_3	y_3	x_5	y_5	x_6	y_6	x_7	y_7	x_8	y_8	x_9	y_9	x_{10}	y_{10}	x_{11}	y_{11}	x_{12}	y_{12}	x_{13}	y_{13}	x_{14}	y_{14}	x_{15}	y_{15}
D^T	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

3.6.7.1 Ex. 3

Available data files: [2D] Leick55∗.∗

Least squares solution $\widehat{\Delta x}_{[\mathrm{cm}]}$ (1. iteration)

	x_1	y_1	x_3	y_3	x_5	y_5	x_6	y_6	x_7	y_7
$\widehat{\Delta x}^{\mathsf{T}}$	0.000	0.000	−33.890	118.661	14.769	24.499	−19.702	0.000	−33.296	−10.809

	x_8	y_8	x_9	y_9	x_{10}	y_{10}	x_{11}	y_{11}	x_{12}	y_{12}
$\widehat{\Delta x}^{\mathsf{T}}$	−3.284	23.317	−2.476	26.135	−0.901	25.769	2.482	8.514	27.545	16.498

	x_{13}	y_{13}	x_{14}	y_{14}	x_{15}	y_{15}
$\widehat{\Delta x}^{\mathsf{T}}$	10.658	10.100	14.774	7.609	−3.336	−284.999

Adjusted coordinates

ID	$\widehat{L}$	$\hat{x}$ [m]	$\hat{x}-x$ [m]	$\lvert\hat{\sigma}\rvert$ [m]	$\widehat{B}$	$\hat{y}$ [m]	$\hat{y}-y$ [m]	$\lvert\hat{\sigma}\rvert$ [m]	$\lvert\hat{\sigma}_{2D}\rvert$ [m]
3	291°22'42.5948"	29 860.4728	−0.340	1.212	44°56'38.7285"	4 976 809.3512	1.186	7.305	7.405
5	291°25'44.3868"	33 925.5241	0.147	0.198	44°48'20.7079"	4 961 461.8040	0.246	0.408	0.454
6	291°12'44.0010"	16 760.0911	−0.197	0.141	44°53' 2.4500"	4 970 087.8491	0.000	0.000	0.141
7	291°14'59.9948"	19 726.5449	−0.334	0.250	44°56' 0.0065"	4 975 575.2195	−0.108	0.114	0.274
8	291°16'48.4985"	22 105.4214	−0.034	0.243	44°55'54.0376"	4 975 398.8047	0.233	0.146	0.283
9	291°17'53.0389"	23 537.3030	−0.025	0.151	44°53'22.1685"	4 970 717.6055	0.262	0.146	0.210
10	291°18'59.9996"	25 001.5438	−0.010	0.174	44°54' 0.0084"	4 971 890.7925	0.258	0.183	0.253
11	291°19'28.5711"	25 647.9263	0.024	0.119	44°51'19.6928"	4 966 946.4205	0.085	0.223	0.253
12	291°23'34.7026"	31 076.8497	0.275	0.197	44°48'20.0553"	4 961 427.2661	0.165	0.345	0.397
13	291°21'39.4249"	28 539.2524	0.106	0.173	44°48'58.6333"	4 962 605.9083	0.101	0.291	0.339
14	291°17'46.3367"	23 428.4521	0.147	0.203	44°47'42.9124"	4 960 248.7328	0.076	0.201	0.286
15	291°13'53.5181"	18 303.3154	−0.033	0.105	44°49'35.4076"	4 963 703.3497	−2.850	0.113	0.154

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
3	739.82	30.73	189.860 489
5	42.23	16.64	17.944 881
6	14.11	0.00	100.000 000
7	25.48	10.20	114.022 807
8	26.59	9.70	128.992 377
9	19.58	7.64	148.643 971
10	23.39	9.52	152.187 742
11	22.57	11.39	188.477 819
12	36.24	16.25	22.452 094
13	30.44	14.86	21.616 705
14	24.93	14.08	50.654 740
15	12.50	9.04	42.456 126

Absolute confidence ellipses ($1-\alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
3	1948.07	80.92	189.860 489
5	111.21	43.83	17.944 881
6	37.15	0.00	100.000 000
7	67.10	26.85	114.022 807
8	70.02	25.55	128.992 377
9	51.56	20.13	148.643 971
10	61.60	25.08	152.187 742
11	59.43	29.99	188.477 819
12	95.44	42.78	22.452 094
13	80.14	39.12	21.616 705
14	65.64	37.06	50.654 740
15	32.92	23.80	42.456 126

Adjusted horizontal distances

Variance component: $\Omega = 7.301,\ r = 17.94, \hat{\sigma}_0^2 = 0.41,\ \alpha_G = 9.26\%,\ k^F_{\alpha_G;r,\infty} = 1.46$

in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
5	11	9929.735	37.663	13.145	84.1	0.8	224.6	+44.784	1.8	0.4	357.2	71.2	357.2	71.2	1.25
3	8	7882.288	0.000	26.193	0.0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
	7	10 208.799	0.000	33.886	0.0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
6	1	4307.854	−3.954	11.542	35.5	0.3	150.7	−11.125	5.6	0.4	971.3	71.7	971.3	71.7	0.46
8	6	7535.171	−19.323	10.200	83.4	0.6	171.4	−23.166	1.8	0.2	284.3	38.4	284.3	38.4	0.84
	7	2385.409	1.108	7.131	20.9	0.2	109.5	+5.295	8.0	0.4	866.3	41.9	866.3	41.9	0.30
9	6	6806.408	−25.836	9.078	83.9	0.8	154.4	−30.787	1.8	0.4	248.3	49.5	248.3	49.5	1.25
	8	4895.295	11.628	8.798	70.9	0.6	121.1	+16.393	2.6	0.4	352.0	47.7	352.0	47.7	0.85
	1	10 759.934	−34.865	11.702	89.3	0.7	236.2	−39.059	1.4	0.2	253.6	41.9	253.6	41.9	1.03
11	9	4321.639	−12.637	9.034	60.8	0.7	115.6	−20.801	3.3	0.6	453.7	81.6	453.7	81.6	1.12
	10	4986.444	12.095	9.783	65.3	0.6	128.5	+18.509	3.0	0.4	445.2	64.1	445.2	64.1	0.90
12	5	2848.884	5.924	8.481	21.2	0.9	129.7	+27.989	8.0	1.7	1022.7	220.6	1022.7	220.6	1.35
	11	7741.723	15.398	12.291	77.2	0.5	183.0	+19.951	2.2	0.2	417.6	45.5	417.6	45.5	0.68
13	5	5506.441	−29.545	10.150	69.4	1.3	137.6	−42.595	2.7	0.9	421.5	130.5	421.5	130.5	1.93
	9	9529.907	38.987	10.695	88.6	0.9	210.1	+44.015	1.5	0.3	240.0	50.3	240.0	50.3	1.31
	11	5215.344	−21.703	10.524	63.3	1.0	136.4	−34.277	3.1	0.8	500.5	125.7	500.5	125.7	1.57
	12	2797.963	2.790	8.179	24.0	0.4	119.6	+11.616	7.3	0.7	909.0	88.3	909.0	88.3	0.61
	10	9936.018	27.085	10.573	89.7	0.6	217.6	+30.187	1.4	0.2	223.6	31.0	223.6	31.0	0.87
14	6	11 885.926	−12.346	12.350	90.2	0.2	259.5	−13.689	1.4	0.1	254.5	13.4	254.5	13.4	0.33
	9	10 469.439	−0.162	9.768	92.1	0.0	226.3	−0.176	1.2	0.0	178.8	0.1	178.8	0.1	0.00
	13	5628.193	−1.194	14.630	39.1	0.1	187.4	−3.057	5.2	0.1	1142.1	18.6	1142.1	18.6	0.10
15	9	8751.823	13.768	10.911	85.9	0.3	196.0	+16.025	1.7	0.1	276.1	22.6	276.1	22.6	0.51
	13	10 294.600	−3.574	13.724	83.9	0.1	233.2	−4.261	1.8	0.0	376.2	6.9	376.2	6.9	0.11
	14	6180.728	−15.546	13.222	58.7	0.7	167.8	−26.495	3.5	0.5	693.4	109.5	693.4	109.5	0.99
	1	6399.111	20.112	9.040	82.0	0.7	146.9	+24.536	1.9	0.3	264.9	44.2	264.9	44.2	1.04
10	8	4549.030	−12.406	8.238	70.5	0.6	112.9	−17.593	2.7	0.4	332.8	51.9	332.8	51.9	0.97
	9	1876.265	−0.851	6.176	5.0	0.4	177.6	−17.111	18.1	1.7	1687.5	162.6	1687.5	162.6	0.60
	7	6434.331	16.699	7.885	86.4	0.6	143.9	+19.320	1.6	0.2	195.2	26.2	195.2	26.2	0.84
7	6	6237.875	12.523	10.917	72.3	0.5	152.5	+17.312	2.6	0.3	421.9	47.9	421.9	47.9	0.71

Adjusted horizontal angles

Variance component: $\Omega = 1.881,\ r = 3.06, \hat{\sigma}_0^2 = 0.61,\ \alpha_G = 0.57\%,\ k^F_{\alpha_G;r,\infty} = 4.14$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
5	12	11	38.025 011	0.452	0.539	22.46	1.03	8.07	+2.01	7.68	1.91	6.3	1.6	1.56
6	7	1	229.448 050	0.283	0.550	19.38	0.69	8.69	+1.46	8.43	1.42	7.0	1.2	1.05
	1	9	233.102 688	0.059	0.497	34.18	0.11	6.54	+0.17	5.73	0.15	4.3	0.1	0.16
8	9	7	123.610 011	0.390	0.565	14.89	1.09	9.92	+2.62	9.88	2.61	8.4	2.2	1.65
9	10	8	324.105 346	0.333	0.572	12.77	1.01	10.71	+2.61	10.80	2.63	9.3	2.3	1.52
	15	6	53.290 147	−0.147	0.494	34.92	0.27	6.47	−0.42	5.64	0.37	4.2	0.3	0.41
11	5	10	254.471 142	0.370	0.583	9.29	1.31	12.55	+3.98	12.91	4.10	11.4	3.6	1.98
12	13	5	171.546 481	0.432	0.559	16.61	1.15	9.39	+2.60	9.26	2.57	7.8	2.2	1.73
13	14	12	255.192 639	0.355	0.573	12.35	1.09	10.89	+2.87	11.01	2.91	9.5	2.5	1.65
14	15	13	134.731 198	0.191	0.570	13.37	0.56	10.46	+1.42	10.52	1.43	9.1	1.2	0.85
15	1	14	195.942 443	0.119	0.560	16.50	0.32	9.42	+0.72	9.30	0.71	7.9	0.6	0.48
	1	9	98.995 727	0.292	0.493	35.27	0.53	6.44	+0.83	5.60	0.72	4.2	0.5	0.80
10	11	9	65.272 776	0.434	0.572	12.59	1.32	10.78	+3.45	10.89	3.48	9.4	3.0	2.00
7	8	6	126.838 088	0.431	0.565	14.88	1.21	9.92	+2.89	9.88	2.88	8.4	2.5	1.82
1	6	15	80.816 814	0.285	0.486	37.00	0.51	6.29	+0.77	5.39	0.66	4.0	0.5	0.76

3.6.7.2 Ex. 4

Available data files: [2D] Leick56∗.∗

Least squares solution $\widehat{\Delta x}_{[\mathrm{cm}]}$ (1. iteration)

	x_1	y_1	x_3	y_3	x_5	y_5	x_6	y_6	x_7	y_7
$\widehat{\Delta x}^{\mathsf{T}}$	0.000	0.000	−33.865	118.510	14.769	24.499	−19.702	0.000	−33.296	−10.809

	x_8	y_8	x_9	y_9	x_{10}	y_{10}	x_{11}	y_{11}	x_{12}	y_{12}
$\widehat{\Delta x}^{\mathsf{T}}$	−3.284	23.317	−2.476	26.135	−0.901	25.769	2.482	8.514	27.545	16.498

	x_{13}	y_{13}	x_{14}	y_{14}	x_{15}	y_{15}
$\widehat{\Delta x}^{\mathsf{T}}$	10.658	10.100	14.774	7.609	−3.336	−284.999

Adjusted coordinates

ID	$\widehat{L}$	$\hat{x}$ [m]	$\hat{x}-x$ [m]	$\lvert\hat{\sigma}\rvert$ [m]	$\widehat{B}$	$\hat{y}$ [m]	$\hat{y}-y$ [m]	$\lvert\hat{\sigma}\rvert$ [m]	$\lvert\hat{\sigma}_{2D}\rvert$ [m]
3	291°22'42.5948"	29 860.4730	−0.340	0.341	44°56'38.7285"	4 976 809.3505	1.186	0.600	0.690
5	291°25'44.3868"	33 925.5241	0.147	0.194	44°48'20.7079"	4 961 461.8040	0.246	0.399	0.443
6	291°12'44.0010"	16 760.0911	−0.197	0.138	44°53' 2.4500"	4 970 087.8491	0.000	0.000	0.138
7	291°14'59.9948"	19 726.5449	−0.334	0.244	44°56' 0.0065"	4 975 575.2195	−0.108	0.111	0.268
8	291°16'48.4985"	22 105.4214	−0.034	0.237	44°55'54.0376"	4 975 398.8047	0.233	0.142	0.277
9	291°17'53.0389"	23 537.3030	−0.025	0.147	44°53'22.1685"	4 970 717.6055	0.262	0.143	0.205
10	291°18'59.9996"	25 001.5438	−0.010	0.170	44°54' 0.0084"	4 971 890.7925	0.258	0.179	0.247
11	291°19'28.5711"	25 647.9263	0.024	0.116	44°51'19.6928"	4 966 946.4205	0.085	0.218	0.247
12	291°23'34.7026"	31 076.8497	0.275	0.193	44°48'20.0553"	4 961 427.2661	0.165	0.337	0.388
13	291°21'39.4249"	28 539.2524	0.106	0.169	44°48'58.6333"	4 962 605.9083	0.101	0.285	0.331
14	291°17'46.3367"	23 428.4521	0.147	0.199	44°47'42.9124"	4 960 248.7328	0.076	0.197	0.280
15	291°13'53.5181"	18 303.3154	−0.033	0.102	44°49'35.4076"	4 963 703.3497	−2.850	0.110	0.151

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
3	63.10	27.93	177.423 142
5	41.26	16.25	17.936 279
6	13.78	0.00	100.000 000
7	24.89	9.96	114.045 025
8	25.98	9.48	128.996 388
9	19.13	7.45	148.657 224
10	22.86	9.29	152.192 130
11	22.05	11.12	188.470 723
12	35.41	15.87	22.441 035
13	29.74	14.51	21.603 611
14	24.35	13.75	50.632 652
15	12.21	8.82	42.383 965

Absolute confidence ellipses ($1-\alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
3	165.58	73.30	177.423 142
5	108.28	42.65	17.936 279
6	36.16	0.00	100.000 000
7	65.32	26.14	114.045 025
8	68.17	24.88	128.996 388
9	50.20	19.56	148.657 224
10	59.98	24.39	152.192 130
11	57.87	29.17	188.470 723
12	92.92	41.64	22.441 035
13	78.03	38.08	21.603 611
14	63.90	36.08	50.632 652
15	32.05	23.15	42.383 965

Adjusted horizontal distances (with additional distance Otter (3) - Chemo (5))

Variance component: $\Omega = 7.301$, $r = 18.93$, $\hat{\sigma}_0^2 = 0.39$, $\alpha_G = 9.92\%$, $k^F_{\alpha_G;r_\infty} = 1.43$

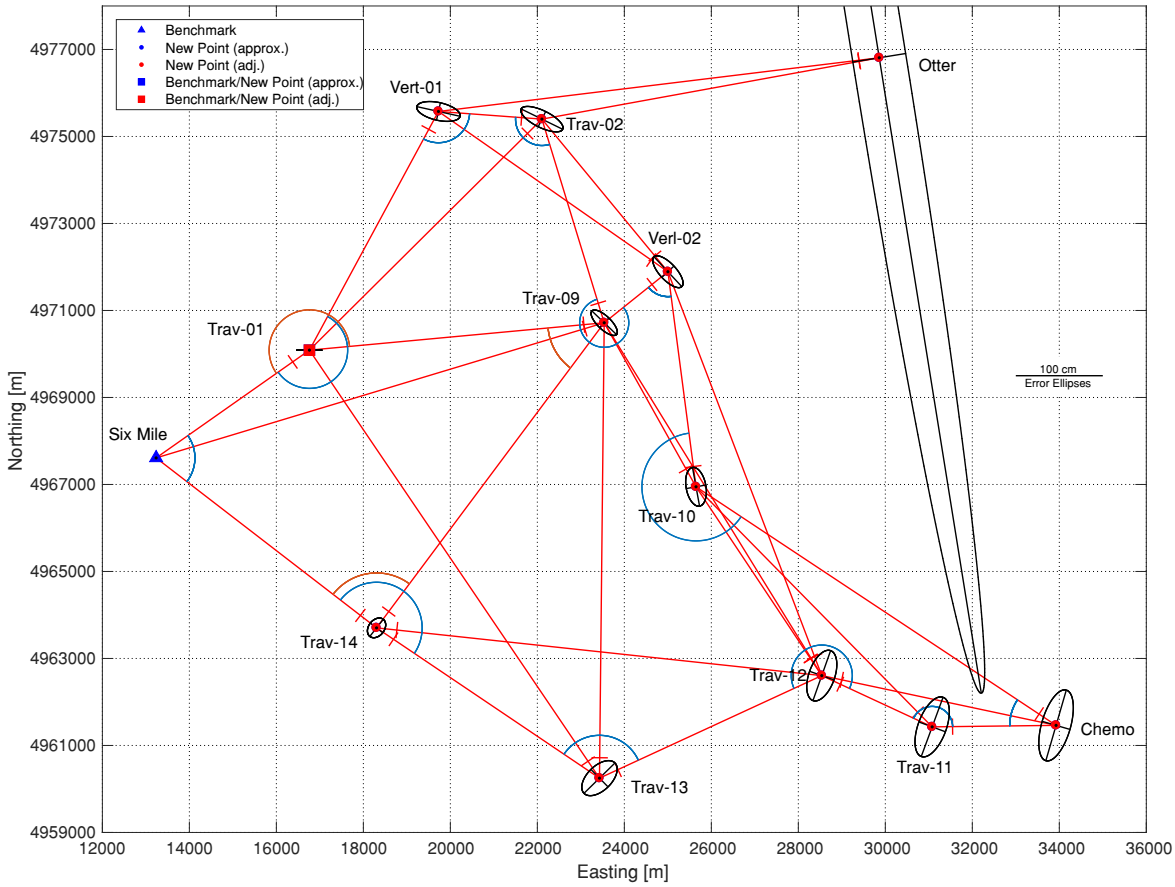
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
5	11	9929.735	37.663	12.843	84.10	0.82	224.62	+44.784	1.80	0.36	35.72	7.12	35.72	7.12	1.28
3	8	7882.288	0.001	20.479	35.96	0.00	272.95	+0.004	5.51	0.00	174.80	0.00	174.80	0.00	0.00
	7	10 208.799	−0.002	20.686	60.96	0.00	271.21	−0.004	3.31	0.00	105.89	0.00	105.89	0.00	0.00
	5	15 876.770	0.000	51.284	0.51	0.00	4605.77	−0.068	57.73	0.00	4582.29	0.07	4582.29	0.07	0.00
6	1	4307.854	−3.954	11.274	35.57	0.30	150.63	−11.117	5.56	0.41	97.06	7.16	97.06	7.16	0.47
8	6	7535.171	−19.323	9.957	83.44	0.56	171.33	−23.158	1.84	0.25	28.37	3.83	28.37	3.83	0.86
	7	2385.409	1.108	6.879	22.91	0.19	104.70	+4.837	7.58	0.35	80.71	3.73	80.71	3.73	0.30
9	6	6806.408	−25.836	8.839	84.03	0.82	154.31	−30.747	1.80	0.36	24.65	4.91	24.65	4.91	1.27
	8	4895.295	11.628	8.590	70.97	0.56	121.04	+16.385	2.64	0.36	35.14	4.76	35.14	4.76	0.87
	1	10 759.934	−34.864	11.423	89.28	0.68	236.14	−39.051	1.43	0.24	25.32	4.19	25.32	4.19	1.06
11	9	4321.639	−12.637	8.826	60.76	0.74	115.61	−20.799	3.32	0.60	45.37	8.16	45.37	8.16	1.15
	10	4986.444	12.096	9.558	65.35	0.60	128.47	+18.508	3.01	0.43	44.51	6.41	44.51	6.41	0.92
12	5	2848.884	5.924	8.286	21.17	0.89	129.73	+27.988	7.97	1.72	102.27	22.06	102.27	22.06	1.38
	11	7741.723	15.398	12.008	77.18	0.45	183.01	+19.950	2.25	0.24	41.76	4.55	41.76	4.55	0.70
13	5	5506.441	−29.545	9.916	69.37	1.28	137.58	−42.594	2.75	0.85	42.15	13.05	42.15	13.05	1.98
	9	9529.907	38.987	10.448	88.58	0.87	210.09	+44.014	1.48	0.31	24.00	5.03	24.00	5.03	1.34
	11	5215.344	−21.703	10.281	63.32	1.04	136.45	−34.277	3.15	0.79	50.05	12.57	50.05	12.57	1.61
	12	2797.963	2.790	7.991	24.02	0.40	119.63	+11.614	7.35	0.71	90.89	8.82	90.89	8.82	0.62
	10	9936.018	27.086	10.329	89.73	0.57	217.60	+30.186	1.40	0.19	22.35	3.10	22.35	3.10	0.89
14	6	11 885.926	−12.346	12.059	90.20	0.22	259.43	−13.688	1.36	0.07	25.42	1.34	25.42	1.34	0.34
	9	10 469.439	−0.162	9.541	92.10	0.00	226.25	−0.176	1.21	0.00	17.87	0.01	17.87	0.01	0.00
	13	5628.193	−1.194	14.294	39.05	0.07	187.39	−3.057	5.16	0.08	114.21	1.86	114.21	1.86	0.10
15	9	8751.823	13.768	10.657	85.92	0.34	195.97	+16.024	1.67	0.14	27.59	2.26	27.59	2.26	0.52
	13	10 294.600	−3.574	13.409	83.87	0.08	233.15	−4.261	1.81	0.03	37.61	0.69	37.61	0.69	0.12
	14	6180.728	−15.546	12.918	58.67	0.65	167.79	−26.495	3.47	0.55	69.34	10.95	69.34	10.95	1.01
	1	6399.111	20.112	8.823	82.01	0.69	146.91	+24.524	1.94	0.32	26.43	4.41	26.43	4.41	1.07
10	8	4549.030	−12.406	8.044	70.55	0.64	112.87	−17.584	2.67	0.42	33.24	5.18	33.24	5.18	1.00
	9	1876.265	−0.851	6.034	4.97	0.40	177.52	−17.099	18.06	1.74	168.69	16.25	168.69	16.25	0.62
	7	6434.331	16.699	7.673	86.54	0.55	143.80	+19.296	1.63	0.22	19.36	2.60	19.36	2.60	0.86
7	6	6237.875	12.523	10.662	72.36	0.47	152.48	+17.307	2.55	0.29	42.15	4.78	42.15	4.78	0.73

