

### Introduction

The global coverage of GRACE allows to monitor the water mass change. However, the coarse spatial resolution of GRACE products limits GRACE largescale applications. The increasing demand of small-scale hydrological studies drives us to improve the spatial resolution of GRACE. Apart from expecting a highly improved resolution from the next generation of GRACE, one possibility is to downscale the GRACE product by assimilation with data in finer resolution. Therefore, in this study, we propose a statistical empirical model for spatial downscaling of GRACE by assimilating GRACE data with terrestrial water storage change from WGHM and multiple hydrological variables from highlyresolved models. In contrast with conventional assimilation approaches, our algorithm is implemented without any dynamic model assumptions.

## Methodology

A moving average partial least-squares regression (MA-PLR) model is applied for assimilation of GRACE and WGHM.

Expand observation matrix *L* by a moving-average ensemble  $\begin{bmatrix} L(t_1) & L(t_1-1) & \cdots & L(t_1-k+1) \end{bmatrix}$ 

$$L_{i} = \begin{bmatrix} L(t_{2}) & L(t_{2}-1) & \cdots & L(t_{2}-k+1) \\ \vdots & \vdots & \ddots & \vdots \\ L(t_{n}) & L(t_{n}-1) & \cdots & L(t_{n}-k+1) \end{bmatrix},$$
$$L = \begin{bmatrix} L_{1} & L_{1} & \cdots & L_{i} \end{bmatrix}$$

- Calculate cross-covariance matrix  $C_{LS}$  between observations L and predictands **S**
- Apply singular value decomposition on covariance matrix:  $C_{LS} = L_0^T S_0 = U_C \Sigma_C V_C^T$

Recast the regression model into  $U_I = L_0 U_C$ ,

$$S_0 = L_0 H = U_I U_C^T H = U_I K$$

- $S_0 = L_0 \Pi = O_L O_C \Pi = O_L K$ Train the prediction matrix **K** on the joint modes from *L*
- $\hat{K} = (U_L^T U_L)^{-1} U_L^T S_0$
- Transform the prediction matrix from mode level to signal level  $\hat{H} = U_C \hat{K}$
- Predict **S** by **H**:  $\hat{S} = L \cdot \hat{H}$

**Table 1:** Observation and predictand matrices in training and predicting.



Figure 1: Illustration of the scenario for GRACE product downscaling. Moving average partial least-squares regression (MA-PLR) model is employed for training and predicting.

# An empirical spatial downscaling of GRACE by statistical assimilation of multiple hydrological variables Jinwei Zhang, Nico Sneeuw







storage change



Amazon basin.



Figure 9: Monthly aggregates of TWS over the Amazon basin from assimilation, comparing with TWS aggregates from GRACE and WGHM. The consistency of aggregated TWS with GRACE satisfies the condition of mass conservation. The correlation (R), NSE and RMSE of assimilated TWS with respect to GRACE are shown in the figure.

## **Conclusion, Discussion and Outlook**

- ation from WGHM.
- done in the future.



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#### Validation

The misclosure  $\varepsilon$  of terrestrial water balance in a catchment is calcu- $\varepsilon = P - ET - R - \frac{dM}{dM}$ 

The water storage flux is calculated by the first derivative of water  $\frac{dM}{dt} = \frac{TWS(t + \Delta t) - TWS(t - \Delta t)}{TWS(t - \Delta t)}$ 

> Figure 8: (a) The misclosure of the water balance in Amazon basin at epoch September 2005, and its (b) mean and (c) RMS from GRA-CE, WGHM and assimilation results. The consistency between downscaled grids and GRA-CE suggests that this assimilation maintains the same level of accuracy as GRACE, although it does not evidently improve the accuracy of TWS observed by GRACE .

The ensemble means of all the models listed in Table 2 for each variable P, ET, R are used to calculate the imbalance of water budget in the

A higher spatial resolution of TWS is achieved with empirical inform-

The assimilated TWS retains the dominant signals from GRACE.

MA-PLR model demonstrates its capability and potential in data assimilation and statistical downscaling.

Further validation with in-situ ground measurements still needs to be

It is still uncertain that our downscaling framework is also applicable for boreal catchments, which contain both solid and liquid mass variation.

