Regional gravity recovery from GRACE using position optimized radial base functions

M. Weigelt, M. Antoni, W. Keller

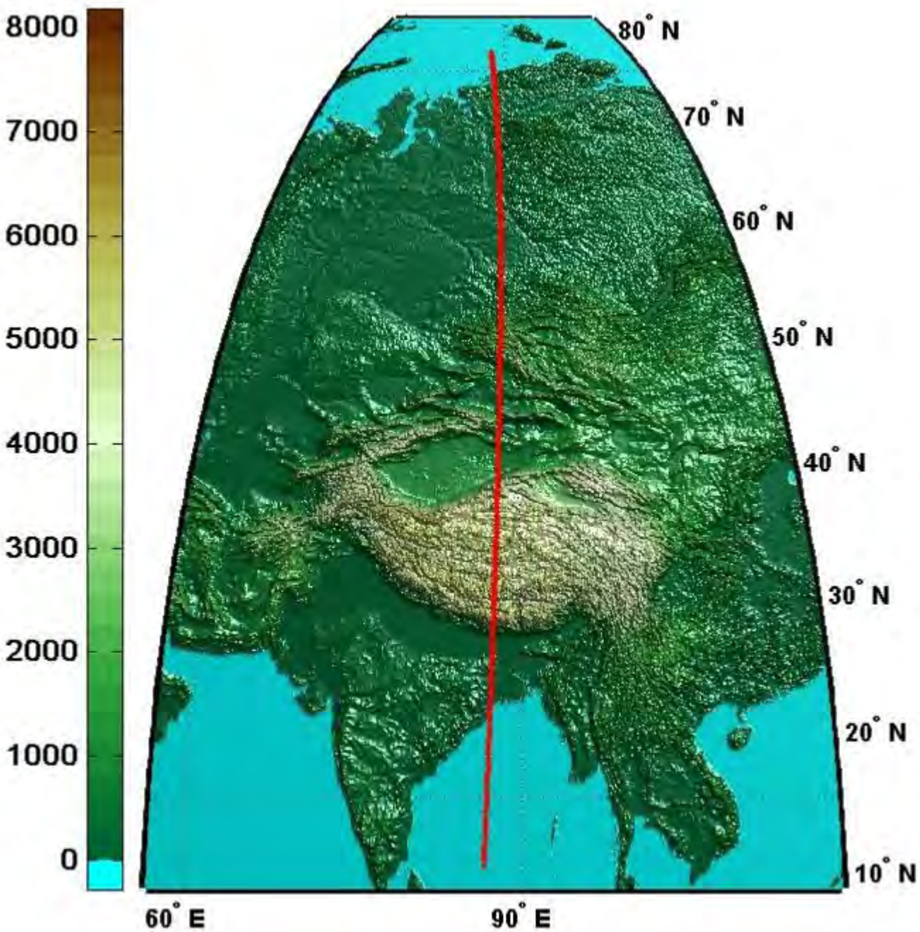
weigelt@gis.uni-stuttgart.de

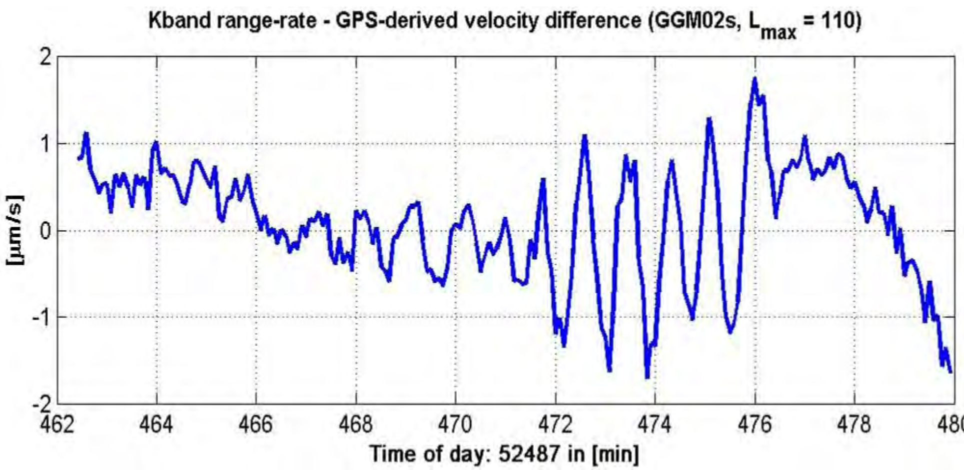
GGEO 2008 Symposium
Session 2
23th June, 2008