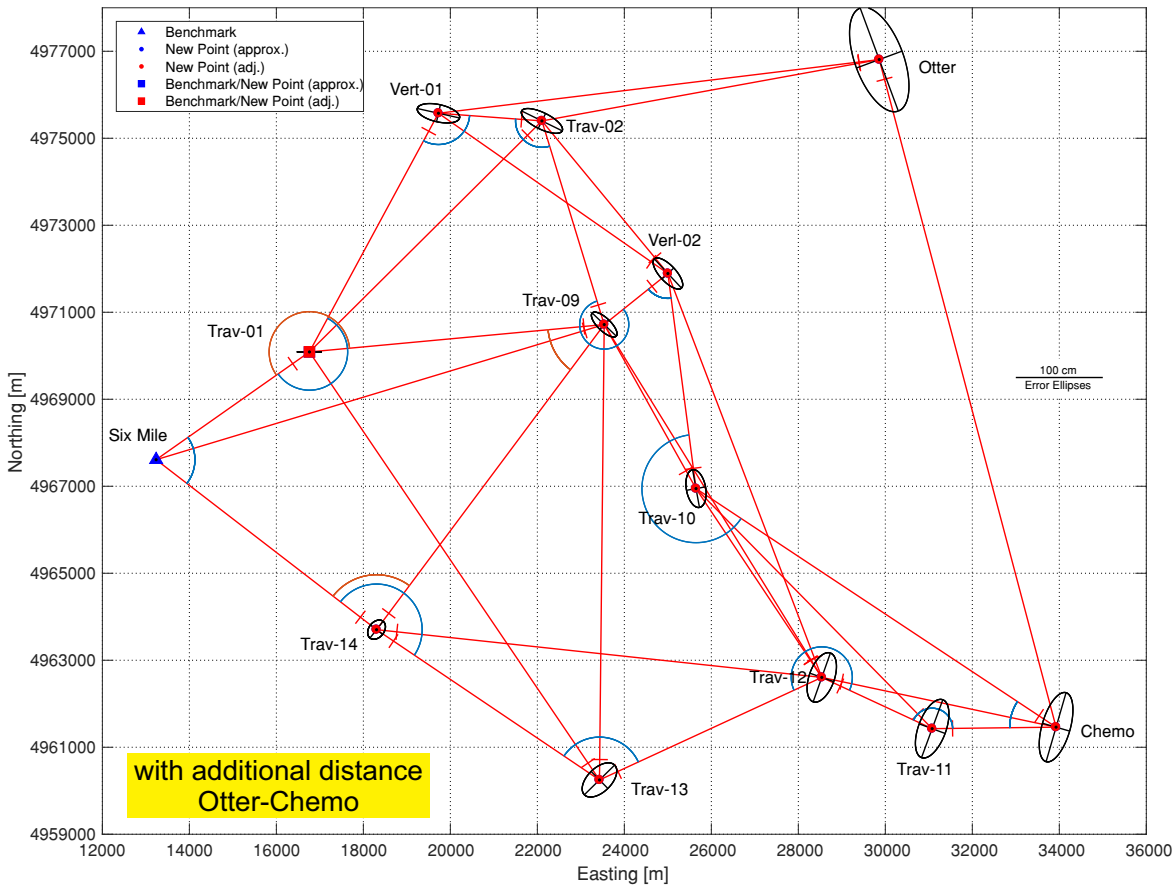
Adjusted horizontal angles

Variance component: $\Omega = 1.881,\ r = 3.07, \hat{\sigma}_0^2 = 0.61,\ \alpha_G = 0.57\%,\ k^F_{\alpha_G;r,\infty} = 4.14$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
5	12	11	38.025 011	0.452	0.527	22.46	1.03	8.07	+2.01	7.68	1.91	6.3	1.6	1.59
6	7	1	229.448 050	0.283	0.537	19.40	0.69	8.69	+1.46	8.42	1.41	7.0	1.2	1.07
	1	9	233.102 688	0.059	0.485	34.20	0.11	6.54	+0.17	5.73	0.15	4.3	0.1	0.17
8	9	7	123.610 011	0.390	0.552	14.93	1.09	9.90	+2.62	9.86	2.60	8.4	2.2	1.69
9	10	8	324.105 346	0.333	0.559	12.77	1.01	10.71	+2.61	10.80	2.63	9.3	2.3	1.56
	15	6	53.290 147	−0.147	0.483	34.92	0.27	6.47	−0.42	5.64	0.37	4.2	0.3	0.42
11	5	10	254.471 142	0.370	0.570	9.30	1.31	12.55	+3.98	12.91	4.09	11.4	3.6	2.03
12	13	5	171.546 481	0.432	0.546	16.61	1.15	9.39	+2.60	9.26	2.57	7.8	2.2	1.77
13	14	12	255.192 639	0.355	0.560	12.35	1.09	10.89	+2.87	11.01	2.91	9.5	2.5	1.69
14	15	13	134.731 198	0.191	0.557	13.38	0.56	10.46	+1.42	10.52	1.43	9.1	1.2	0.87
15	1	14	195.942 443	0.119	0.547	16.50	0.32	9.42	+0.72	9.30	0.71	7.9	0.6	0.49
	1	9	98.995 727	0.292	0.481	35.27	0.53	6.44	+0.83	5.60	0.72	4.2	0.5	0.82
10	11	9	65.272 776	0.434	0.559	12.60	1.32	10.78	+3.45	10.89	3.48	9.4	3.0	2.04
7	8	6	126.838 088	0.431	0.552	14.88	1.21	9.92	+2.89	9.88	2.88	8.4	2.5	1.87
1	6	15	80.816 814	0.285	0.475	37.01	0.51	6.29	+0.77	5.39	0.66	4.0	0.5	0.78

Network graphs





Supplementary information

Observed distances	:	29	30
Observed angles	:	15	15
Coordinate unknowns	:	23	23
Datum defect	:	3	3
Datum definition	:	fix	fix
Number of datum constraints	:	3	3
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	11.3	11.9
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	3.0	3.0
Number of iterations (Max=20)	:	20	20
Stop criterion (actual)	:	$8.4 \cdot 10^{-10}$	$8.7 \cdot 10^{-10}$
Redundancy r	:	21	22
Redundancy distances	:	17.94	18.93
Redundancy angles	:	3.06	3.07
Weighted square sum of residuals Ω [-]	:	9.18254	9.18254
(a priori) standard deviation σ_0 [-]	:	1	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	0.6613	0.6461
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.6613	0.6461
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	0.4373	0.4174
Critical value $k^F_{\alpha_G;r,\infty}$	:	1.38	1.36
Number of outliers (Data snooping & τ-criterion)	:	0	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [m]	:	3.2005	3.2002
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [m^2]	:	55.759	1.365

3.6.8 Ghilani & Wolf (2012), Ch. 16.9.5

Charles D. Ghilani and Paul R. Wolf (2012): Elementary Surveying. An Introduction to Geomatics. 13th Edition, Ch. 16.9.5, p 422 & 450-456

Available data files: [2D] Ghilani_Wolf_Distance_Angle∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
A	415.2730 (D)	929.8680 (D)
B	507.9340	764.6520
C	618.9520	815.3530
D	723.8520	753.2870
E	826.1280	856.4380
F	794.6590	1021.6550
G	578.7410	1103.8260
H	652.2210	980.2450
J	600.5950	899.2720
K	713.3620	877.4180

Datum: fix, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
A	B	189.4360	7	20 408.16
B	C	122.0500	7	20 408.16
C	D	121.9010	7	20 408.16
D	E	145.2560	7	20 408.16
E	F	168.1800	7	20 408.16
F	G	231.0210	7	20 408.16
G	A	238.7140	7	20 408.16
	H	143.7800	7	20 408.16
H	J	96.0360	7	20 408.16
	K	119.6310	7	20 408.16
J	C	85.9080	7	20 408.16
K	E	114.6950	7	20 408.16

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
A	G	B	119.438 271 6	2.7469	537 118 676
B	A	C	105.266 666 7	3.6111	310 798 234
C	B	D	261.285 802 5	4.2284	226 677 875
D	C	E	115.716 049 4	3.9198	263 780 583
E	D	F	138.288 888 9	3.4568	339 167 493
	K	F	76.307 407 4	3.7963	281 216 011
F	E	G	135.132 098 8	2.9321	471 414 629
G	F	A	124.870 370 4	2.5617	617 581 221
	F	H	42.699 382 7	3.0556	434 090 096
H	G	J	270.283 950 6	4.5062	199 592 655
	J	G	129.717 284 0	4.5062	199 592 655
	J	K	329.715 432 1	4.6296	189 089 646
J	H	C	150.157 407 4	5.5556	131 312 254
K	H	E	145.861 111 1	4.4136	208 055 016

Grid bearings

in	to	T [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
A	B	167.460 185 2	0.000 31	$4.2545 \cdot 10^{16}$

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ grid bearings (1. iteration)

A	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	x_F	y_F	x_G	y_G	x_H	y_H	x_J	y_J	x_K	y_K	Δy
$T_{A,B}$	293.1237	164.3977	−293.1237	−164.3977	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−0.06

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{cm}]}$ horizontal distances (1. iteration)

Too large to be displayed !

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ horizontal angles (1. iteration)

Too large to be displayed !

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E	x_F	y_F	x_G	y_G	x_H	y_H	x_J	y_J	x_K	y_K
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (1. iteration)

	x_A	y_A	x_B	y_B	x_C	y_C	x_D	y_D	x_E	y_E
$\widehat{\Delta x}^T$	0	0	0.404	−0.687	0.272	−0.310	1.465	−0.150	0.512	0.288

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (continued)

	x_F	y_F	x_G	y_G	x_H	y_H	x_J	y_J	x_K	y_K
$\widehat{\Delta x}^T$	0.210	−0.100	0.452	0.121	0.528	−0.004	0.413	−0.239	0.831	−0.012

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
B	507.9380	0.404	0.214	764.6451	−0.687	0.382	0.438
C	618.9547	0.272	0.459	815.3499	−0.310	0.493	0.674
D	723.8666	1.465	0.642	753.2855	−0.150	0.685	0.939
E	826.1331	0.512	0.528	856.4409	0.288	0.923	1.063
F	794.6611	0.210	0.581	1021.6540	−0.100	0.859	1.037
G	578.7455	0.452	0.578	1103.8272	0.121	0.451	0.733
H	652.2263	0.528	0.493	980.2450	−0.004	0.609	0.784
J	600.5991	0.413	0.497	899.2696	−0.239	0.575	0.760
K	713.3703	0.831	0.558	877.4179	−0.012	0.733	0.921

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
B	0.44	0.00	167.460 185
C	0.50	0.45	176.765 367
D	0.74	0.58	41.058 901
E	0.93	0.52	8.385 491
F	0.91	0.49	173.670 278
G	0.60	0.42	124.105 169
H	0.63	0.47	175.160 712
J	0.58	0.50	195.805 562
K	0.73	0.56	196.564 810

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
B	1.28	0.00	167.460 185
C	1.45	1.32	176.765 367
D	2.15	1.70	41.058 901
E	2.71	1.51	8.385 491
F	2.66	1.43	173.670 278
G	1.75	1.23	124.105 169
H	1.84	1.36	175.160 712
J	1.68	1.45	195.805 562
K	2.14	1.63	196.564 810

Adjusted grid bearings

Variance component: $\Omega = 0.000,\ r = 0.00, \hat{\sigma}_0^2 = 0.00,\ \alpha_G = 0.00\%,\ k^F_{\alpha_G;r,\infty} = \infty$

in	to	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\widehat{T}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
A	B	167.460 185 2	0	0.014	0.00	∞	34.88	0	∞	0	103.8	0	103.8	0	∞

Adjusted horizontal distances

Variance component: $\Omega = 2.143,\ r = 2.89, \hat{\sigma}_0^2 = 0.74,\ \alpha_G = 0.52\%,\ k^F_{\alpha_G;r,\infty} = 4.34$

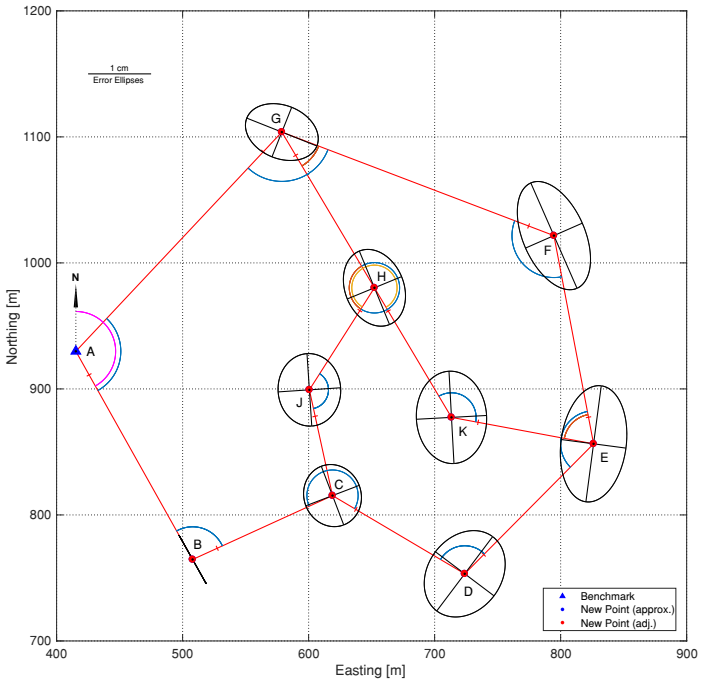
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
A	B	189.4344	0.157	0.438	19.5	0.5	6.6	+0.804	8.4	1.0	52.8	6.5	52.8	6.5	0.73
B	C	122.0478	0.215	0.452	14.2	0.8	7.7	+1.521	10.2	2.0	66.0	13.1	66.0	13.1	1.17
C	D	121.8955	0.554	0.435	20.6	1.7	6.4	+2.690	8.1	3.4	50.6	21.4	50.6	21.4	2.50
D	E	145.2565	−0.055	0.436	20.3	0.2	6.4	−0.270	8.2	0.3	51.1	2.1	51.1	2.1	0.25
E	F	168.1840	−0.401	0.424	24.7	1.2	5.8	−1.627	7.2	2.0	43.9	12.3	43.9	12.3	1.65
F	G	231.0237	−0.275	0.423	25.0	0.8	5.8	−1.099	7.2	1.4	43.4	8.2	43.4	8.2	1.13
G	A	238.7155	−0.147	0.450	14.9	0.5	7.5	−0.984	9.9	1.3	63.7	8.4	63.7	8.4	0.78
	H	143.7776	0.241	0.396	34.4	0.6	4.9	+0.701	5.7	0.8	32.3	4.6	32.3	4.6	0.84
H	J	96.0332	0.283	0.422	25.5	0.8	5.7	+1.109	7.1	1.4	42.7	8.3	42.7	8.3	1.15
	K	119.6328	−0.178	0.415	27.9	0.5	5.5	−0.636	6.6	0.8	39.4	4.6	39.4	4.6	0.69
J	C	85.9037	0.431	0.411	29.2	1.1	5.4	+1.473	6.4	1.8	37.9	10.4	37.9	10.4	1.63
K	E	114.6974	−0.237	0.402	32.3	0.6	5.1	−0.733	6.0	0.9	34.4	5.0	34.4	5.0	0.85

Adjusted horizontal angles

Variance component: $\Omega = 2.237,\ r = 6.11, \hat{\sigma}^2 = 0.37,\ \alpha_G = 1.83\%,\ k^F_{\alpha_G;r,\infty} = 2.53$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
A	G	B	119.438 033	0.239	1.541	35.32	0.15	19.10	+0.68	5.59	0.20	12.4	0.4	0.21
B	A	C	105.264 646	2.021	1.971	38.82	0.90	23.95	+5.21	5.19	1.13	14.7	3.2	1.29
C	B	D	261.284 006	1.796	2.447	31.18	0.76	31.29	+5.76	6.14	1.13	21.5	4.0	1.09
D	C	E	115.715 684	0.365	2.185	36.16	0.16	26.94	+1.01	5.49	0.21	17.2	0.6	0.22
E	D	F	138.291 862	−2.973	1.827	42.58	1.32	21.89	−6.98	4.80	1.53	12.6	4.0	1.89
	K	F	76.307 344	0.064	1.785	54.58	0.02	21.23	+0.12	3.77	0.02	9.6	0.1	0.03
F	E	G	135.134 550	−2.451	1.571	41.03	1.31	18.92	−5.97	4.95	1.56	11.2	3.5	1.87
G	F	A	124.871 220	−0.849	1.363	41.80	0.51	16.37	−2.03	4.88	0.61	9.5	1.2	0.74
	F	H	42.698 748	0.635	1.415	55.96	0.28	16.88	+1.13	3.67	0.25	7.4	0.5	0.40
H	G	J	270.283 883	0.068	1.937	62.05	0.02	23.64	+0.11	3.23	0.01	9.0	0.0	0.03
	J	G	129.716 117	1.167	1.937	62.05	0.33	23.64	+1.88	3.23	0.26	9.0	0.7	0.47
	J	K	329.714 249	1.183	2.246	51.67	0.36	26.61	+2.29	4.00	0.34	12.9	1.1	0.51
J	H	C	150.157 679	−0.272	3.396	23.21	0.10	47.65	−1.17	7.52	0.18	36.6	0.9	0.15
K	H	E	145.861 227	−0.116	2.482	35.03	0.04	30.81	−0.33	5.63	0.06	20.0	0.2	0.06

Network graph



Supplementary information

Observed grid bearings	:	1
Observed distances	:	12
Observed angles	:	14
Coordinate unknowns	:	18
Datum defect	:	2
Datum definition	:	fix
Number of datum constraints	:	2
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	3.4
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.6
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$3.4 \cdot 10^{-11}$
Redundancy r	:	9
Redundancy grid bearings	:	0.00
Redundancy distances	:	2.89
Redundancy angles	:	6.11
Weighted square sum of residuals Ω [-]	:	4.38065
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	0.6977
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.6977
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.4867 ($k^F_{\alpha_G;r,\infty}$ =2.01)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	2.188
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	6.3113

3.7 Distance-Direction-Angle networks

3.7.1 Wolf (1979), Ex. I3.2-11

Wolf W (1979): Ausgleichungsrechnung II. Dümmlers, pp. 66

Available data files: [2D] Wolf_DistanceDirectionAngle_free∗.∗

Coordinates

Point name	Easting x [m]	Northing y [m]
1	184 423.2800 (D)	726 419.3300 (D)
2	186 444.1800 (D)	726 476.6600 (D)
3	183 257.8400 (D)	725 490.3500 (D)
4	184 292.0000 (D)	723 313.0000 (D)
5	185 487.0000 (D)	721 829.0000 (D)
6	186 708.7200 (D)	722 104.5800 (D)
7	184 868.2000 (D)	725 139.7000 (D)
8	186 579.3000 (D)	725 336.6000 (D)
9	185 963.0700 (D)	723 322.0200 (D)

Datum: free, (D)...Datum coordinate

Horizontal distances

in	to	s [m]	$\lvert\sigma\rvert$ [mm]	p [rad²/m²]
7	9	2121.9000	30	$1.713\,47\cdot10^{-6}$

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]
8	7	2	99.781 000 0	3.5	$5.102\,04\cdot10^{-1}$

Directions and approximate orientations

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
1	2	0.000 000	2.5	1	98
	7	80.500 000	2.5	1	
	3	158.961 000	2.5	1	
2	8	0.000 000	2.5	1	192
	7	62.726 000	2.5	1	
	1	105.712 000	2.5	1	
3	1	0.000 000	2.5	1	57
	7	56.496 000	2.5	1	
	9	85.845 000	2.5	1	
	4	114.595 000	2.5	1	

Directions and approximate orientations (continued)

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
4	7	0.000 000	2.5	1	19
	8	34.450 000	2.5	1	
	9	80.211 000	2.5	1	
	5	137.402 000	2.5	1	
	3	352.309 000	2.5	1	
5	9	0.000 000	2.5	1	19
	6	66.245 000	2.5	1	
	4	337.216 000	2.5	1	
6	5	0.000 000	2.5	1	285
	9	79.169 000	2.5	1	
	8	111.582 000	2.5	1	
7	2	0.000 000	2.5	1	55
	8	37.498 000	2.5	1	
	9	110.258 000	2.5	1	
	4	164.232 000	2.5	1	
	3	258.441 000	2.5	1	
	1	323.486 000	2.5	1	
8	6	0.000 000	2.5	1	197
	9	21.445 000	2.5	1	
	4	56.442 000	2.5	1	
9	8	0.000 000	2.5	1	18
	6	146.143 000	2.5	1	
	5	200.733 000	2.5	1	
	4	280.756 000	2.5	1	
	3	324.105 000	2.5	1	
	7	346.569 000	2.5	1	

Design matrices and reduced observation vectors (1. iteration)

Too large to be displayed !

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints (1. iteration)

Too large to be displayed !

Least squares solution $\widehat{\Delta x}_{[\text{m,mgon}]}$ (1. iteration)

	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5
$\widehat{\Delta x}^{\mathsf{T}}$	−0.2465	0.3316	0.1744	0.1348	−0.5272	0.2304	0.0766	0.2968	0.3937	−0.4777

Least squares solution $\widehat{\Delta x}_{[\text{m,mgon}]}$ (continued)

	x_6	y_6	x_7	y_7	x_8	y_8	x_9	y_9
$\widehat{\Delta x}^{\mathsf{T}}$	−0.0638	−0.5967	−0.1910	−0.0377	0.1918	−0.1407	0.1919	0.2593

Least squares solution $\widehat{\Delta x}_{[\mathrm{m,mgon}]}$ (continued)

	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
$\widehat{\Delta x}^{\mathsf{T}}$	200.664	489.671	160.946	447.156	632.997	876.144	214.669	456.583	901.610

Adjusted coordinates

Point name	$\hat{x}$ [m]	$\hat{x} - x$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\hat{y}$ [m]	$\hat{y} - y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{2D}\rvert$ [cm]
1	184 423.0335	−24.648	2.183	726 419.6616	33.165	3.117	3.805
2	186 444.3543	17.433	2.510	726 476.7948	13.484	3.512	4.317
3	183 257.3128	−52.720	3.557	725 490.5804	23.041	2.099	4.130
4	184 292.0767	7.667	2.172	723 313.2969	29.691	2.190	3.085
5	185 487.3938	39.385	1.780	721 828.5221	−47.787	3.704	4.110
6	186 708.6561	−6.392	2.975	722 103.9831	−59.694	3.388	4.509
7	184 868.0090	−19.096	1.254	725 139.6623	−3.770	1.249	1.770
8	186 579.4918	19.177	2.793	725 336.4593	−14.068	2.547	3.780
9	185 963.2619	19.195	1.060	723 322.2794	25.938	1.438	1.786

Absolute error ellipses

in	A [cm]	B [cm]	ϕ [gon]
1	3.21	2.04	179.690 890
2	3.82	2.01	30.576 400
3	3.66	1.92	117.480 524
4	2.24	2.12	154.760 361
5	3.72	1.75	5.895 903
6	4.10	1.87	156.312 869
7	1.29	1.22	52.257 621
8	3.18	2.04	57.061 588
9	1.44	1.05	6.684 932

Absolute confidence ellipses ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	ϕ [gon]
1	8.79	5.58	179.690 890
2	10.44	5.51	30.576 400
3	9.99	5.26	117.480 524
4	6.13	5.80	154.760 361
5	10.16	4.80	5.895 903
6	11.22	5.12	156.312 869
7	3.52	3.33	52.257 621
8	8.70	5.58	57.061 588
9	3.94	2.88	6.684 932

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 1.969,\ r = 13.59,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.14,\ \alpha_G = 6.37\%,\ k^F_{\alpha_G;r,\infty} = 1.64$

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	2	0.000 392	98.201 056	−0.392	0.892	23.60	0.32	21.26	−1.662	2022	12.5	5.58	0.44	516.0	40.3	331.4	25.9	0.79
	7	80.500 050	178.700 713	−0.050	0.824	34.83	0.03	17.50	−0.142	1355	1.1	3.95	0.03	242.8	2.0	108.2	0.9	0.08
	3	158.960 558	257.161 222	0.442	0.876	26.27	0.34	20.15	+1.681	1491	10.3	5.12	0.43	348.0	29.0	175.5	14.6	0.84
2	8	0.000 980	192.490 650	−0.980	0.911	20.18	0.87	23.00	−4.855	1148	17.7	6.27	1.32	331.1	69.9	225.5	47.6	2.14
	7	62.725 635	255.215 305	0.365	0.805	37.76	0.24	16.81	+0.968	2067	11.9	3.62	0.21	339.7	19.6	97.8	5.6	0.58
	1	105.711 386	298.201 056	0.614	0.854	29.92	0.45	18.89	+2.053	2022	19.5	4.58	0.50	420.4	45.7	239.0	26.0	1.10
3	1	0.000 276	57.161 222	−0.276	0.877	26.16	0.22	20.20	−1.055	1491	6.5	5.65	0.30	349.2	18.2	257.3	13.4	0.53
	7	56.495 516	113.656 461	0.484	0.720	50.25	0.27	14.57	+0.964	1648	12.5	2.90	0.19	187.7	12.4	112.7	7.5	0.67
	9	85.845 172	143.006 118	−0.172	0.629	61.96	0.09	13.12	−0.278	3468	9.4	1.90	0.04	271.9	5.8	29.8	0.6	0.21
	4	114.595 036	171.755 982	−0.036	0.765	43.81	0.02	15.61	−0.083	2411	1.4	3.49	0.02	332.1	1.8	173.6	0.9	0.05
4	7	399.999 875	19.447 031	0.125	0.689	54.38	0.07	14.01	+0.230	1915	3.8	2.84	0.05	192.2	3.2	115.2	1.9	0.17
	8	34.450 657	53.897 812	−0.657	0.646	59.92	0.34	13.35	−1.096	3054	31.5	2.39	0.20	256.6	21.1	118.1	9.7	0.83
	9	80.210 670	99.657 826	0.330	0.771	42.96	0.20	15.76	+0.768	1671	8.7	3.84	0.19	236.0	11.5	167.3	8.1	0.49
	5	137.401 972	156.849 127	0.028	0.866	28.01	0.02	19.52	+0.101	1906	0.9	5.63	0.03	420.7	2.2	305.8	1.6	0.05
	3	352.308 826	371.755 982	0.174	0.810	36.96	0.11	16.99	+0.470	2411	6.6	4.46	0.12	405.7	11.2	252.6	7.0	0.28
5	9	0.000 763	19.633 759	−0.763	0.824	34.78	0.52	17.52	−2.193	1568	18.8	3.96	0.50	281.3	35.2	135.5	17.0	1.27
	6	66.244 106	85.877 103	0.894	0.880	25.55	0.71	20.44	+3.498	1252	17.6	5.24	0.90	299.2	51.2	188.4	32.2	1.73
	4	337.216 131	356.849 127	−0.131	0.870	27.22	0.10	19.80	−0.482	1906	3.9	4.97	0.12	431.5	10.5	202.6	4.9	0.25
6	5	0.000 959	285.877 103	−0.959	0.892	23.63	0.79	21.25	−4.058	1252	18.9	5.58	1.06	319.2	60.9	199.6	38.1	1.93
	9	79.168 658	365.044 801	0.342	0.769	43.13	0.21	15.73	+0.793	1428	7.7	3.05	0.15	200.7	10.1	94.5	4.8	0.51
	8	111.581 383	397.457 527	0.617	0.805	37.70	0.40	16.82	+1.636	3235	31.3	3.62	0.35	532.6	51.8	179.5	17.4	0.98
7	2	0.000 637	55.215 305	−0.637	0.862	28.65	0.48	19.30	−2.223	2067	20.7	5.71	0.66	447.1	51.5	360.8	41.6	1.17
	8	37.497 088	92.711 755	0.912	0.867	27.71	0.69	19.63	+3.293	1723	24.7	5.86	0.98	384.0	64.4	332.3	55.8	1.70
	9	110.257 074	165.471 742	0.926	0.748	46.25	0.54	15.19	+2.003	2122	30.9	3.70	0.49	272.2	35.9	139.4	18.4	1.34
	4	164.232 363	219.447 031	−0.363	0.828	34.16	0.25	17.67	−1.063	1915	10.9	4.96	0.30	350.0	21.0	228.1	13.7	0.61
	3	258.441 794	313.656 461	−0.794	0.844	31.54	0.57	18.39	−2.516	1648	20.5	5.29	0.72	326.0	44.6	243.5	33.3	1.38
	1	323.486 045	378.700 713	−0.045	0.860	28.95	0.03	19.20	−0.156	1355	1.0	5.66	0.05	290.4	2.4	249.5	2.0	0.08
8	6	0.000 944	197.457 527	−0.944	0.749	46.15	0.56	15.21	−2.045	3235	48.0	2.76	0.37	416.2	56.0	178.2	24.0	1.36
	9	21.444 827	218.901 410	0.173	0.670	56.91	0.09	13.69	+0.304	2106	5.7	1.71	0.04	195.2	4.3	74.6	1.7	0.22
	4	56.441 229	253.897 812	0.771	0.772	42.70	0.47	15.81	+1.805	3054	37.0	3.10	0.35	434.5	49.6	111.7	12.8	1.16
9	8	399.999 801	18.901 410	0.199	0.818	35.65	0.13	17.30	+0.558	2106	6.6	4.78	0.15	368.4	11.9	260.3	8.4	0.33
	6	146.143 192	165.044 801	−0.192	0.868	27.59	0.15	19.67	−0.696	1428	4.3	5.87	0.21	319.4	11.3	261.3	9.3	0.36
	5	200.732 150	219.633 759	0.850	0.864	28.27	0.64	19.43	+3.009	1568	20.9	5.77	0.89	343.2	53.1	283.6	43.9	1.57
	4	280.756 217	299.657 826	−0.217	0.785	40.84	0.14	16.16	−0.530	1671	5.7	4.21	0.14	251.0	8.2	201.7	6.6	0.33
	3	324.104 508	343.006 118	0.492	0.644	60.14	0.25	13.32	+0.817	3468	26.8	2.57	0.16	289.2	17.7	127.6	7.8	0.62
	7	346.570 132	365.471 742	−1.132	0.690	54.32	0.61	14.02	−2.084	2122	37.7	3.02	0.45	213.4	31.7	112.6	16.7	1.51

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
1	98.200 664	200.66	0.93
2	192.489 670	489.67	1.00
3	57.160 946	160.95	0.71
4	19.447 156	447.16	0.61
5	19.632 996	633.00	0.87
6	285.876 144	876.14	0.87
7	55.214 668	214.67	0.51
8	197.456 583	456.58	0.80
9	18.901 609	901.61	0.50

Adjusted horizontal distances

Variance component: $\Omega/\sigma_0^2 = 0.000,\ r = 0.00,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.00,\ \alpha_G = 0.00‰,\ k^F_{\alpha_G;r,\infty} = \infty$

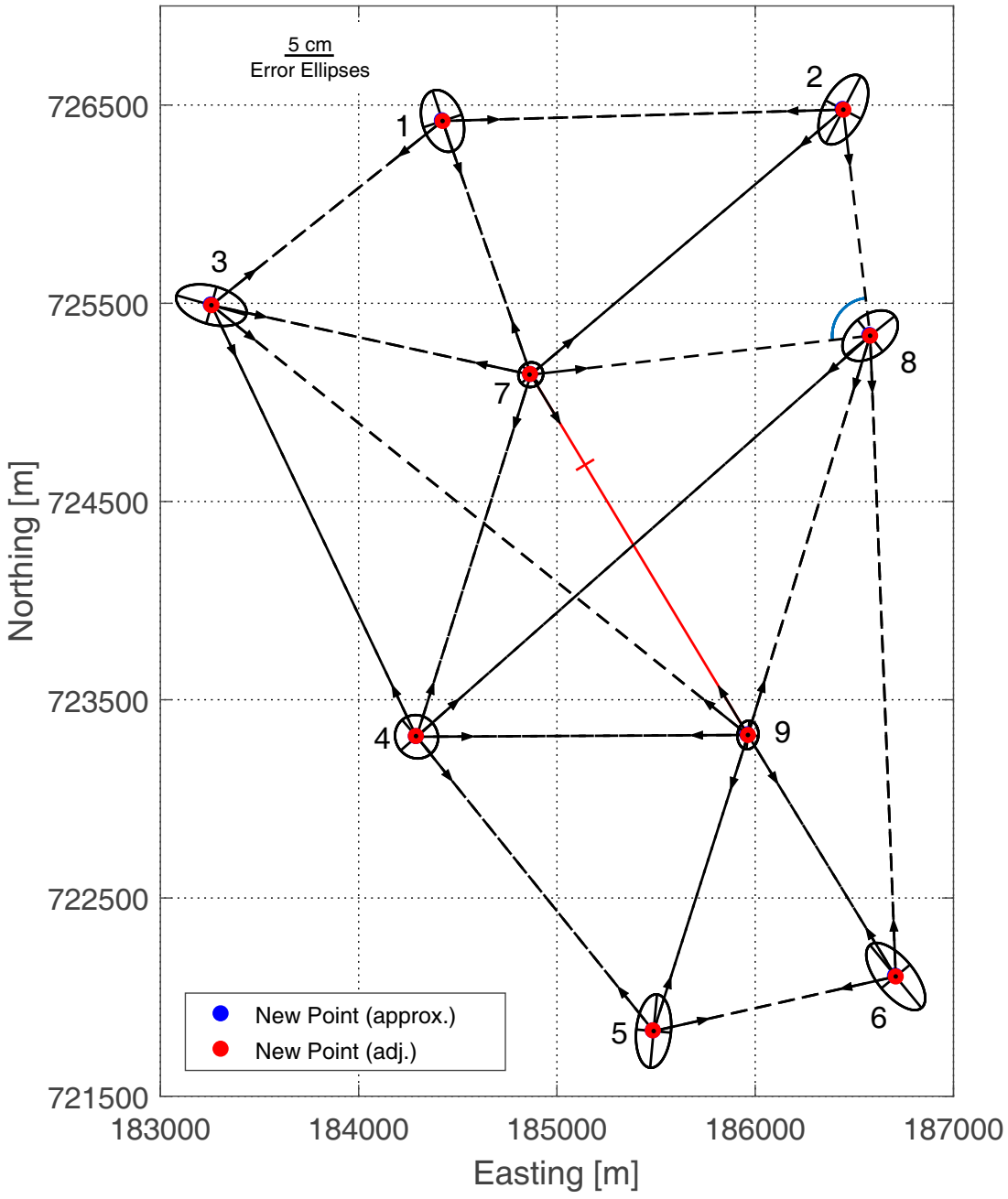
in	to	$\hat{s}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{s}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
7	9	2121.9000	0	1.224	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 0.362,\ r = 0.41,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.88,\ \alpha_G = 0.03\%,\ k^F_{\alpha_G;r,\infty} = 25.00$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
8	7	2	99.778 894	2.106	1.095	41.19	0.94	0.35	+5.11	4.94	1.12	13.3	3.0	2.3

Network graph



Supplementary information

Observed directions	:	36
Observed distances	:	1
Observed angles	:	1
Orientation unknowns	:	9
Coordinate unknowns	:	18
Datum defect	:	3
Datum definition	:	free
Number of datum constraints	:	3
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	6.6
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.8
Number of iterations (Max=20)	:	5
Stop criterion (actual)	:	$6.4 \cdot 10^{-11}$
Redundancy r	:	14
Redundancy directions	:	13.59
Redundancy distances	:	0.00
Redundancy angles	:	0.41
Weighted square sum of residuals Ω [mgon^2]	:	14.572
(a priori) standard deviation σ_0 [mgon]	:	2.5
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	1.0202
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.4081
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.1665 ($k^F_{\alpha_G;r,\infty}$ =1.62)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	125.971

4 Three-dimensional networks (Spatial networks)

4.1 Baumann (1995), Ch. 23.3.4

Baumann Eberhard (1995): Vermessungskunde, Lehr- und Übungsbuch für Ingenieure. Band 2: Punktbestimmung nach Höhe und Lage. Fünfte, bearbeitete und erweiterte Auflage Bonn. Ch. 23.3.4, pp. 224

Available data files: [3D] Baumann23_3_4_fix∗.∗

Coordinates

Point name	X [m]	Y [m]	Z [m]
1	1000.0000 (D)	1201.1710 (D)	108.6800 (D)
2	1371.2170 (D)	1072.8950 (D)	111.9740 (D)
3	1016.4370 (D)	952.3520 (D)	117.3120 (D)
N	1181.7660	1071.6740	94.2580

Datum: fix, (D)...Datum coordinate

Approximate orientations

Point name	ω [gon]
N	339.409 000

Spatial distances

in	to	S [m]	$\lvert\sigma\rvert$ [mm]	p [rad²/m²]
N	1	223.6428	5	$3.947\,84{\cdot}10^{-5}$
	2	190.2878	5	$3.947\,84{\cdot}10^{-5}$
	3	205.1894	5	$3.947\,84{\cdot}10^{-5}$

Directions

in	to	r [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	ω [gon]
N	1	0.000 000	2	1	339.409 000
	2	160.183 800	2	1	
	3	320.788 400	2	1	

Zenith angles

in	to	z [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]	i_h [m]	t_h [m]
N	1	95.901 500 0	2.5	0.64	1.600	1.572
	2	94.045 000 0	2.5	0.64	1.600	1.650
	3	92.839 000 0	2.5	0.64	1.600	1.588

Design matrix $A_{[\text{mgon/m,-}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ directions (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_N	Y_N	Z_N	ω_N	Δy
$r_{N,1}$	165.5151	232.3221	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0026	−0.0036	0.0000	−1	0.65
$r_{N,2}$	0.0000	0.0000	0.0000	2.1656	−336.0200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0053	0.0000	−1	3.09
$r_{N,3}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−182.7283	253.1829	0.0000	0.0029	−0.0040	0.0000	−1	−3.81

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[\text{mm}]}$ spatial distances (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_N	Y_N	Z_N	Δy
$S_{N,1}$	−0.8128	0.5790	0.0644	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8128	−0.5790	−0.0644	1.26
$S_{N,2}$	0.0000	0.0000	0.0000	0.9956	0.0064	0.0934	0.0000	0.0000	0.0000	−0.9956	−0.0064	−0.0934	1.69
$S_{N,3}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.8057	−0.5815	0.1123	0.8057	0.5815	−0.1123	0.82

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ zenith angles (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_N	Y_N	Z_N	Δy
$z_{N,1}$	−14.9217	10.6308	−284.0706	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	14.9217	−10.6308	284.0706	1.74
$z_{N,2}$	0.0000	0.0000	0.0000	31.2354	0.2013	−333.0979	0.0000	0.0000	0.0000	−31.2354	−0.2013	333.0979	−2.55
$z_{N,3}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−28.2518	−20.3900	−308.2983	28.2518	20.3900	308.2983	3.14

Matrix $D^{T}_{[-]}$ of datum constraints

	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_N	Y_N	Z_N	ω_N
	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0
D^{T}	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{cm,mgon}]}$ (1. iteration)

	X_N	Y_N	Z_N	ω_N
$\widehat{\Delta x}^{T}$	−0.148	0.552	0.183	−0.259

Adjusted coordinates

Point name	$\widehat{X}$ [m]	$\widehat{X} - X$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y} - Y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z} - Z$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
N	1181.7645	−0.148	0.348	1071.6795	0.552	0.396	94.2598	0.183	0.526	0.745

Absolute error ellipsoids

in	A [cm]	B [cm]	C [cm]
N	0.53	0.40	0.35

Absolute confidence ellipsoids ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	C [m]
N	2.12	1.60	1.40

Adjusted orientation unknowns

in	$\hat{\omega}$ [gon]	$\hat{\omega} - \omega$ [mgon]	$\lvert\hat{\sigma}\rvert$ [mgon]
N	339.408 741	−0.26	1.33

Adjusted directions

Variance component: $\Omega/\sigma_0^2 = 2.191,\ r = 1.19,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.85,\ \alpha_G = 0.13\%,\ k^F_{\alpha_G;r,\infty} = 9.28$

in	to	$\hat{r}$ [gon]	$\widehat{T}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{r}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	s [m]	Q_f [mm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
N	1	399.998 571	339.407 312	1.429	1.619	49.55	1.02	11.74	+2.884	223	5.0	2.43	0.60	20.8	5.1	8.1	2.0	0.89
	2	160.182 826	99.591 567	0.974	2.016	21.77	1.04	17.71	+4.472	189	2.9	5.93	1.50	41.2	10.4	20.6	5.2	0.92
	3	320.790 803	260.199 543	−2.403	1.656	47.19	1.75	12.03	−5.091	204	7.7	2.65	1.12	20.3	8.6	8.7	3.7	1.53

Adjusted spatial distances

Variance component: $\Omega/\sigma_0^2 = 1.381,\ r = 1.80,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.77,\ \alpha_G = 0.24\%,\ k^F_{\alpha_G;r,\infty} = 6.42$

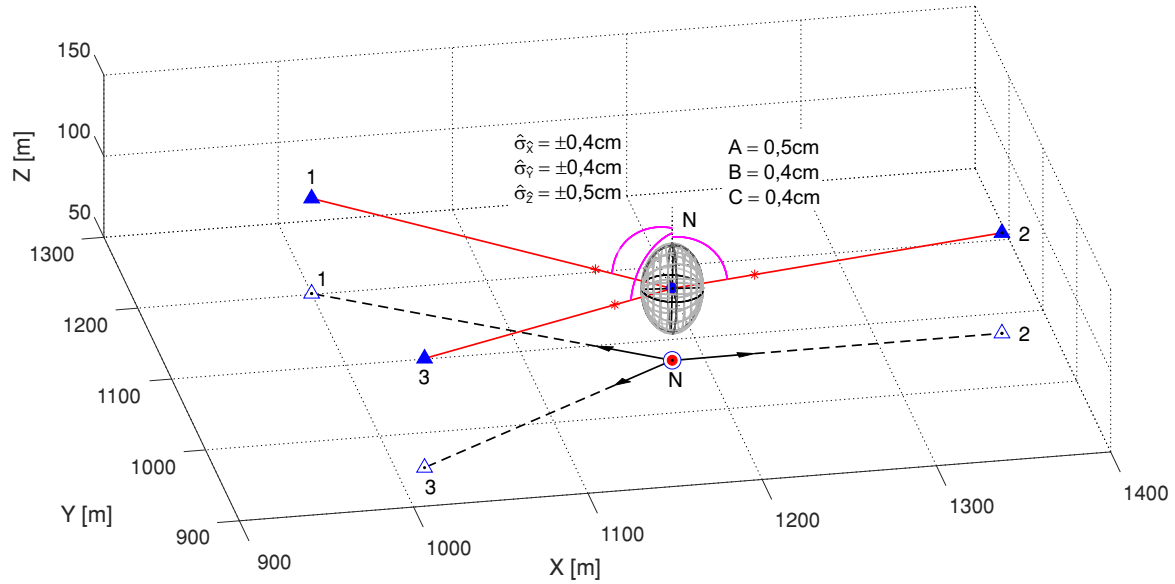
in	to	$\widehat{S}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{S}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
N	1	223.6370	0.578	0.362	59.7	1.5	2.7	+0.968	3.4	1.2	10.8	3.9	10.8	3.9	1.31
	2	190.2874	0.042	0.351	62.1	0.1	2.6	+0.068	3.2	0.1	9.9	0.3	9.9	0.3	0.09
	3	205.1904	−0.099	0.369	58.0	0.3	2.7	−0.171	3.5	0.2	11.4	0.7	11.4	0.7	0.23

Adjusted zenith angles

Variance component: $\Omega/\sigma_0^2 = 2.921,\ r = 2.02,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.45,\ \alpha_G = 0.29\%,\ k^F_{\alpha_G;r,\infty} = 5.82$

in	to	$\hat{z}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{z}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
N	1	95.900 197 4	1.303	1.498	72.37	0.61	12.14	+1.80	2.55	0.38	11.8	1.7	11.8	1.7	0.54
	2	94.048 205 6	−3.206	1.754	62.08	1.63	13.11	−5.16	3.23	1.27	14.9	5.9	14.9	5.9	1.43
	3	92.836 493 4	2.507	1.631	67.23	1.22	12.60	+3.73	2.89	0.85	13.3	3.9	13.3	3.9	1.07

Network graph



Supplementary information

Observed directions	:	3
Observed spatial distances	:	3
Observed zenith angles	:	3
Orientation unknowns	:	1
Coordinate unknowns	:	3
Datum defect	:	4
Datum definition	:	fix
Number of datum constraints	:	9
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	1.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.2
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$3 \cdot 10^{-12}$
Redundancy r	:	5
Redundancy directions	:	1.19
Redundancy spatial distances	:	1.80
Redundancy zenith angles	:	2.02
Weighted square sum of residuals Ω [mgon^2]	:	25.972
(a priori) standard deviation σ_0 [mgon]	:	2
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	2.2791
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.1396
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	1.2986 ($k^F_{\alpha_G;r,\infty}$ =2.89)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.572
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.55466

4.2 Blankenbach & Willert (2009)

Blankenbach J and V Willert (2009): Robuster räumlicher Bogenschnitt - Ein Ansatz zur robusten Positionsberechnung in Indoor-Szenarien, AVN, 8-9, pp. 320-327

Available data files: [3D] BlankenbachWillert3D_Distance_fix∗.∗

Coordinates

Point number	ID	X [m]	Y [m]	Z [m]
51	51	−8.6200 (D)	27.4200 (D)	10.2900 (D)
37	37	−4.9500 (D)	27.3000 (D)	10.0400 (D)
31	31	−4.8500 (D)	20.6900 (D)	10.5100 (D)
102	10	0.5200 (D)	22.5900 (D)	9.7700 (D)
331	33	−6.1200 (D)	25.4800 (D)	10.9200 (D)
35	35	0.4500 (D)	27.3900 (D)	10.7500 (D)
103	01	0.6100 (D)	17.1900 (D)	9.7100 (D)
101	02	−7.8800 (D)	22.4500 (D)	10.1600 (D)
MS	MS	−2.5900	24.2200	9.6200

Datum: fix, (D)...Datum coordinate

Spatial distances

in	to	S [m]	$\lvert\sigma\rvert$ [cm]	p [−]
MS	31	4.3500	5	1
	35	4.3700	5	1
	37	3.8500	5	1
	51	8.2000	5	1
	02	5.7300	5	1
	10	3.5100	5	1
	01	8.1700	5	1
	33	3.9800	5	1

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{m}]}$ spatial distances (1. iteration)

Too large to displayed!

Matrix $D^{\mathsf{T}}{}_{[\text{-}]}$ of datum constraints

Too large to displayed!

Least squares solution $\widehat{\Delta x}_{[\text{m}]}$ (1. iteration)

	X_{MS}	Y_{MS}	Z_{MS}
$\widehat{\Delta x}^{\mathsf{T}}$	0.2848	0.0756	−0.0363

Adjusted coordinates

ID	$\widehat{X}$ [m]	$\widehat{X} - X$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y} - Y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z} - Z$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
MS	−2.3042	28.576	29.914	24.3101	9.009	33.222	9.5248	−9.522	119.427	127.52

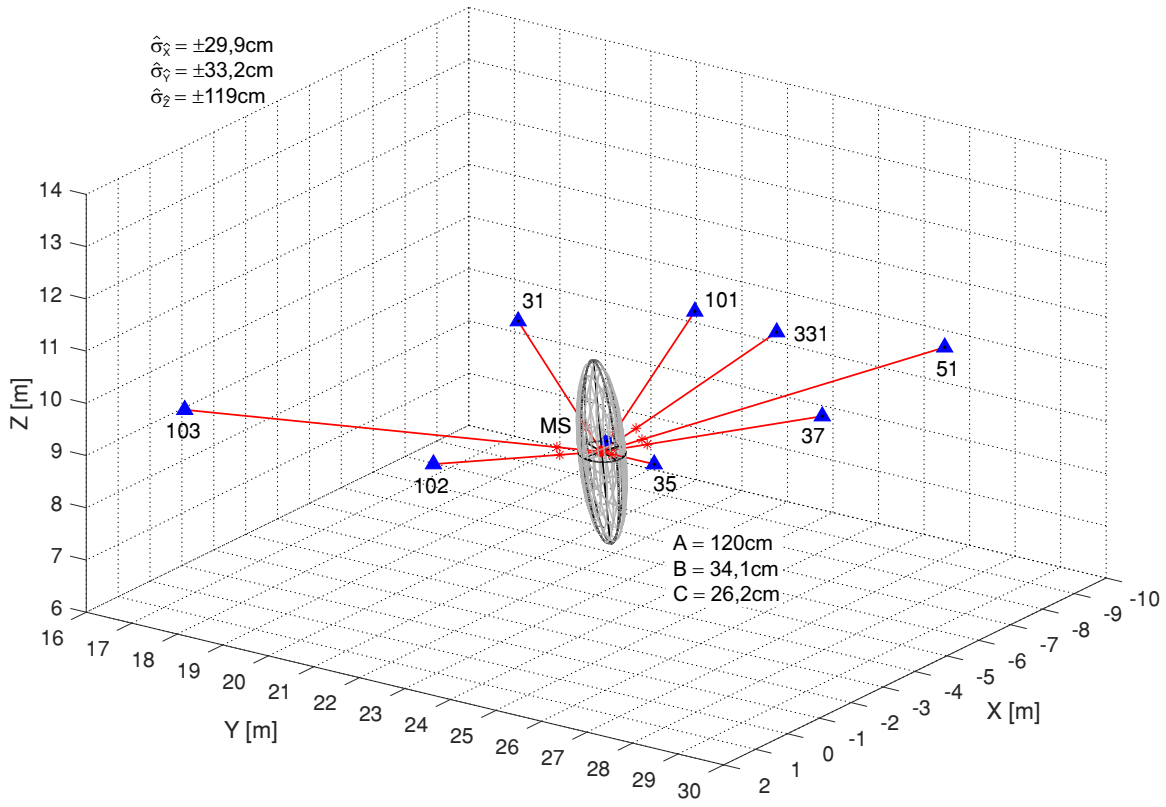
Absolute error ellipsoids

in	A [cm]	B [cm]	C [cm]
MS	120.04	34.13	26.19

Absolute confidence ellipsoids ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	C [m]
MS	483.58	137.50	105.52

Network graph



Adjusted spatial distances

in	to	$\widehat{S}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{S}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
MS	31	4.5339	−18.394	40.084	52.09	5.10	28.63	−35.310*	3.96	4.89	13.71	16.92	13.71	16.92	0.44
	35	4.3096	6.038	50.043	25.33	2.40	41.05	+23.837	7.09	4.12	30.65	17.80	30.65	17.80	0.21
	37	4.0255	−17.555	28.103	76.45	4.02	23.63	−22.962*	2.29	2.23	5.56	5.41	5.56	5.41	0.35
	51	7.0814	111.862	26.220	79.50	25.09	23.17	+140.703#	2.10	12.74	4.75	28.84	4.75	28.84	2.17
	02	5.9121	−18.207	30.109	72.97	4.26	24.19	−24.951*	2.51	2.59	6.54	6.74	6.54	6.74	0.37
	10	3.3159	19.410	33.348	66.84	4.75	25.27	+29.040*	2.91	3.34	8.38	9.63	8.38	9.63	0.41
	01	7.6956	47.436	32.161	69.16	11.41	24.84	+68.589*	2.76	7.62	7.66	21.15	7.66	21.15	0.98
	33	4.2279	−24.793	37.688	57.65	6.53	27.21	−43.006*	3.54	5.60	11.52	18.21	11.52	18.21	0.56

Supplementary information

Observed spatial distances	:	8
Coordinate unknowns	:	3
Datum defect	:	6
Datum definition	:	fix
Number of datum constraints	:	24
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	1.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.2
Number of iterations (Max=20)	:	20
Stop criterion (actual)	:	$2.2 \cdot 10^{-7}$
Redundancy r	:	5
Redundancy spatial distances	:	5.00
Weighted square sum of residuals Ω [m^2]	:	1.67692
(a priori) standard deviation σ_0 [m]	:	$5 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [m]	:	$5.79124 \cdot 10^{-1}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	11.5825
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	134.1540 ($k^F_{\alpha_G;r,\infty}$ =2.89)
Number of outliers (Data snooping)	:	7 (Remove outliers or scale standard deviations by the factor 11.58)
Number of outliers (τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	29.963
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	16261.22370

4.3 Wolf H (1979), Ex. I3.2-14

Wolf H (1979): Ausgleichungsrechnung II. Dümmlers, pp. 87

Coordinates

Point number	X [m]	Y [m]	Z [m]
1	1200.0000 (D)	900.0000 (D)	900.0000 (D)
2	900.0000 (D)	600.0000 (D)	900.0000 (D)
3	600.0000 (D)	900.0000 (D)	900.0000 (D)
4	900.0000 (D)	1200.0000 (D)	900.0000 (D)
P	900.0000	900.0000	1300.0000

Datum: fix, (D)...Datum coordinate

Spatial distances

in	to	S [m]	$\lvert\sigma\rvert$ [cm]	p [1/m^2]
1	P	499.9900	1	1
2	P	500.0000	1	1
3	P	500.0100	1	1
4	P	500.0200	1	1

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[cm]}$ spatial distances (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_P	Y_P	Z_P	Δy
$S_{1,P}$	0.6	0.0	−0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−0.6	0.0	0.8	−1.0
$S_{2,P}$	0.0	0.0	0.0	0.0	−0.6	−0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.8	0.0
$S_{3,P}$	0.0	0.0	0.0	0.0	0.0	0.0	−0.6	0.0	−0.8	0.0	0.0	0.0	0.6	0.0	0.8	1.0
$S_{4,P}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	−0.8	0.0	−0.6	0.8	2.0

Matrix $D^T{}_{[-]}$ of datum constraints

	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_P	Y_P	Z_P
D^T	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

4.3.1 Spatial distance observations only

Available data files: [3D] Wolf_3D_Distance_fix∗.∗

Least squares solution $\widehat{\Delta x}_{[\mathrm{m}]}$ (1. iteration)

	X_P	Y_P	Z_P
$\widehat{\Delta x}^\mathsf{T}$	0.0167	−0.0167	0.0062

Adjusted coordinates

Point name	$\widehat{X}$ [m]	$\widehat{X}-X$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y}-Y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z}-Z$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
P	900.0167	1.667	1.179	899.9833	−1.667	1.179	1300.0062	0.625	0.625	1.780

Absolute error ellipsoids

in	A [cm]	B [cm]	C [cm]
P	1.18	1.18	0.62

Absolute confidence ellipsoids $(1-\alpha = 95\%)$

in	A [cm]	B [cm]	C [m]
P	29.98	29.98	15.90

4.3.2 Spatial distance and vertical angle observations

Available data files: [3D] Wolf_3D_DistanceVerticalAngle_fix∗.∗

Vertical angles

	to	β [gon]	$\lvert\sigma\rvert$ [gon]	p [1/rad^2]
1	P	59.033 271 6	0.012 73	2500
2	P	59.033 302 5	0.012 73	2500
3	P	59.033 333 3	0.012 73	2500
4	P	59.033 364 2	0.012 73	2500

Design matrix $A_{[\mathrm{mgon/m}]}$ **and reduced observation vector** $\Delta y_{[\mathrm{mgon}]}$ **vertical angles** (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_P	Y_P	Z_P	Δy
$\beta_{1,\mathrm{P}}$	−101.8592	0	−76.3944	0	0.0000	0.0000	0.0000	0	0.0000	0	0.0000	0.0000	101.8592	0.0000	76.3944	−0.18
$\beta_{2,\mathrm{P}}$	0.0000	0	0.0000	0	101.8592	−76.3944	0.0000	0	0.0000	0	0.0000	0.0000	0.0000	−101.8592	76.3944	−0.14
$\beta_{3,\mathrm{P}}$	0.0000	0	0.0000	0	0.0000	0.0000	101.8592	0	−76.3944	0	0.0000	0.0000	−101.8592	0.0000	76.3944	−0.11
$\beta_{4,\mathrm{P}}$	0.0000	0	0.0000	0	0.0000	0.0000	0.0000	0	0.0000	0	−101.8592	−76.3944	0.0000	101.8592	76.3944	−0.08

Least squares solution $\widehat{\Delta x}_{[\mathrm{m}]}$ (1. iteration)

	X_P	Y_P	Z_P
$\widehat{\Delta x}^\mathsf{T}$	0.0164	−0.0164	0.0062

Adjusted coordinates

Point name	$\widehat{X}$ [m]	$\widehat{X}-X$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y}-Y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z}-Z$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
P	900.0164	1.637	0.543	899.9836	−1.637	0.543	1300.0062	0.621	0.290	0.821

Absolute error ellipsoids

in	A [cm]	B [cm]	C [cm]
P	0.54	0.54	0.29

Absolute confidence ellipsoids $(1-\alpha = 95\%)$

in	A [cm]	B [cm]	C [m]
P	2.19	2.19	1.17

Adjusted spatial distances (solely distance observations)

in	to	$\widehat{S}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{S}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	P	499.9950	−0.500	0.866	25.0	1.0	8.3	−2.000	7.2	1.7	62.0	15.0	62.0	15.0	1.00
2	P	499.9950	0.500	0.866	25.0	1.0	8.3	+2.000	7.2	1.7	62.0	15.0	62.0	15.0	1.00
3	P	500.0150	−0.500	0.866	25.0	1.0	8.3	−2.000	7.2	1.7	62.0	15.0	62.0	15.0	1.00
4	P	500.0150	0.500	0.866	25.0	1.0	8.3	+2.000	7.2	1.7	62.0	15.0	62.0	15.0	1.00

Adjusted spatial distances (in combination with vertical angles)

Variance component: $\Omega/\sigma_0^2 = 1.001,\ r = 1.04,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.96,\ \alpha_G = 0.11\%,\ k^F_{\alpha_G;r,\infty} = 10.44$

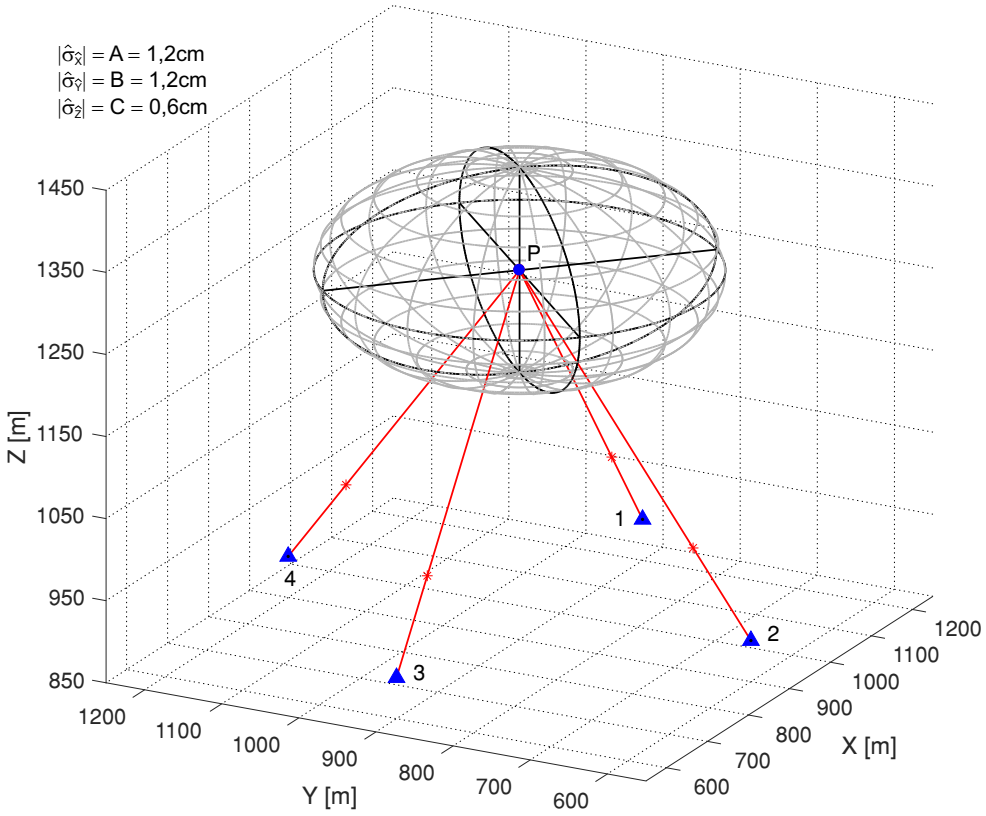
in	to	$\widehat{S}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{S}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	P	499.9951	−0.514	0.400	26.0	1.0	8.1	−1.977	7.0	1.7	59.9	14.6	59.9	14.6	2.17
2	P	499.9951	0.486	0.400	26.0	1.0	8.1	+1.867	7.0	1.6	59.9	13.8	59.9	13.8	2.05
3	P	500.0148	−0.479	0.400	26.0	0.9	8.1	−1.840	7.0	1.6	59.9	13.6	59.9	13.6	2.02
4	P	500.0148	0.521	0.400	26.0	1.0	8.1	+2.004	7.0	1.7	59.9	14.8	59.9	14.8	2.20*

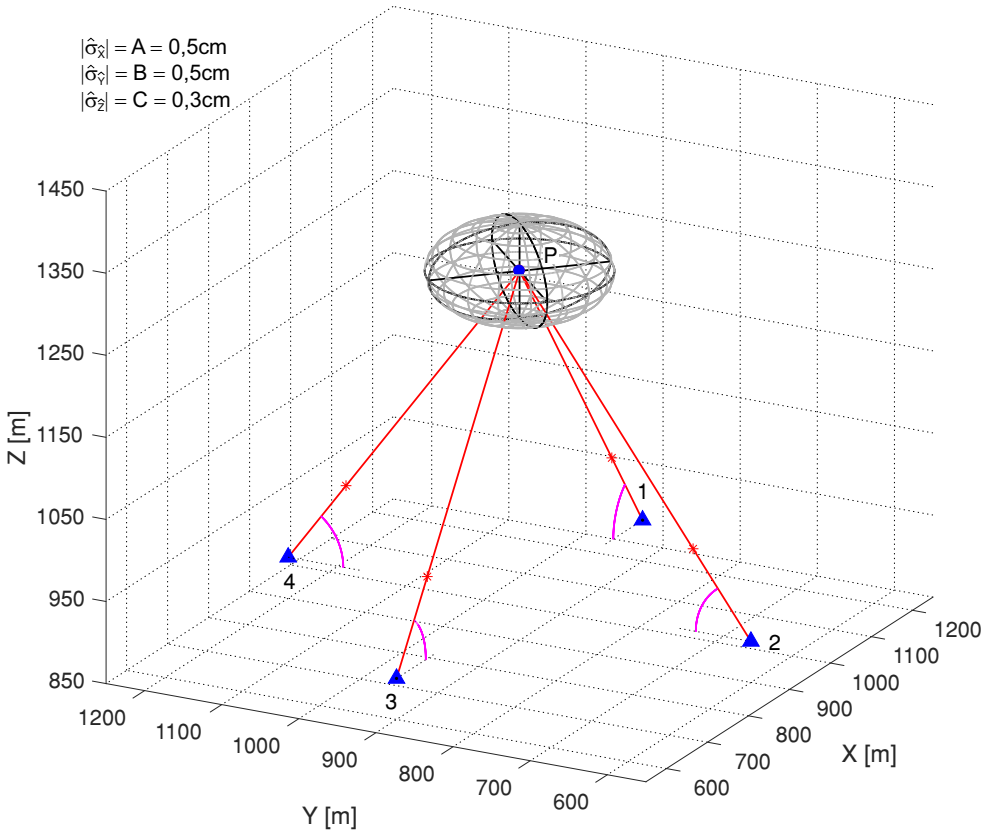
Adjusted vertical angles

Variance component: $\Omega/\sigma_0^2 = 0.080,\ r = 3.96,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.02,\ \alpha_G = 0.88\%,\ k^F_{\alpha_G;r,\infty} = 3.41$

in	to	$\hat{\beta}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\beta}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
1	P	59.035 588 6	−2.317	0.596	98.99	0.18	52.88	−2.34	0.42	0.02	4.2	0.2	4.2	0.2	0.39
2	P	59.035 588 5	−2.286	0.596	98.99	0.18	52.88	−2.31	0.42	0.02	4.2	0.2	4.2	0.2	0.39
3	P	59.032 253 6	1.080	0.596	98.99	0.09	52.88	+1.09	0.42	0.01	4.2	0.1	4.2	0.1	0.18
4	P	59.032 253 6	1.111	0.596	98.99	0.09	52.88	+1.12	0.42	0.01	4.2	0.1	4.2	0.1	0.19

Network graphs





Supplementary information

Observed spatial distances	:	4	4
Observed vertical angles	:	0	4
Coordinate unknowns	:	3	3
Datum defect	:	6	4
Datum definition	:	fix	fix
Number of datum constraints	:	12	12
Type-I-error probability α_L [%] (Baarda)	:	0.1	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.1	1.3
Test value $k^N_{\alpha_L/2}$	:	3.29	3.29
χ^2-Noncentrality parameter λ_0	:	17.1	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	∞	2.2
Number of iterations (Max=20)	:	3	3
Stop criterion (actual)	:	$3.2 \cdot 10^{-12}$	$3.5 \cdot 10^{-12}$
Redundancy r	:	1	5
Redundancy spatial distances	:	1.00	1.04
Redundancy vertical angles	:	n.a.	3.96
Weighted square sum of residuals Ω [-]	:	$1 \cdot 10^{-4}$	$1.08147 \cdot 10^{-4}$
(a priori) standard deviation σ_0 [-]	:	$1 \cdot 10^{-2}$	$1 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	$1 \cdot 10^{-2}$	$4.65073 \cdot 10^{-3}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.0000	0.4651
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	1.0000	0.2163
Critical value $k^F_{\alpha_G;r,\infty}$	:	10.83	2.89
Number of outliers (Data snooping & τ-criterion)	:	0	1
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	2.357	2.315
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	3.16845	0.67435

4.4 Wolf H (1979), Ex. I3.2-15

Wolf H (1979): Ausgleichungsrechnung II. Dümmlers, pp. 90

Available data files: [3D] Wolf_SpatialPolygonTraverse_fix*.*

Coordinates

Point name	X [m]	Y [m]	Z [m]
A	−2000 (D)	1000 (D)	0 (D)
S1	0	1000	1000
S2	0	−1000	1000
B	2000 (D)	−1000 (D)	0 (D)

Datum: fix, (D)...Datum coordinate

Spatial distances

in	to	S [m]	$\lvert\sigma\rvert$ [mm]	p [rad²/m²]
A	S1	2236.0680	35.120	$8.001\,86\cdot10^{-7}$
	S2	3000.0000	47.120	$4.445\,18\cdot10^{-7}$
B	S2	2236.0680	35.120	$8.001\,86\cdot10^{-7}$
S1	S2	2000.0000	31.420	$9.997\,41\cdot10^{-7}$

Horizontal angles

in	from	to	α [gon]	$\lvert\sigma\rvert$ [mgon]	p [−]
S1	S2	A	100	1	4
S2	B	S1	300	1	4

Vertical Angles

in	to	β [gon]	$\lvert\sigma\rvert$ [mgon]	p [−]
A	S1	29.516 700 0	2	1
B	S2	29.516 700 0	2	1

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ vertical angles (1. iteration)

A	X_A	Y_A	Z_A	X_{S1}	Y_{S1}	Z_{S1}	X_{S2}	Y_{S2}	Z_{S2}	X_B	Y_B	Z_B	Δy
$\beta_{A,S1}$	12.7324	0	−25.4648	−12.7324	0	25.4648	0.0000	0	0.0000	0.0000	0	0.0000	−0.02
$\beta_{B,S2}$	0.0000	0	0.0000	0.0000	0	0.0000	12.7324	0	25.4648	−12.7324	0	−25.4648	−0.02

Design matrix $A_{[\text{-}]}$ and reduced observation vector $\Delta y_{[\text{mm}]}$ spatial distances (1. iteration)

A	X_A	Y_A	Z_A	X_{S1}	Y_{S1}	Z_{S1}	X_{S2}	Y_{S2}	Z_{S2}	X_B	Y_B	Z_B	Δy
$S_{A,S1}$	−0.8944	0.0000	−0.4472	0.8944	0	0.4472	0.0000	0.0000	0.0000	0.0000	0	0.0000	0
$S_{A,S2}$	−0.6667	0.6667	−0.3333	0.0000	0	0.0000	0.6667	−0.6667	0.3333	0.0000	0	0.0000	0
$S_{B,S2}$	0.0000	0.0000	0.0000	0.0000	0	0.0000	−0.8944	0.0000	0.4472	0.8944	0	−0.4472	0
$S_{S1,S2}$	0.0000	0.0000	0.0000	0.0000	1	0.0000	0.0000	−1.0000	0.0000	0.0000	0	0.0000	0

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ horizontal angles (1. iteration)

A	X_A	Y_A	Z_A	X_S1	Y_S1	Z_S1	X_S2	Y_S2	Z_S2	X_B	Y_B	Z_B	Δy
$\alpha_{\text{S1,S2,A}}$	0	31.8310	0	−31.8310	−31.8310	0	31.8310	0.0000	0	0	0.0000	0	0
$\alpha_{\text{S2,B,S1}}$	0	0.0000	0	31.8310	0.0000	0	−31.8310	−31.8310	0	0	31.8310	0	0

Matrix $D^\mathsf{T}_{[\text{-}]}$ of datum constraints

	X_A	Y_A	Z_A	X_S1	Y_S1	Z_S1	X_S2	Y_S2	Z_S2	X_B	Y_B	Z_B
D^T	1	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	1

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (1. iteration)

	X_A	Y_A	Z_A	X_S1	Y_S1	Z_S1	X_S2	Y_S2	Z_S2	X_B	Y_B	Z_B
$\widehat{\Delta x}^\mathsf{T}$	0	0	0	0.010	−0.004	−0.053	0.001	0.001	−0.039	0	0	0

Adjusted coordinates

Point name	$\widehat{X}$ [m]	$\widehat{X} - X$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y} - Y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z} - Z$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
S1	0.0001	0.010	0.026	1000.0000	−0.004	0.022	999.9995	−0.053	0.049	0.060
S2	0.0000	0.001	0.023	−1000.0000	0.001	0.021	999.9996	−0.039	0.043	0.053

Absolute error ellipsoids

in	A[cm]	B[cm]	C[cm]
S1	0.05	0.02	0.02
S2	0.04	0.02	0.02

Absolute confidence ellipsoids ($1 - \alpha = 95\%$)

in	A[cm]	B[cm]	C[m]
S1	0.38	0.18	0.16
S2	0.33	0.17	0.15

Adjusted vertical angles

Variance component: $\Omega/\sigma_0^2 = 0.000,\ r = 0.69,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.00,\ \alpha_G = 0.06\%,\ k^F_{\alpha_G;r,\infty} = 15.26$

in	to	$\hat{\beta}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\beta}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
A	S1	29.516 708 7	−0.009	0.014	24.11	0.01	16.83	−0.04	7.33	0.02	448.6	1.0	448.6	1.0	1.09
B	S2	29.516 713 6	−0.014	0.012	45.03	0.01	12.32	−0.03	4.57	0.01	237.8	0.6	237.8	0.6	1.25

Adjusted spatial distances

Variance component: $\Omega/\sigma_0^2 = 0.000,\ r = 1.13,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.00,\ \alpha_G = 0.12\%,\ k^F_{\alpha_G;r,\infty} = 9.66$

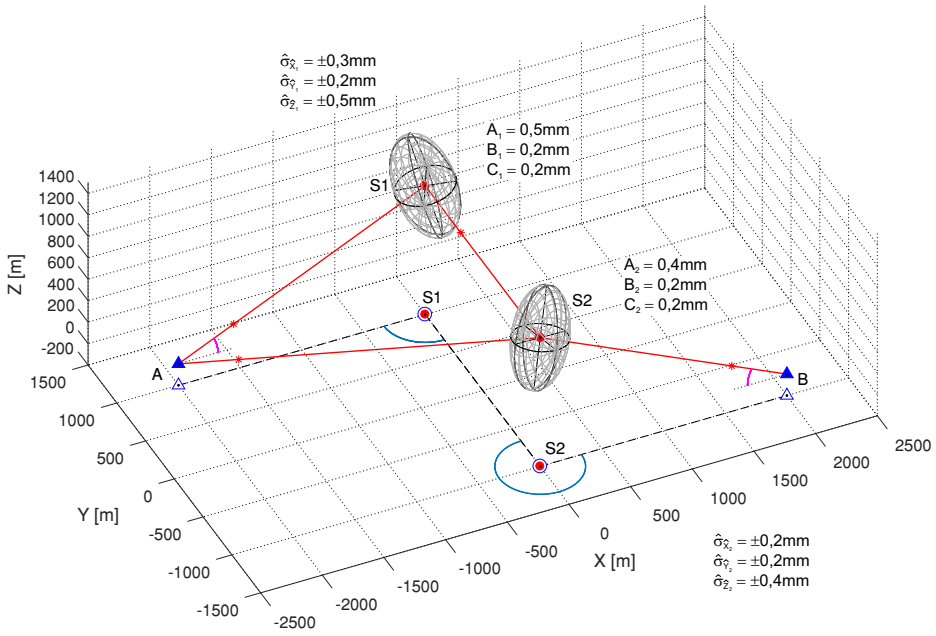
in	to	$\widehat{S}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{S}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
A	S1	2236.0678	0.015	0.025	24.1	0.0	29.6	+0.063	7.3	0.0	224.3	0.5	224.3	0.5	1.09
	S2	2999.9999	0.013	0.025	55.9	0.0	26.0	+0.023	3.7	0.0	114.9	0.1	114.9	0.1	0.46
B	S2	2236.0678	0.018	0.025	21.6	0.0	31.2	+0.085	7.9	0.0	244.8	0.7	244.8	0.7	1.39
S1	S2	2000.0000	0.005	0.024	11.7	0.0	38.0	+0.042	11.4	0.0	335.2	0.4	335.2	0.4	0.56

Adjusted horizontal angles

Variance component: $\Omega/\sigma_0^2 = 0.000,\ r = 0.18,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.00,\ \alpha_G = 0.01\%,\ k^F_{\alpha_G;r,\infty} = 57.38$

in	from	to	$\hat{\alpha}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\alpha}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
S1	S2	A	99.999 998	0.002	0.000	11.69	0.00	12.08	+0.01	11.36	0.01	10.7	0.0	0.56
S2	B	S1	300.000 003	−0.003	0.000	5.88	0.01	17.04	−0.05	16.53	0.05	16.0	0.0	1.41

Network graph



Supplementary information

Observed spatial distances	:	4
Observed angles	:	2
Observed vertical angles	:	2
Coordinate unknowns	:	6
Datum defect	:	4
Datum definition	:	fix
Number of datum constraints	:	6
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	0.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	1.4
Redundancy r	:	2
Redundancy spatial distances	:	1.13
Redundancy angles	:	0.18
Redundancy vertical angles	:	0.69
Weighted square sum of residuals Ω [mgon^2]	:	0.00052659
(a priori) standard deviation σ_0 [mgon]	:	2
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	0.016226
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.0081
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.0001 ($k^F_{\alpha_G;r,\infty}$ =5.87)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.011
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.00640

4.5 Wolf H (1979), Ex. I3.2-18

Wolf H (1979): Ausgleichungsrechnung II. Dümmlers, pp. 103

Available data files: [3D] Wolf_PosAngle_and_Dist*.*

Coordinates

Point name	X [m]	Y [m]	Z [m]
A	500 (D)	500 (D)	500 (D)
B	1000 (D)	1000 (D)	500 (D)
C	1200 (D)	1700 (D)	500 (D)
D	750 (D)	2000 (D)	500 (D)
N	900	1200	1500

Datum: fix, (D)...Datum coordinate

Spatial distances

in	to	S [m]	$\lvert\sigma\rvert$ [mm]	p [rad^2/m^2]
N	A	1284.5200	30	$2.741\,56\cdot10^{-7}$
	B	1024.7000	30	$2.741\,56\cdot10^{-7}$
	C	1157.5800	30	$2.741\,56\cdot10^{-7}$
	D	1289.3800	30	$2.741\,56\cdot10^{-7}$

Position angles

in	from	to	π [gon]	$\lvert\sigma\rvert$ [mgon]	p [–]
N	B	A	37.011 185 2	1	1
	C	B	42.631 567 9	1	1
	D	C	27.547 095 7	1	1
	A	D	80.476 564 8	1	1

Design matrix A[-] and reduced observation vector Δy[mm] spatial distances (1. iteration)

A	X_A	Y_A	Z_A	X_B	Y_B	Z_B	X_C	Y_C	Z_C	X_D	Y_D	Z_D	X_N	Y_N	Z_N	Δy
$S_{N,A}$	−0.3114	−0.5449	−0.7785	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3114	0.5449	0.7785	−3.26
$S_{N,B}$	0.0000	0.0000	0.0000	0.0976	−0.1952	−0.9759	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0976	0.1952	0.9759	4.92
$S_{N,C}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2592	0.4319	−0.8639	0.0000	0.0000	0.0000	−0.2592	−0.4319	0.8639	−3.69
$S_{N,D}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.1163	0.6205	−0.7756	0.1163	−0.6205	0.7756	0.30

Design matrix A[mgon/m] and reduced observation vector Δy[mgon] position angles (1. iteration)

A	X_A	Y_A	Z_A	X_B	Y_B	Z_B	X_C	Y_C	Z_C	X_D	Y_D	Z_D	X_N	Y_N	Z_N	Δy
$\pi_{N,B,A}$	−32.293	−23.486	29.357	44.455	43.197	−4.194	0.000	0.000	0.000	0.000	0.000	0.000	−12.162	−19.711	−25.163	0.0008
$\pi_{N,C,B}$	0.000	0.000	0.000	−18.281	−58.549	9.882	9.356	47.298	26.456	0.000	0.000	0.000	8.925	11.252	−36.337	0.0016
$\pi_{N,D,C}$	0.000	0.000	0.000	0.000	0.000	0.000	46.114	−29.946	−1.139	−42.950	15.464	18.814	−3.164	14.482	−17.675	0.0009
$\pi_{N,A,D}$	1.161	−40.808	28.101	0.000	0.000	0.000	0.000	0.000	0.000	14.309	37.924	28.193	−15.469	2.884	−56.294	0.0012

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	X_A	Y_A	Z_A	X_B	Y_B	Z_B	X_C	Y_C	Z_C	X_D	Y_D	Z_D	X_N	Y_N	Z_N
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Least squares solution $\widehat{\Delta x}_{[mm]}$ (1. iteration)

	X_N	Y_N	Z_N
$\widehat{\Delta x}^{\mathsf{T}}$	−1.6	0.8	0.1

Adjusted coordinates

Point name	$\widehat{X}$ [m]	$\widehat{X} - X$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y} - Y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z} - Z$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
N	899.9984	−0.157	0.498	1200.0008	0.085	0.293	1500.0001	0.012	0.128	0.592

Absolute error ellipsoids

in	A [cm]	B [cm]	C [cm]
N	0.53	0.23	0.11

Absolute confidence ellipsoids ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	C [m]
N	2.14	0.95	0.44

Adjusted spatial distances

Variance component: $\Omega/\sigma_0^2 = 0.052$, $r = 2.57$, $\hat{\sigma}_0^2/\sigma_0^2 = 0.02$, $\alpha_G = 0.43\%$, $k^F_{\alpha_G;r,\infty} = 4.76$

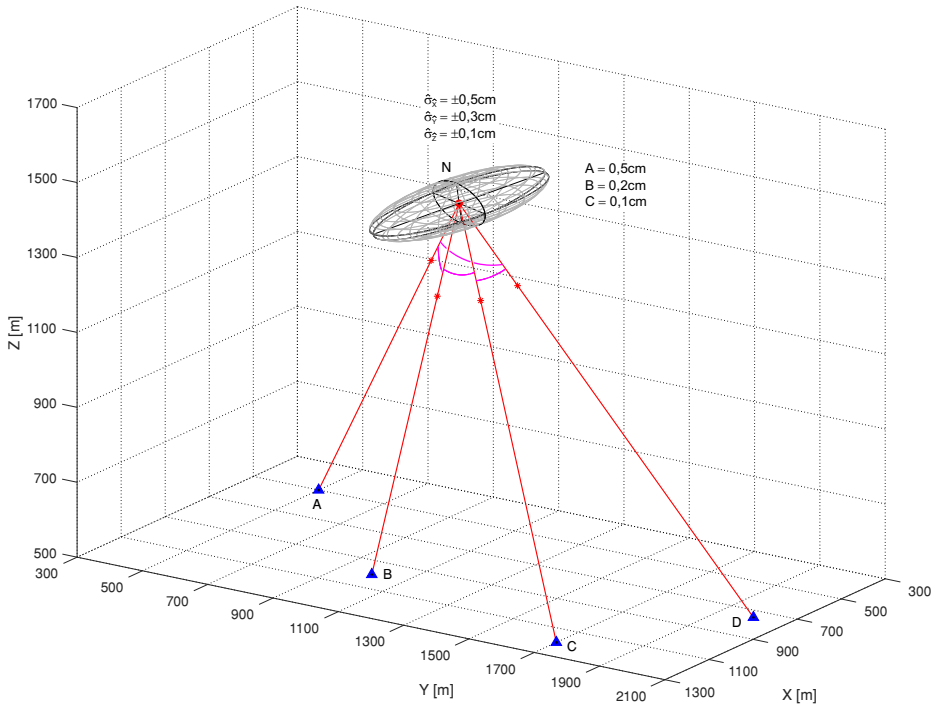
in	to	$\widehat{S}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{S}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
N	A	1284.5233	−0.332	0.173	68.3	0.1	15.0	−0.486	2.8	0.1	47.5	1.5	47.5	1.5	1.31
	B	1024.6955	0.449	0.189	62.4	0.2	15.7	+0.720	3.2	0.1	59.1	2.7	59.1	2.7	1.85
	C	1157.5838	−0.384	0.177	66.8	0.2	15.2	−0.575	2.9	0.1	50.4	1.9	50.4	1.9	1.53
	D	1289.3791	0.092	0.195	59.8	0.0	16.0	+0.154	3.4	0.0	64.3	0.6	64.3	0.6	0.39

Adjusted position angles

Variance component: $\Omega/\sigma_0^2 = 0.001$, $r = 2.43$, $\hat{\sigma}_0^2/\sigma_0^2 = 0.00$, $\alpha_G = 0.39\%$, $k^F_{\alpha_G;r,\infty} = 4.99$

in	from	to	$\hat{\pi}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{\pi}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mgon]	IP_2 [mgon]	T_τ
N	B	A	37.011 184	0.001	0.001	63.70	0.00	5.18	0.00	3.12	0.00	1.9	0.0	0.02
	C	B	42.631 557	0.010	0.001	56.23	0.01	5.51	+0.02	3.65	0.01	2.4	0.0	0.14
	D	C	27.547 110	−0.014	0.001	79.23	0.02	4.64	−0.02	2.12	0.01	1.0	0.0	0.16
	A	D	80.476 584	−0.019	0.001	43.59	0.03	6.26	−0.04	4.70	0.03	3.5	0.0	0.28

Network graph



Supplementary information

Observed spatial distances	:	4
Observed position angles	:	4
Coordinate unknowns	:	3
Datum defect	:	6
Datum definition	:	fix
Number of datum constraints	:	12
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	1.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.2
Redundancy r	:	5
Redundancy spatial distances	:	2.57
Redundancy position angles	:	2.43
Weighted square sum of residuals Ω [mgon^2]	:	0.052599
(a priori) standard deviation σ_0 [mgon]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [mgon]	:	0.10257
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.1026
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.0105 ($k^F_{\alpha_G;r,\infty}$ =2.89)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.178
Trace coordinate covariance matrix, tr$\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	0.35003

4.6 Ghilani (2010)

Ghilani Charles D. (2010): Adjustment Computations. Spatial Data Analysis. Fifth Edition, John Wiley & Sons, Inc., ISBN 978-0-470-46491-5, Ch. 17.6, p 337-352

Available data files: [3D] Ghilani_GNSS_Baselines∗.∗

Coordinates

Point name	X [m]	Y [m]	Z [m]
A	402.3509 (D)	−4 652 995.3011 (D)	4 349 760.7775 (D)
B	8086.0318 (D)	−4 642 712.8474 (D)	4 360 439.0833 (D)
C	12 046.5808	−4 649 394.0824	4 353 160.0645
D	−3081.5831	−4 643 107.3692	4 359 531.1234
E	−4919.3388	−4 649 361.2199	4 352 934.4548
F	1518.8012	−4 648 399.1454	4 354 116.6914

Datum: fix, (D)...Datum coordinate

3D-GNSS baseline components

in	to	ΔX [m]	ΔY [m]	ΔZ [m]	$\sigma^2_{\Delta X}$ [cm²]	$\sigma_{\Delta X\Delta Y}$ [cm²]	$\sigma_{\Delta X\Delta Z}$ [cm²]	$\sigma^2_{\Delta Y}$ [cm²]	$\sigma_{\Delta Y\Delta Z}$ [cm²]	$\sigma^2_{\Delta Z}$ [cm²]
A	C	11 644.2232	3601.2165	3399.2550	9.884	−0.096	0.095	9.377	−0.095	9.827
	E	−5321.7164	3634.0754	3173.6652	2.158	−0.021	0.022	1.919	−0.021	2.005
	F	1116.4577	4596.1553	4355.9141	0.662	−0.008	0.009	0.811	−0.008	0.938
B	C	3960.5442	−6681.2467	−7279.0148	2.305	−0.022	0.021	2.546	−0.022	2.252
	D	−11 167.6076	−394.5204	−907.9593	2.700	−0.028	0.028	2.721	−0.027	2.670
	F	−6567.2310	−5686.3033	−6322.3807	0.551	−0.006	0.006	0.747	−0.006	0.663
D	C	15 128.1647	−6286.7054	−6371.0583	1.461	−0.014	0.013	1.614	−0.014	1.308
	E	−1837.7459	−6253.8534	−6596.6697	1.231	−0.012	0.012	1.277	−0.012	1.283
F	A	−1116.4523	−4596.1610	−4355.9062	0.748	−0.008	0.009	0.659	−0.008	0.762
	B	6567.2311	5686.2926	6322.3917	0.664	−0.007	0.007	0.746	−0.006	0.605
	C	10 527.7852	−994.9377	−956.6246	2.567	−0.022	0.024	2.163	−0.023	2.397
	D	−4600.3787	5291.7785	5414.4311	0.933	−0.010	0.009	0.988	−0.010	1.204
	E	−6438.1364	−962.0694	−1182.2305	0.944	−0.009	0.010	0.996	−0.009	0.883

Design matrix $A_{[-]}$ and reduced observation vector $\Delta y_{[cm]}$ GNSS Baselines (1. iteration)

A	X_A	Y_A	Z_A	X_B	Y_B	Z_B	X_C	Y_C	Z_C	X_D	Y_D	Z_D	X_E	Y_E	Z_E	X_F	Y_F	Z_F	Δy
$\Delta X_{A,C}$	−1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−0.673
$\Delta Y_{A,C}$	0	−1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	−0.219
$\Delta Z_{A,C}$	0	0	−1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	−3.197
$\Delta X_{A,E}$	−1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−2.673
$\Delta Y_{A,E}$	0	−1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	−0.579
$\Delta Z_{A,E}$	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1.207
$\Delta X_{A,F}$	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0.737
$\Delta Y_{A,F}$	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−0.039
$\Delta Z_{A,F}$	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.023
$\Delta X_{B,C}$	0	0	0	−1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−0.482
$\Delta Y_{B,C}$	0	0	0	0	−1	0	0	1	0	0	0	0	0	0	0	0	0	0	−1.169
$\Delta Z_{B,C}$	0	0	0	0	0	−1	0	0	1	0	0	0	0	0	0	0	0	0	0.396
$\Delta X_{B,D}$	0	0	0	−1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0.728
$\Delta Y_{B,D}$	0	0	0	0	−1	0	0	0	0	0	1	0	0	0	0	0	0	0	0.141
$\Delta Z_{B,D}$	0	0	0	0	0	−1	0	0	0	0	0	1	0	0	0	0	0	0	0.056
$\Delta X_{B,F}$	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−0.042
$\Delta Y_{B,F}$	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	1	0	−0.529
$\Delta Z_{B,F}$	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	1	1.116
$\Delta X_{D,C}$	0	0	0	0	0	0	1	0	0	−1	0	0	0	0	0	0	0	0	0.080
$\Delta Y_{D,C}$	0	0	0	0	0	0	0	1	0	0	−1	0	0	0	0	0	0	0	0.780
$\Delta Z_{D,C}$	0	0	0	0	0	0	0	0	1	0	0	−1	0	0	0	0	0	0	0.060
$\Delta X_{D,E}$	0	0	0	0	0	0	0	0	0	−1	0	0	1	0	0	0	0	0	0.980
$\Delta Y_{D,E}$	0	0	0	0	0	0	0	0	0	0	−1	0	0	1	0	0	0	0	−0.270
$\Delta Z_{D,E}$	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	1	0	0	0	−0.110
$\Delta X_{F,A}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	−0.197
$\Delta Y_{F,A}$	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	−0.531
$\Delta Z_{F,A}$	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0.767
$\Delta X_{F,B}$	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0.052
$\Delta Y_{F,B}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	−0.541
$\Delta Z_{F,B}$	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−1	−0.016
$\Delta X_{F,C}$	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	−1	0	0	0.560
$\Delta Y_{F,C}$	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	−1	0	−0.070
$\Delta Z_{F,C}$	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	−1	0.230
$\Delta X_{F,D}$	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1	0	0	0.560
$\Delta Y_{F,D}$	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1	0	0.230
$\Delta Z_{F,D}$	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1	−0.090
$\Delta X_{F,E}$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−1	0	0	0.360
$\Delta Y_{F,E}$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−1	0	0.510
$\Delta Z_{F,E}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−1	0.610

Matrix $D^{\mathsf{T}}{}_{[-]}$ of datum constraints

	X_A	Y_A	Z_A	X_B	Y_B	Z_B	X_C	Y_C	Z_C	X_D	Y_D	Z_D	X_E	Y_E	Z_E	X_F	Y_F	Z_F
D^{T}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

Least squares solution $\widehat{\Delta x}_{[mm]}$ (1. iteration)

	X_A	Y_A	Z_A	X_B	Y_B	Z_B	X_C	Y_C	Z_C	X_D	Y_D	Z_D	X_E	Y_E	Z_E	X_F	Y_F	Z_F
$\widehat{\Delta x}^{\mathsf{T}}$	0	0	0	0	0	0	−0.04	−0.16	−0.07	−0.03	0.05	−0.07	−0.28	0.03	0.00	−0.01	0.07	0.01

Adjusted coordinates

Point name	$\widehat{X}$ [m]	$\widehat{X} - X$ [mm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y} - Y$ [mm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z} - Z$ [mm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
C	12 046.5808	−0.04	0.608	−4 649 394.0826	−0.16	0.612	4 353 160.0644	−0.07	0.597	1.049
D	−3081.5831	−0.03	0.494	−4 643 107.3692	0.05	0.506	4 359 531.1233	−0.07	0.514	0.874
E	−4919.3391	−0.28	0.523	−4 649 361.2199	0.03	0.526	4 352 934.4548	0.00	0.517	0.905
F	1518.8012	−0.01	0.267	−4 648 399.1453	0.07	0.282	4 354 116.6914	0.01	0.280	0.478

Absolute error ellipsoids

in	A [cm]	B [cm]	C [cm]
C	0.61	0.61	0.60
D	0.51	0.51	0.49
E	0.53	0.52	0.52
F	0.28	0.28	0.27

Absolute confidence ellipsoids $(1 - \alpha = 95\%)$

in	A [cm]	B [cm]	C [m]
C	1.83	1.81	1.78
D	1.53	1.51	1.47
E	1.58	1.56	1.54
F	0.84	0.83	0.79

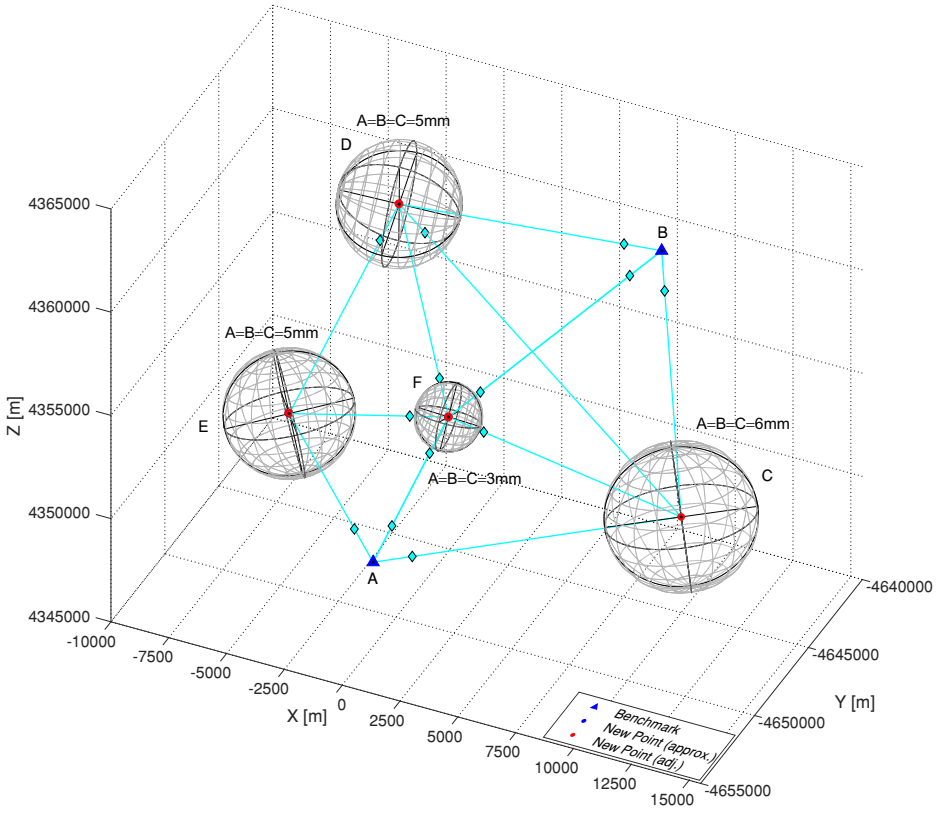
Adjusted 3D-GNSS baseline components

in	to	Variable	Value [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
A	C	$\widehat{\Delta X}$	11 644.2299	−6.69	6.08	92.53	0.22	13.51	−0.72	1.17	0.06	1.01	0.05	1.01	0.05	0.31
		$\widehat{\Delta Y}$	3601.2185	−2.03	6.12	92.01	0.07	13.19	−0.22	1.22	0.02	1.05	0.02	1.05	0.02	0.10
		$\widehat{\Delta Z}$	3399.2869	−31.90	5.97	92.75	1.06	13.45	−3.44	1.16	0.30	0.98	0.25	0.98	0.25	1.49
	E	$\widehat{\Delta X}$	−5321.6900	−26.45	5.23	74.64	2.08	7.03	−3.54	2.41	1.21	1.78	0.90	1.78	0.90	2.95
		$\widehat{\Delta Y}$	3634.0812	−5.82	5.26	71.14	0.50	6.79	−0.82	2.63	0.32	1.96	0.24	1.96	0.24	0.70
		$\widehat{\Delta Z}$	3173.6773	−12.07	5.17	73.33	1.00	6.83	−1.65	2.49	0.60	1.82	0.44	1.82	0.44	1.41
	F	$\widehat{\Delta X}$	1116.4503	7.38	2.67	78.49	1.02	3.79	+0.94	2.16	0.54	0.82	0.20	0.82	0.20	1.45
		$\widehat{\Delta Y}$	4596.1558	−0.46	2.82	80.42	0.06	4.15	−0.06	2.04	0.03	0.81	0.01	0.81	0.01	0.08
		$\widehat{\Delta Z}$	4355.9139	0.22	2.80	83.35	0.02	4.38	+0.03	1.85	0.01	0.73	0.00	0.73	0.00	0.04
B	C	$\widehat{\Delta X}$	3960.5490	−4.78	6.08	67.98	0.38	7.61	−0.70	2.84	0.26	2.44	0.23	2.44	0.23	0.54
		$\widehat{\Delta Y}$	−6681.2352	−11.53	6.12	70.58	0.86	7.85	−1.63	2.67	0.56	2.31	0.48	2.31	0.48	1.22
		$\widehat{\Delta Z}$	−7279.0188	4.03	5.97	68.36	0.32	7.50	+0.59	2.81	0.22	2.37	0.19	2.37	0.19	0.46
	D	$\widehat{\Delta X}$	−11 167.6149	7.31	4.94	81.91	0.49	7.50	+0.89	1.94	0.23	1.36	0.16	1.36	0.16	0.69
		$\widehat{\Delta Y}$	−394.5218	1.36	5.06	81.19	0.09	7.56	+0.17	1.99	0.04	1.42	0.03	1.42	0.03	0.13
		$\widehat{\Delta Z}$	−907.9599	0.63	5.14	80.26	0.04	7.54	+0.08	2.05	0.02	1.49	0.02	1.49	0.02	0.06
	F	$\widehat{\Delta X}$	−6567.2306	−0.41	2.67	74.17	0.06	3.56	−0.05	2.44	0.04	0.92	0.01	0.92	0.01	0.09
		$\widehat{\Delta Y}$	−5686.2979	−5.36	2.82	78.76	0.70	4.02	−0.68	2.15	0.36	0.85	0.14	0.85	0.14	0.99
		$\widehat{\Delta Z}$	−6322.3919	11.15	2.80	76.45	1.57	3.85	+1.46	2.29	0.87	0.91	0.34	0.91	0.34	2.21
D	C	$\widehat{\Delta X}$	15 128.1639	0.81	6.18	47.69	0.10	7.23	+0.17	4.33	0.10	3.78	0.09	3.78	0.09	0.14
		$\widehat{\Delta Y}$	−6286.7134	8.01	6.32	50.61	0.89	7.38	+1.58	4.08	0.88	3.65	0.78	3.65	0.78	1.25
		$\widehat{\Delta Z}$	−6371.0589	0.60	6.02	44.58	0.08	7.08	+0.14	4.61	0.09	3.92	0.07	3.92	0.07	0.11
	E	$\widehat{\Delta X}$	−1837.7560	10.05	5.52	50.60	1.27	6.44	+1.99	4.08	1.26	3.18	0.98	3.18	0.98	1.80
		$\widehat{\Delta Y}$	−6253.8507	−2.68	5.59	51.04	0.33	6.54	−0.53	4.05	0.33	3.20	0.26	3.20	0.26	0.47
		$\widehat{\Delta Z}$	−6596.6685	−1.17	5.60	51.12	0.14	6.55	−0.23	4.04	0.14	3.20	0.11	3.20	0.11	0.20

Adjusted 3D-GNSS baseline components (continued)

in	to	Variable	Value [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
F	A	$\widehat{\Delta X}$	−1116.4503	−1.98	2.67	80.95	0.25	3.97	−0.24	2.00	0.12	0.76	0.05	0.76	0.05	0.36
		$\widehat{\Delta Y}$	−4596.1558	−5.24	2.82	75.92	0.74	3.85	−0.69	2.33	0.42	0.93	0.17	0.93	0.17	1.05
		$\widehat{\Delta Z}$	−4355.9139	7.68	2.80	79.50	0.99	4.04	+0.97	2.10	0.50	0.83	0.20	0.83	0.20	1.39
	B	$\widehat{\Delta X}$	6567.2306	0.51	2.67	78.57	0.07	3.80	+0.06	2.16	0.04	0.81	0.01	0.81	0.01	0.10
		$\widehat{\Delta Y}$	5686.2979	−5.34	2.82	78.74	0.70	4.02	−0.68	2.15	0.36	0.86	0.14	0.86	0.14	0.98
		$\widehat{\Delta Z}$	6322.3919	−0.15	2.80	74.19	0.02	3.73	−0.02	2.44	0.01	0.96	0.01	0.96	0.01	0.03
	C	$\widehat{\Delta X}$	10 527.7796	5.63	6.04	71.59	0.42	7.82	+0.79	2.60	0.26	2.22	0.22	2.22	0.22	0.59
		$\widehat{\Delta Y}$	−994.9372	−0.47	6.05	66.20	0.04	7.47	−0.07	2.95	0.03	2.52	0.02	2.52	0.02	0.06
		$\widehat{\Delta Z}$	−956.6270	2.38	5.94	70.59	0.18	7.61	+0.34	2.67	0.12	2.24	0.10	2.24	0.10	0.26
	D	$\widehat{\Delta X}$	−4600.3843	5.61	4.68	53.11	0.80	5.48	+1.06	3.88	0.75	2.57	0.50	2.57	0.50	1.13
		$\widehat{\Delta Y}$	5291.7762	2.33	4.78	53.77	0.32	5.60	+0.43	3.83	0.30	2.59	0.20	2.59	0.20	0.45
		$\widehat{\Delta Z}$	5414.4319	−0.82	4.92	59.85	0.10	5.86	−0.14	3.39	0.08	2.35	0.06	2.35	0.06	0.14
	E	$\widehat{\Delta X}$	−6438.1403	3.87	4.97	47.77	0.58	5.81	+0.81	4.32	0.60	3.03	0.42	3.03	0.42	0.81
		$\widehat{\Delta Y}$	−962.0745	5.14	5.01	49.62	0.73	5.85	+1.04	4.16	0.74	2.95	0.52	2.95	0.52	1.03
		$\widehat{\Delta Z}$	−1182.2366	6.11	4.90	45.68	0.96	5.74	+1.34	4.51	1.05	3.12	0.73	3.12	0.73	1.36

Network graph



Supplementary information

Observed GNSS Baseline Components	:	39
Coordinate unknowns	:	12
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	6
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	14.9
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	3.1
Number of iterations (Max=20)	:	20
Stop criterion (actual)	:	$6.7 \cdot 10^{-10}$
Redundancy r	:	27
Redundancy GNSS Baselines	:	27.00
Weighted square sum of residuals Ω [-]	:	13.5145
(a priori) standard deviation σ_0 [-]	:	1
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	0.707486
Ratio $\hat{\sigma}_0/\sigma_0$	:	0.7075
Global test ($\hat{\sigma}_0^2/\sigma_0^2$)	:	0.5005 ($k^F_{\alpha_G;r,\infty}$ =1.28)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	0.034
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	2.91322

4.7 Caspary W (2013)

Caspary W (2013): Fehlertolerante Auswertung von Messdaten: Daten- und Modellanalyse, robuste Schätzung. 1. Auflage, ISBN 978-3486727715, pp. 9

Available data files: [3D] Caspary*.*

Coordinates

Point name	X [m]	Y [m]	Z [m]
1	4000 (D)	−10 000 (D)	1600 (D)
2	3000 (D)	11 000 (D)	1500 (D)
3	700 (D)	−700 (D)	800 (D)
4	0 (D)	0 (D)	700 (D)
N	5000	2000	1800

Datum: fix, (D)...Datum coordinate

3D-GNSS baseline components

in	to	ΔX [m]	$\lvert\sigma\rvert$ [cm]	p [1/m^2]	ΔY [m]	$\lvert\sigma\rvert$ [cm]	p [1/m^2]	ΔZ [m]	$\lvert\sigma\rvert$ [cm]	p [1/m^2]
4	N	5000.0200	1.600	3.906 25	1999.9800	1.600	3.906 25	1099.9400	6.200	0.260 14

Spatial distances

in	to	S [m]	$\lvert\sigma\rvert$ [mm]	p [1/m^2]
1	N	12 043.3050	37.500	0.711 11
2	N	9224.4040	29.400	1.156 93
3	N	5174.9100	18.500	2.921 84
4	N	5496.4020	19.800	2.550 76

Zenith angles

in	to	z [gon]	$\lvert\sigma\rvert$ [mgon]	p [1/rad^2]
4	N	87.172 600 0	0.300	45 031 637.2

Design matrix A[-] and reduced observation vector Δy[cm] 3D-GNSS Baselines (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_N	Y_N	Z_N	Δy
$\Delta X_{4,N}$	0	0	0	0	0	0	0	0	0	−1	0	0	1	0	0	2.000
$\Delta Y_{4,N}$	0	0	0	0	0	0	0	0	0	0	−1	0	0	1	0	−2.000
$\Delta Z_{4,N}$	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	1	−6.000

Design matrix A[-] and reduced observation vector Δy[cm] spatial distances (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_N	Y_N	Z_N	Δy
$S_{1,N}$	−0.0830	−0.9964	−0.0166	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0830	0.9964	0.0166	4.963
$S_{2,N}$	0.0000	0.0000	0.0000	−0.2168	0.9757	−0.0325	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2168	−0.9757	0.0325	−2.010
$S_{3,N}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.8309	−0.5217	−0.1932	0.0000	0.0000	0.0000	0.8309	0.5217	0.1932	−2.961
$S_{4,N}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.9097	−0.3639	−0.2001	0.9097	0.3639	0.2001	3.957

Design matrix $A_{[\text{mgon/m}]}$ and reduced observation vector $\Delta y_{[\text{mgon}]}$ zenith angles (1. iteration)

A	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_N	Y_N	Z_N	Δy
$z_{4,\text{N}}$	0	0	0	0	0	0	0	0	0	−2.1523	−0.8609	11.3482	2.1523	0.8609	−11.3482	0.04

Matrix $D^\mathsf{T}_{[\text{-}]}$ of datum constraints

	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_N	Y_N	Z_N
D^T	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Least squares solution $\widehat{\Delta x}_{[\text{cm}]}$ (1. iteration)

	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	X_N	Y_N	Z_N
$\widehat{\Delta x}^\mathsf{T}$	0	0	0	0	0	0	0	0	0	0	0	0	1.482	−0.766	−1.319

Adjusted coordinates

Point name	$\widehat{X}$ [m]	$\widehat{X} - X$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Y}$ [m]	$\widehat{Y} - Y$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\widehat{Z}$ [m]	$\widehat{Z} - Z$ [cm]	$\lvert\hat{\sigma}\rvert$ [cm]	$\lvert\hat{\sigma}_{3D}\rvert$ [cm]
N	5000.0148	1.482	1.720	1999.9923	−0.766	1.856	1799.9868	−1.319	3.450	4.278

Absolute error ellipsoids

in	A [cm]	B [cm]	C [cm]
N	3.46	2.01	1.51

Absolute confidence ellipsoids ($1 - \alpha = 95\%$)

in	A [cm]	B [cm]	C [m]
N	13.95	8.08	6.08

Adjusted 3D-GNSS baseline components

Variance component: $\Omega/\sigma_0^2 = 1.269,\ r = 1.72,\ \hat{\sigma}_0^2/\sigma_0^2 = 0.74,\ \alpha_G = 0.22\%,\ k^F_{\alpha_G;r,\infty} = 6.68$

in	to	Variable	Value [m]	$\hat{e}$ [mm]	$\lvert\hat{\sigma}\rvert$ [mm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
4	N	$\widehat{\Delta X}$	5000.0148	5.18	17.20	47.35	0.47	9.61	+1.09	4.36	0.50	5.06	0.58	5.06	0.58	0.32
		$\widehat{\Delta Y}$	1999.9923	−12.34	18.56	38.69	1.24	10.63	−3.19	5.20	1.56	6.52	1.95	6.52	1.95	0.84
		$\widehat{\Delta Z}$	1099.9868	−46.81	34.50	85.88	0.81	27.64	−5.45	1.68	0.33	3.90	0.77	3.90	0.77	0.55

Adjusted spatial distances

Variance component: $\Omega/\sigma_0^2 = 9.507,\ r = 3.09,\ \hat{\sigma}_0^2/\sigma_0^2 = 3.08,\ \alpha_G = 0.58\%,\ k^F_{\alpha_G;r,\infty} = 4.12$

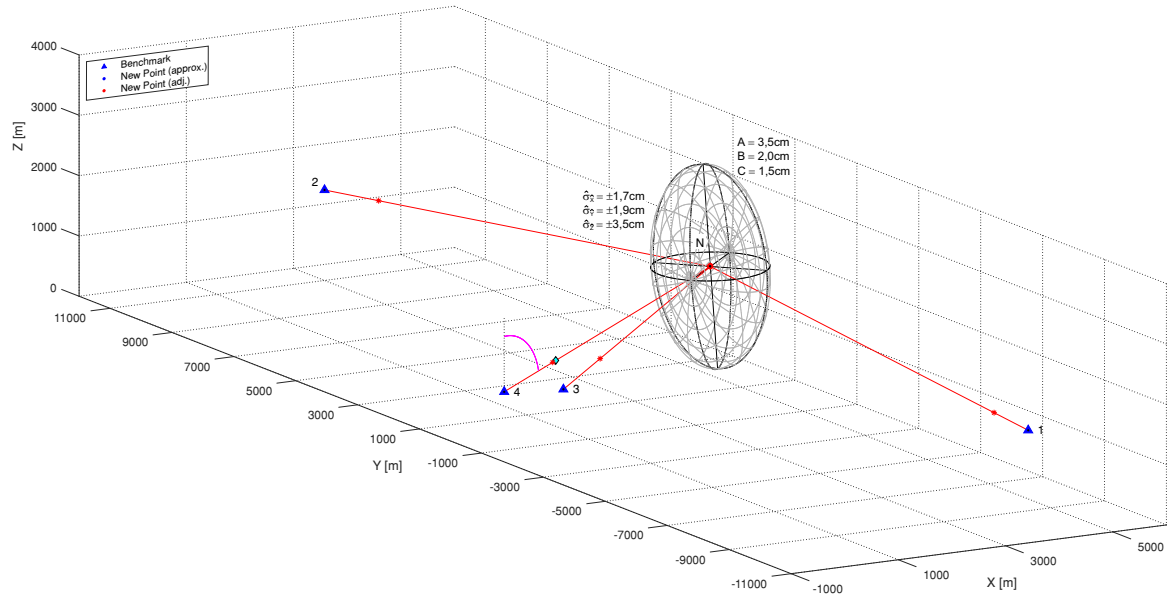
in	to	$\widehat{S}$ [m]	$\hat{e}$ [cm]	$\lvert\hat{\sigma}_{\hat{S}}\rvert$ [cm]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [cm]	$\widehat{\nabla}$ [cm]	IF_1	IF_2	IP_1 [cm]	IP_2 [cm]	IK_1 [cm]	IK_2 [cm]	T_τ
1	N	12 043.249	5.625	1.816	89.31	1.59	16.40	+6.298	1.43	0.55	1.75	0.67	1.75	0.67	1.07
2	N	9224.434	−3.036	1.943	80.09	1.15	13.57	−3.791	2.06	0.58	2.70	0.75	2.70	0.75	0.78
3	N	5174.945	−3.538	1.542	68.35	2.31	9.25	−5.177	2.81	1.57	2.93	1.64	2.93	1.64	1.56
4	N	5496.370	3.151	1.574	71.20	1.89	9.70	+4.425	2.63	1.20	2.79	1.27	2.79	1.27	1.27

Adjusted zenith angles

Variance component: $\Omega/\sigma_0^2 = 0.191,\ r = 0.19,\ \hat{\sigma}_0^2/\sigma_0^2 = 1.00,\ \alpha_G = 0.01\%,\ k^F_{\alpha_G;r,\infty} = 52.82$

in	to	$\hat{z}$ [gon]	$\hat{e}$ [mgon]	$\lvert\hat{\sigma}_{\hat{z}}\rvert$ [mgon]	IR [%]	$\lvert w\rvert$	$\lvert\nabla\rvert$ [mgon]	$\widehat{\nabla}$ [mgon]	IF_1	IF_2	IP_1 [mm]	IP_2 [mm]	IK_1 [mm]	IK_2 [mm]	T_τ
4	N	87.172 731 2	−0.131	0.400	19.12	1.00	2.84	−0.69	8.50	2.06	198.0	47.9	198.0	47.9	0.68

Network graph



Supplementary information

Observed 3D-GNSS Baseline Components	:	3
Observed spatial distances	:	4
Observed zenith angles	:	1
Coordinate unknowns	:	3
Datum defect	:	3
Datum definition	:	fix
Number of datum constraints	:	12
Type-I-error probability α_L [%] (Baarda)	:	0.1
Type-I-error probability α_G [%] (Baarda)	:	1.3
Test value $k^N_{\alpha_L/2}$	:	3.29
χ^2-Noncentrality parameter λ_0	:	17.1
Type-I-error probability α_τ [%] (Pope)	:	0.1
Critical value $k^\tau_{\alpha_\tau}$	:	2.2
Number of iterations (Max=20)	:	3
Stop criterion (actual)	:	$5.7 \cdot 10^{-13}$
Redundancy r	:	5
Redundancy 3D-GNSS Baselines	:	1.72
Redundancy spatial distances	:	3.09
Redundancy zenith angles	:	0.19
Weighted square sum of residuals Ω [-]	:	$1.09680 \cdot 10^{-2}$
(a priori) standard deviation σ_0 [-]	:	$3.16228 \cdot 10^{-2}$
(a posteriori) estimated standard deviation $\hat{\sigma}_0$ [-]	:	$4.68358 \cdot 10^{-2}$
Ratio $\hat{\sigma}_0/\sigma_0$	:	1.4811
Global test $(\hat{\sigma}_0^2/\sigma_0^2)$	:	2.1936 ($k^F_{\alpha_G;r,\infty}$ =2.89)
Number of outliers (Data snooping & τ-criterion)	:	0
$\|\widehat{\mathbf{X}} - \mathbf{X}\|$ [cm]	:	1.669
Trace coordinate covariance matrix, $\mathrm{tr}\widehat{\Sigma}_{\widehat{X}}$ [cm^2]	:	18.30207

A Abbreviations

Observable and/or unknown quantities ("$\widehat{\ }$" indicate L_2-norm, "$\widetilde{\ }$" L_1-norm estimates)

$h; \hat{h}, \tilde{h}$	Height difference (levelled, trigonometric); estimates
$r; \hat{r}, \tilde{r}$	Direction; estimates
$s; \hat{s}, \tilde{s}$	Distance (2D) or levelling line length; estimates
$S; \widehat{S}, \tilde{S}$	Distance (3D); estimates
$z; \hat{z}, \tilde{z}$	Zenith angle; estimates
$x, y, z; \hat{x}, \hat{y}, \hat{z}; \tilde{x}, \tilde{y}, \tilde{z}$	(Approx.) Cartesian coordinates, estimates
$\Delta x, \Delta y; \widehat{\Delta x}, \widehat{\Delta y}, \widetilde{\Delta x}, \widetilde{\Delta y}$	2D-GNSS baseline vector components; estimates
$\Delta X, \Delta Y, \Delta Z; \widehat{\Delta X}, \widehat{\Delta Y}, \widehat{\Delta Z}, \widetilde{\Delta X}, \widetilde{\Delta Y}, \widetilde{\Delta Z}$	3D-GNSS baseline vector components; estimates
$\mathbf{X}; \widehat{\mathbf{X}}, \widetilde{\mathbf{X}}$	Vector of coordinates x, y or X, Y, Z; estimates
A	Azimuth
$L, B, H; \widehat{L}, \widehat{B}, \widehat{H}; \widetilde{L}, \widetilde{B}, \widetilde{H}$	(Approx.) longitude, latitude, height; estimates
$T; \widehat{T}, \widetilde{T}$	Grid bearing (clockwise from North); estimates
$\alpha; \hat{\alpha}, \tilde{\alpha}$	Horizontal angle; estimates
$\beta; \hat{\beta}, \tilde{\beta}$	Vertical angle; estimates
$\pi; \hat{\pi}, \tilde{\pi}$	Position angle; estimates
$a; \hat{a}, \tilde{a}$	(Approx.) additive constant of distance/height difference observation; estimates
$m; \widehat{m}, \widetilde{m}$	(Approx.) scale of distance/height difference observation; estimates
$\omega; \widehat{\omega}, \widetilde{\omega}$	(Approx.) orientation unknown; estimates
$e; \hat{e}, \tilde{e}$	Observational residual, estimates
$x, \Delta x, \xi, \Delta\xi$	Vector of unknown parameters in linear and linearized systems
$\hat{x}, \widehat{\Delta x}, \hat{\xi}, \widehat{\Delta\xi}$	Least squares (L_2-norm) estimates of unknown parameters
$\tilde{x}, \widetilde{\Delta x}, \tilde{\xi}, \widetilde{\Delta\xi}$	Least absolute (L_1-norm) estimates of unknown parameters

Other quantities

A	Design matrix, major semi axis (error or confidence ellipse, ellipsoid)
A_x	Design matrix in partitioned systems, submatrix of A (coordinates/heights)
A_z	Design matrix in partitioned systems, submatrix of A (orientations/scale/...)
$\bar{A}_x$	Design matrix after elimination of part A_z, $\bar{A}_x = A_x - A_z N_{zz}^{-1} N_{zx}$
B	Minor semi axis (error or confidence ellipse), middle semi axis (error ellipsoid)
B^T, c	Matrix of constraints for external restrictions, vector of inhomogeneities ($B^\mathsf{T}\Delta x = c$)
C	Major semi axis (error ellipsoid)
D^T, c	Matrix of constraints for resolving datum problem, vector of inhomogeneities ($D^\mathsf{T}\Delta x = c$)
i_h, t_h	Instrument height, target height
$k^N, k^{\chi^2}, k^F, k^\tau$	Critical value (of the Normal, χ^2, Fisher, τ (Pope) distribution, for the given Type-I-error probability α, and the given degrees of freedom)
N	Normal equation matrix ($N = A^\mathsf{T}PA$)
N_{xx}, N_{zz}	Normal equation submatrices ($N_{xx} = A_x^\mathsf{T}PA_x$, $N_{zz} = A_z^\mathsf{T}PA_z$)
N_{xz}, N_{zx}	Normal equation submatrices ($N_{xz} = A_x^\mathsf{T}PA_z$, $N_{zx} = A_z^\mathsf{T}PA_x$)
$P \equiv P_y, p$	Diagonal observational weight matrix, diagonal elements of P
P_x	Weight matrix of coordinates/heights
P_A	Projection matrix onto $\mathcal{R}(A)$ along $\mathcal{R}$ $(PA)^\perp$, ($P_A = AN^{-1}A^\mathsf{T}P$)
$P_A^\perp$	Redundancy matrix = Projection matrix onto $\mathcal{R}(PA)^\perp$ along $\mathcal{R}$(PA), ($P_A^\perp = I - P_A$)
P_{A_x}	Projection matrix onto $\mathcal{R}(A_x)$, ($P_{A_x} = \bar{A}_x(\bar{A}_x^\mathsf{T}P\bar{A}_x)^{-1}\bar{A}_x^\mathsf{T}P$)
Q	Cofactor matrix ($Q = N^{-1}$)
Q_f	Lateral deviation caused by residual estimate in direction observation, $Q_f = s\hat{e}$
r	(local) redundancy number, redundancy
$\mathcal{R}(\cdot)$	Column (range) space of matrix
$T_N, T_{\chi^2}, T_F, T_\tau$	Test statistic (wrt. the Normal, χ^2, Fisher, τ (Pope) distribution)
w	Standardized observational residual $\hat{e}/\sigma_{\hat{e}}$
$y, \Delta y$	Vector of observations, reduced vector
α_K	Type-I-error error probability
α_L	Type-I-error probability local test (data snooping, Baarda)
α_G	Type-I-error probability global test
α_τ	Type-I-error probability local test (Pope)
γ	Power of test ($\gamma = 80\%$ throughout all examples)
Ω	Weighted square sum of residuals, ($\Omega = \hat{e}^\mathsf{T}P\hat{e}$)
$\widetilde{\Omega}$	Sum of absolute weighted residuals, ($\widetilde{\Omega} = \lvert\hat{e}^\mathsf{T}G\rvert 1$, 1=summation vector, $P = GG^\mathsf{T}$, G=regular lower triangular matrix)
ϕ	Bearing of major semi axis (error/confidence ellipse), clockwise from North
$\sigma_c, \sigma_l, \sigma_d$	(A priori) observational standard deviation: constant (c) and distance dependent parts (l,d)
$\sigma, \hat{\sigma}$	(A priori) observational standard deviation, (a posteriori) estimated value
$\sigma_0, \hat{\sigma}_0$	(A priori) standard deviation of unit weight, (a posteriori) estimated value
$\widehat{\Sigma}$	(Estimated) Variance-covariance matrix ($\widehat{\Sigma} = \hat{\sigma}_0^2 Q$)

Internal Reliability

IR	Impact [%] of possible blunder on residual $\hat{e}$, ($IR = 100[\%] \cdot \mathrm{r}$)
	$0 \leq IR < 1$ not controlled
	$1 \leq IR < 10$ badly controlled
	$10 \leq IR < 30$ weakly – sufficiently controlled
	$30 \leq IR < 70$ well controlled
	$70 \leq IR \leq 100$ observation can be omitted without loss of reliability
$\lvert w \rvert$	Absolute standardized observational residual $\lvert \hat{e} \rvert / \sigma_{\hat{e}}$
	< 2.6 supposedly outlier
	$2.6 - 3.3$ supposedly outlier
	$3.3 - 4.1$ outlier probable
	> 4.1 outlier very probable
λ_0	Noncentrality parameter χ^2 distribution
$\lvert \nabla \rvert$	Minimal detectable bias (MDB: $\sigma\sqrt{\lambda_0 / r}$)
$\widehat{\nabla}$	Estimate for possible blunder ($\hat{e}/r$ using $\lvert w \rvert \leq k^N_{\alpha_L/2}$)
$\widehat{\nabla}^*, \widehat{\nabla}^\#$	$\lvert w \rvert > k^N_{\alpha_L/2}$, $\widehat{\nabla}^\# = \widehat{\nabla}^*(\max(\lvert w \rvert)) \rightarrow$ subtract $\widehat{\nabla}^\#$!
T_τ	Absolute, internally studentized observational residual $\lvert \hat{e} \rvert / \hat{\sigma}_{\hat{e}}$, possibly corrupted by blunder in observation
$T^*_\tau, T^\#_\tau$	Test statistic $T_\tau > k^\tau$, $T^\#_\tau = \max(T^*_\tau) \rightarrow$ subtract $\widehat{\nabla}^\#$!

External Reliability

IF_1	(Theoretical) impact factor of MDB $\lvert \nabla \rvert$ on coordinates/heights ("Net distortion", $\lvert \nabla \rvert_i \sqrt{(P \cdot P_{A_x})_{ii}} / \sigma_0$, target $\leq (6-8)$)
IF_2	(Empirical) impact factor of blunder estimate $\widehat{\nabla}$ on coordinates/heights ("Net distortion", $\widehat{\nabla}_i \sqrt{(P \cdot P_{A_x})_{ii}} / \sigma_0$)
IP_1	(Theoretical) impact factor of MDB $\lvert \nabla \rvert$ on the estimate ($\lvert \nabla \rvert_i \, (P_A)_{ii}$)
IP_2	(Empirical) impact factor of blunder estimate $\widehat{\nabla}$ on the estimate ($\widehat{\nabla}_i (P_A)_{ii}$)
IK_1	(Theoretical) impact factor of MDB $\lvert \nabla \rvert$ on the relative point position ($\lvert \nabla \rvert_i \, (A_x (\bar{A}'_x P \bar{A}_x)^{-1} \bar{A}'_x P)_{ii}$)
IK_2	(Empirical) impact factor of blunder estimate $\widehat{\nabla}$ on the relative point position ($\widehat{\nabla}_i (A_x (\bar{A}'_x P \bar{A}_x)^{-1} \bar{A}'_x P)_{ii}$, impact on point position if observation would be omitted)
Remark 1	$IP = IK$ if no parameters like orientation unknowns, scale... are present in the corresponding observation equation (then $A_z = 0 \Rightarrow A = A_x = \bar{A}_x$)
Remark 2	In case of directions, zenith angles or vertical angles IP and IK are scaled by the distance s between involved points

B Legend

Point markers

- Benchmark
- New point (approximate location)
- New point (adjusted)
- Benchmark/New point (approximate location)
- Benchmark/New point (adjusted)

Observations

Coordinate(s) x,y,z; Height H

P Q Levelling height difference h_{PQ} from P to Q

P Q Trigonometric height difference h_{PQ} from P to Q

P Q 2D-GNSS baseline vector components between P and Q

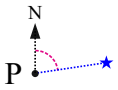
P Q 3D-GNSS baseline vector components between P and Q

P Q Planar distance s_{PQ} in P to Q

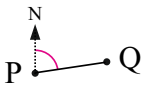
P Q Spatial distance S_{PQ} in P to Q

P Q Direction r_{PQ} in P to Q

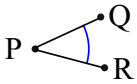
P Q Direction r_{PQ} and distance s_{PQ} in P to Q



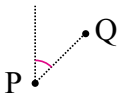
Azimuth A_P in P to azimuth mark



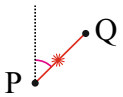
Grid bearing T_{PQ} in P to Q



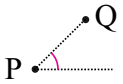
Horizontal angle α_{PQR}/position angle π_{PQR} in P from Q to R



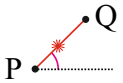
Zenith angle z_{PQ} in P to Q



Zenith angle z_{PQ} and spatial distance S_{PQ} in P to Q



Vertical angle β_{PQ} in P to Q



Vertical angle β_{PQ} and spatial distance S_{PQ} in P to Q

C Observation equations and design matrices

See also Sneeuw et al. (20xx) [20].

C.1 One dimensional network (Height network)

C.1.1 Levelled height difference

Stand point i with instrument height i_h, target point j with target height t_h, scale m, additive constant a. Constant standard deviation σ_c [m] , distance dependent standard deviations σ_l [m/$\sqrt{km}$] and σ_d [m/km] for a levelling line length s of 1 km

$$h_{ij} = m[H_j + t_h - (H_i + i_h)] + a \quad , \quad \sigma_h = \sqrt{\sigma_c^2 + \sigma_l^2 s_{[km]} + (\sigma_d s_{[km]})^2}$$

$$A_h = \left(\frac{\partial h_{ij}}{\partial H_i} \;\; \frac{\partial h_{ij}}{\partial H_j} \;\; \frac{\partial h_{ij}}{\partial m} \;\; \frac{\partial h_{ij}}{\partial a} \right)\Big|_0 = \left(-m \,, \;\; m \,, \;\; H_j + t_h - (H_i + i_h) \,, \;\; 1 \right)\Big|_0$$

C.1.2 Trigonometric height difference

Stand point i with instrument height i_h, target point j with target height t_h, standard deviation σ_h

$$h_{ij} = H_j + t_h - (H_i + i_h)$$

$$A_h = \left(\frac{\partial h_{ij}}{\partial H_i} \;\; \frac{\partial h_{ij}}{\partial H_j} \right)\Big|_0 = \left(-1 \,, \;\; 1 \right)$$

C.1.3 Height

Stand point i, standard deviation σ_H

$$H_i = H_i$$

C.2 Two dimensional network (Planar network)

C.2.1 2D distance

Stand point i, target point j, scale m, additive constant a. Constant standard deviation σ_c, distance dependent standard deviations σ_l and σ_d

$$s_{ij} = m\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} + a \quad , \quad \sigma_s = \sqrt{\sigma_c^2 + \sigma_l^2 s + (\sigma_d s)^2}$$

$$A_s = \left(\frac{\partial s_{ij}}{\partial x_i} \;\; \frac{\partial s_{ij}}{\partial y_i} \;\; \frac{\partial s_{ij}}{\partial x_j} \;\; \frac{\partial s_{ij}}{\partial x_j} \;\; \frac{\partial s_{ij}}{\partial m} \;\; \frac{\partial s_{ij}}{\partial a} \right)\Big|_0 = \left(\frac{m^2(x_i - x_j)}{s_{ij} - a} \,, \;\; \frac{m^2(y_i - y_j)}{s_{ij} - a} \,, \;\; -\frac{m^2(x_i - x_j)}{s_{ij} - a} \,, \;\; -\frac{m^2(y_i - y_j)}{s_{ij} - a} \,, \;\; \frac{s_{ij} - a}{m} \,, \;\; 1 \right)\Big|_0$$

C.2.2 2D GNSS baseline vector components

Stand point i , target point j, with 2×2 variance-covariance matrix $\Sigma_{[\Delta x, \Delta y]}$

$$\Delta x_{ij} = x_j - x_i \quad , \quad \Delta y_{ij} = y_j - y_i$$

$$A_{\Delta x} = \left(\frac{\partial \Delta x_{ij}}{\partial x_i} \;\; \frac{\partial \Delta x_{ij}}{\partial x_j} \right)\Big|_0 = \left(-1 \,, \;\; 1 \right), \quad A_{\Delta y} = \left(\frac{\partial \Delta y_{ij}}{\partial y_i} \;\; \frac{\partial \Delta y_{ij}}{\partial y_j} \right)\Big|_0 = \left(-1 \,, \;\; 1 \right)$$

C.2.3 Bearing

Stand point i, target point j, standard deviation σ_T

$$T_{ij} = \arctan \frac{x_j - x_i}{y_j - y_i}$$

$$A_T = \left(\frac{\partial T_{ij}}{\partial x_i} \;\; \frac{\partial T_{ij}}{\partial y_i} \;\; \frac{\partial T_{ij}}{\partial x_j} \;\; \frac{\partial T_{ij}}{\partial x_j} \right)\Big|_0 = \left(\frac{y_i - y_j}{(s_{ij})^2} \,, -\frac{x_i - x_j}{(s_{ij})^2} \,, \frac{y_i - y_j}{(s_{ij})^2} \,, -\frac{x_i - x_j}{(s_{ij})^2} \right)\Big|_0$$

C.2.4 Direction

Stand point i, target point j, orientation ω_i, constant standard deviation σ_c, distance dependent standard deviations σ_l and σ_d

$$r_{ij} = T_{ij} - \omega_i \quad , \quad \sigma_r = \sqrt{\sigma_c^2 + \sigma_l^2 / s + \sigma_d^2 / s^2}$$

$$A_r = \left(\frac{\partial r_{ij}}{\partial x_i} \;\; \frac{\partial r_{ij}}{\partial y_i} \;\; \frac{\partial r_{ij}}{\partial x_j} \;\; \frac{\partial r_{ij}}{\partial x_j} \;\; \frac{\partial r_{ij}}{\partial \omega_i} \right)\Big|_0 = \left(\frac{y_i - y_j}{(s_{ij})^2} \,, -\frac{x_i - x_j}{(s_{ij})^2} \,, \frac{y_i - y_j}{(s_{ij})^2} \,, -\frac{x_i - x_j}{(s_{ij})^2} \,, \; -1 \right)\Big|_0$$

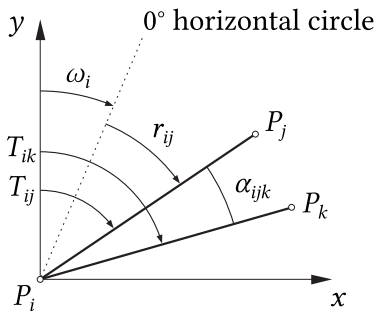
C.2.5 Horizontal angle

Stand point i, left target point j (backsight station), right target point k (foresight station), standard deviation σ_α

$$\alpha_{ijk} = T_{ik} - T_{ij} = \arctan \frac{x_k - x_i}{y_k - y_i} - \arctan \frac{x_j - x_i}{y_j - y_i}$$

$$A_\alpha = \left(\frac{\partial \alpha_{ijk}}{\partial x_i} \;\; \frac{\partial \alpha_{ijk}}{\partial y_i} \;\; \frac{\partial \alpha_{ijk}}{\partial x_j} \;\; \frac{\partial \alpha_{ijk}}{\partial x_j} \;\; \frac{\partial \alpha_{ijk}}{\partial x_k} \;\; \frac{\partial \alpha_{ijk}}{\partial x_k} \right)\Big|_0$$

$$= \left(-\frac{y_k - y_i}{(s_{ik})^2} + \frac{y_j - y_i}{(s_{ij})^2} \,, \; \frac{x_k - x_i}{(s_{ik})^2} - \frac{x_j - x_i}{(s_{ij})^2} \,, \; -\frac{y_j - y_i}{(s_{ij})^2} \,, \; \frac{x_j - x_i}{(s_{ij})^2} \,, \; \frac{y_k - y_i}{(s_{ik})^2} \,, \; -\frac{x_k - x_i}{(s_{ik})^2} \right)\Big|_0$$



Bearings T_{ij}, T_{ik}, direction r_{ij}, orientation ω_i and horizontal angle α_{ijk}.

C.2.6 Coordinate

Standpoint i, with standard deviations σ_x, σ_y or 2×2 variance-covariance matrix $\Sigma_{[x,y]}$

$$x_i = x_i \text{ and/or } y_i = y_i$$

C.3 Three dimensional network (Spatial network)

C.3.1 3D distance (Slant distance)

Stand point i with instrument height i_h, target point j with target height t_h, scale m, additive constant a. Constant standard deviation σ_c, distance dependent standard deviations σ_l and σ_d

$$S_{ij} = m\sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j + t_h - (Z_i + i_h))^2} + a \quad , \quad \sigma_S = \sqrt{\sigma_c^2 + \sigma_l^2 S + (\sigma_d S)^2}$$

$$A_s = \left(\frac{\partial S_{ij}}{\partial X_i} \; \frac{\partial S_{ij}}{\partial Y_i} \; \frac{\partial S_{ij}}{\partial Z_i} \; \frac{\partial S_{ij}}{\partial X_j} \; \frac{\partial S_{ij}}{\partial Y_j} \; \frac{\partial S_{ij}}{\partial Z_j} \; \frac{\partial S_{ij}}{\partial m} \; \frac{\partial S_{ij}}{\partial a} \right)\Big|_0$$

$$= \left(\frac{m^2(X_i - X_j)}{S_{ij} - a} \, , \; \frac{m^2(Y_i - Y_j)}{S_{ij} - a} \, , \; \frac{m^2(Z_i - Z_j)}{S_{ij} - a} \, , \; -\frac{m^2(X_i - X_j)}{S_{ij} - a} \, , \; -\frac{m^2(Y_i - Y_j)}{S_{ij} - a} \, , \; -\frac{m^2(Z_i - Z_j)}{S_{ij} - a} \, , \; \frac{S_{ij} - a}{m} \, , \; 1 \right)\Big|_0$$

C.3.2 3D GNSS baseline vector components

Stand point i , target point j, with 3×3 variance-covariance matrix $\Sigma_{[\Delta X, \Delta Y, \Delta Z]}$

$$\Delta X_{ij} = X_j - X_i \quad , \quad \Delta Y_{ij} = Y_j - Y_i \quad , \quad \Delta Z_{ij} = Z_j - Z_i$$

$$A_{\Delta X} = \left(\frac{\partial \Delta X_{ij}}{\partial X_i} \; \frac{\partial \Delta X_{ij}}{\partial X_j} \right)\Big|_0 = \left(-1 \, , \; 1 \right), \quad A_{\Delta Y} = \left(\frac{\partial \Delta Y_{ij}}{\partial Y_i} \; \frac{\partial \Delta Y_{ij}}{\partial Y_j} \right)\Big|_0 = \left(-1 \, , \; 1 \right)$$

$$A_{\Delta Z} = \left(\frac{\partial \Delta Z_{ij}}{\partial Y_i} \; \frac{\partial \Delta Y_{ij}}{\partial Y_j} \right)\Big|_0 = \left(-1 \, , \; 1 \right)$$

C.3.3 Vertical angle

Stand point i with instrument height i_h, target point j with target height t_h, constant standard deviation σ_c, distance dependent standard deviations σ_l and σ_d

$$\beta_{ij} = \operatorname{arccot} \frac{\sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2}}{Z_j + t_h - (Z_i + i_h)} = \operatorname{arccot} \frac{s_{ij}}{\Delta Z_{ij}}, \quad , \quad \sigma_\beta = \sqrt{\sigma_c^2 + \sigma_l^2/s + \sigma_d^2/s^2}$$

$$A_\beta = \left(\frac{\partial \beta_{ij}}{\partial X_i} \; \frac{\partial \beta_{ij}}{\partial Y_i} \; \frac{\partial \beta_{ij}}{\partial Z_i} \; \frac{\partial \beta_{ij}}{\partial X_j} \; \frac{\partial \beta_{ij}}{\partial Y_j} \; \frac{\partial \beta_{ij}}{\partial Z_j} \right)\Big|_0$$

$$= \frac{1}{(s_{ij})^2 + (\Delta Z_{ij})^2} \left(-\frac{(X_i - X_j)\Delta Z_{ij}}{s_{ij}} \, , \; -\frac{(Y_i - Y_j)\Delta Z_{ij}}{s_{ij}} \, , \; -s_{ij} \, , \; \frac{(X_i - X_j)\Delta Z_{ij}}{s_{ij}} \, , \; \frac{(Y_i - Y_j)\Delta Z_{ij}}{s_{ij}} \, , \; s_{ij} \right)\Big|_0$$

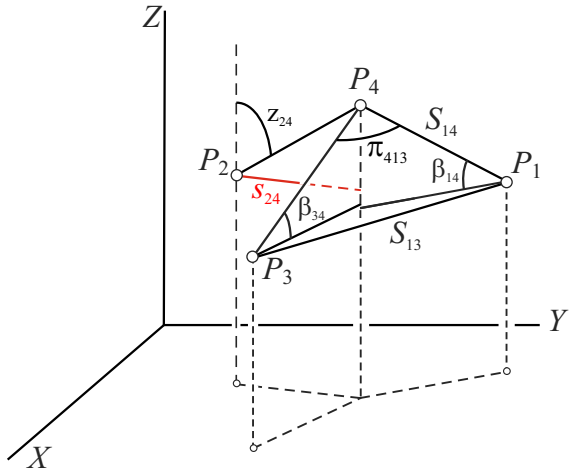
C.3.4 Zenith angle

Stand point i with instrument height i_h, target point j with target height t_h, constant standard deviation σ_c, distance dependent standard deviations σ_l and σ_d

$$z_{ij} = \arctan \frac{\sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2}}{Z_j + t_h - (Z_i + i_h)} = \arctan \frac{s_{ij}}{\Delta Z_{ij}} \quad , \quad \sigma_z = \sqrt{\sigma_c^2 + \sigma_l^2/s + \sigma_d^2/s^2}$$

$$A_z = \left(\frac{\partial z_{ij}}{\partial X_i} \; \frac{\partial z_{ij}}{\partial Y_i} \; \frac{\partial z_{ij}}{\partial Z_i} \; \frac{\partial z_{ij}}{\partial X_j} \; \frac{\partial z_{ij}}{\partial Y_j} \; \frac{\partial z_{ij}}{\partial Z_j} \right)\Big|_0$$

$$= \frac{-1}{(s_{ij})^2 + (\Delta Z_{ij})^2} \left(-\frac{(X_i - X_j)\Delta Z_{ij}}{s_{ij}} \; , \; -\frac{(Y_i - Y_j)\Delta Z_{ij}}{s_{ij}} \; , \; -s_{ij} \; , \; \frac{(X_i - X_j)\Delta Z_{ij}}{s_{ij}} \; , \; \frac{(Y_i - Y_j)\Delta Z_{ij}}{s_{ij}} \; , \; s_{ij} \right)\Big|_0$$



Vertical angle β, zenith angle z, position angle π, planar and spatial distance, s and S

C.3.5 Position angle

Stand point i, left target point j (backsight station), right target point k (foresight station), standard deviation σ_π

$$\pi_{ijk} = \arccos \frac{S_{ij}^2 + S_{ik}^2 - S_{jk}^2}{2 S_{ij} S_{ik}}$$

$$A_\pi = \left(\frac{\partial \pi_{ijk}}{\partial X_i} \; \frac{\partial \pi_{ijk}}{\partial Y_i} \; \frac{\partial \pi_{ijk}}{\partial Z_i} \; \frac{\partial \pi_{ijk}}{\partial X_j} \; \frac{\partial \pi_{ijk}}{\partial Y_j} \; \frac{\partial \pi_{ijk}}{\partial Z_j} \; \frac{\partial \pi_{ijk}}{\partial X_k} \; \frac{\partial \pi_{ijk}}{\partial Y_k} \; \frac{\partial \pi_{ijk}}{\partial Z_k} \right)\Big|_0$$

with

$$\frac{\partial \pi_{ijk}}{\partial X_j} = \frac{1}{S_{ij} sin\pi_{ijk}} [\frac{1}{S_{ij}} (cos\pi_{ijk} - \frac{S_{ij}}{S_{ik}})(X_i - X_j) + \frac{X_k - X_j}{S_{ik}}]$$

$$\frac{\partial \pi_{ijk}}{\partial Y_j} = \frac{1}{S_{ij} sin\pi_{ijk}} [\frac{1}{S_{ij}} (cos\pi_{ijk} - \frac{S_{ij}}{S_{ik}})(Y_i - Y_j) + \frac{Y_k - Y_j}{S_{ik}}]$$

$$\frac{\partial \pi_{ijk}}{\partial Z_j} = \frac{1}{S_{ij} sin\pi_{ijk}} [\frac{1}{S_{ij}} (cos\pi_{ijk} - \frac{S_{ij}}{S_{ik}})(Z_i - Z_j) + \frac{Z_k - Z_j}{S_{ik}}]$$

$$\frac{\partial \pi_{ijk}}{\partial X_k} = \frac{1}{S_{ik} sin\pi_{ijk}} [\frac{1}{S_{ik}} (cos\pi_{ijk} - \frac{S_{ik}}{S_{ij}})(X_k - X_i) + \frac{X_k - X_j}{S_{ij}}]$$

$$\frac{\partial \pi_{ijk}}{\partial Y_k} = \frac{1}{S_{ik} sin\pi_{ijk}} [\frac{1}{S_{ik}} (cos\pi_{ijk} - \frac{S_{ik}}{S_{ij}})(Y_k - Y_i) + \frac{Y_k - Y_j}{S_{ij}}]$$

$$\frac{\partial \pi_{ijk}}{\partial Z_k} = \frac{1}{S_{ik} sin\pi_{ijk}} [\frac{1}{S_{ik}} (cos\pi_{ijk} - \frac{S_{ik}}{S_{ij}})(Z_k - Z_i) + \frac{Z_k - Z_j}{S_{ij}}]$$

$$\frac{\partial \pi_{ijk}}{\partial X_i} = -(\frac{\partial \pi_{ijk}}{\partial X_j} + \frac{\partial \pi_{ijk}}{\partial X_k}) \quad , \quad \frac{\partial \pi_{ijk}}{\partial Y_i} = -(\frac{\partial \pi_{ijk}}{\partial Y_j} + \frac{\partial \pi_{ijk}}{\partial Y_k}) \quad , \quad \frac{\partial \pi_{ijk}}{\partial Z_i} = -(\frac{\partial \pi_{ijk}}{\partial Z_j} + \frac{\partial \pi_{ijk}}{\partial Z_k})$$

C.3.6 Coordinate

Standpoint i, with standard deviations $\sigma_X, \sigma_Y, \sigma_Z$ or 3×3 variance-covariance matrix $\Sigma_{[X,Y,Z]}$

$$X_i = X_i \text{ and/or } Y_i = Y_i \text{ and/or } Z_i = Z_i$$

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