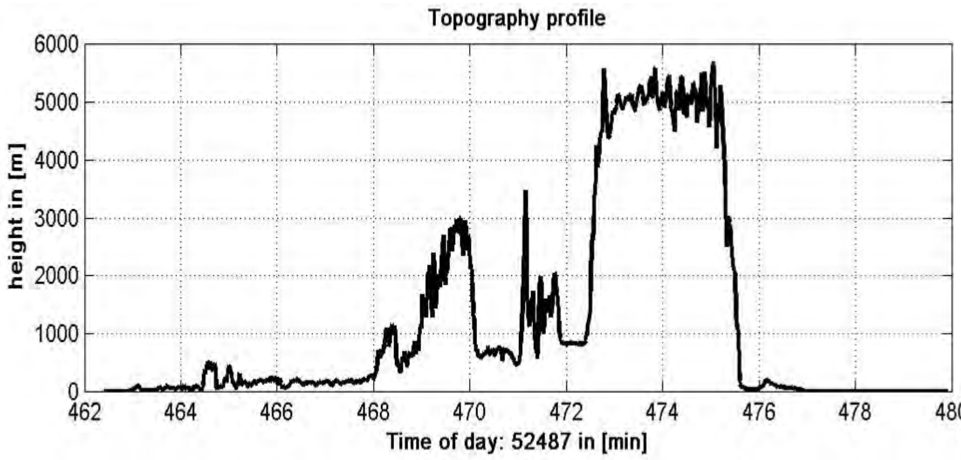
Motivation

- Comparison: K-Band range-rate - integrated velocity difference

  Integration key points: GGM02s (L_{max} = 110), 30 min arc







Approach

Previous studies:

- *Eicker/Mayer-Gürr*: short arcs + radial base functions/grid
- *Han/Simons*: energy integral + Slepian functions
- *Schmidt*: short arcs/energy integral + multi-resolution analysis

Approach:

line-of-sight gradiometry +
position optimized radial base functions

Advantages:

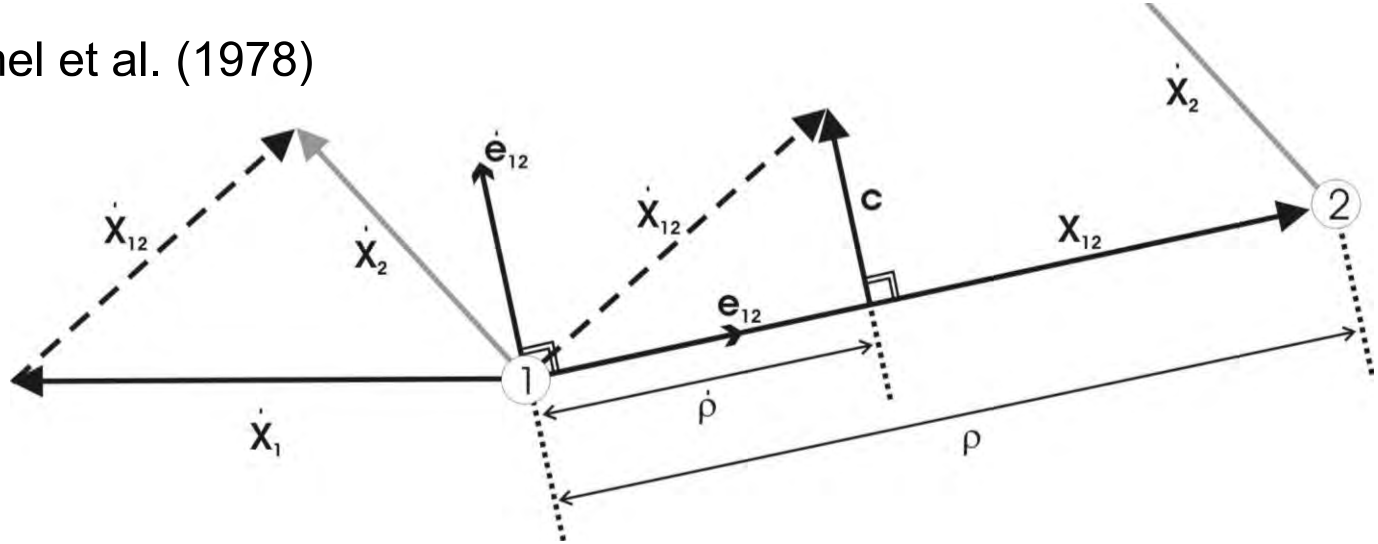
- increased sensitivity in the medium to short wavelength
- direct link to the barycenter of the satellites
- no overparametrization

Challenges:

- mixed approach: K-Band- and GPS-measurements need to be combined
- non-linear relation between position and pseudo-observable

Approach

from Rummel et al. (1978)



$$\dot{\rho} = \dot{\mathbf{X}}_{12}^T \mathbf{e}_{12}$$

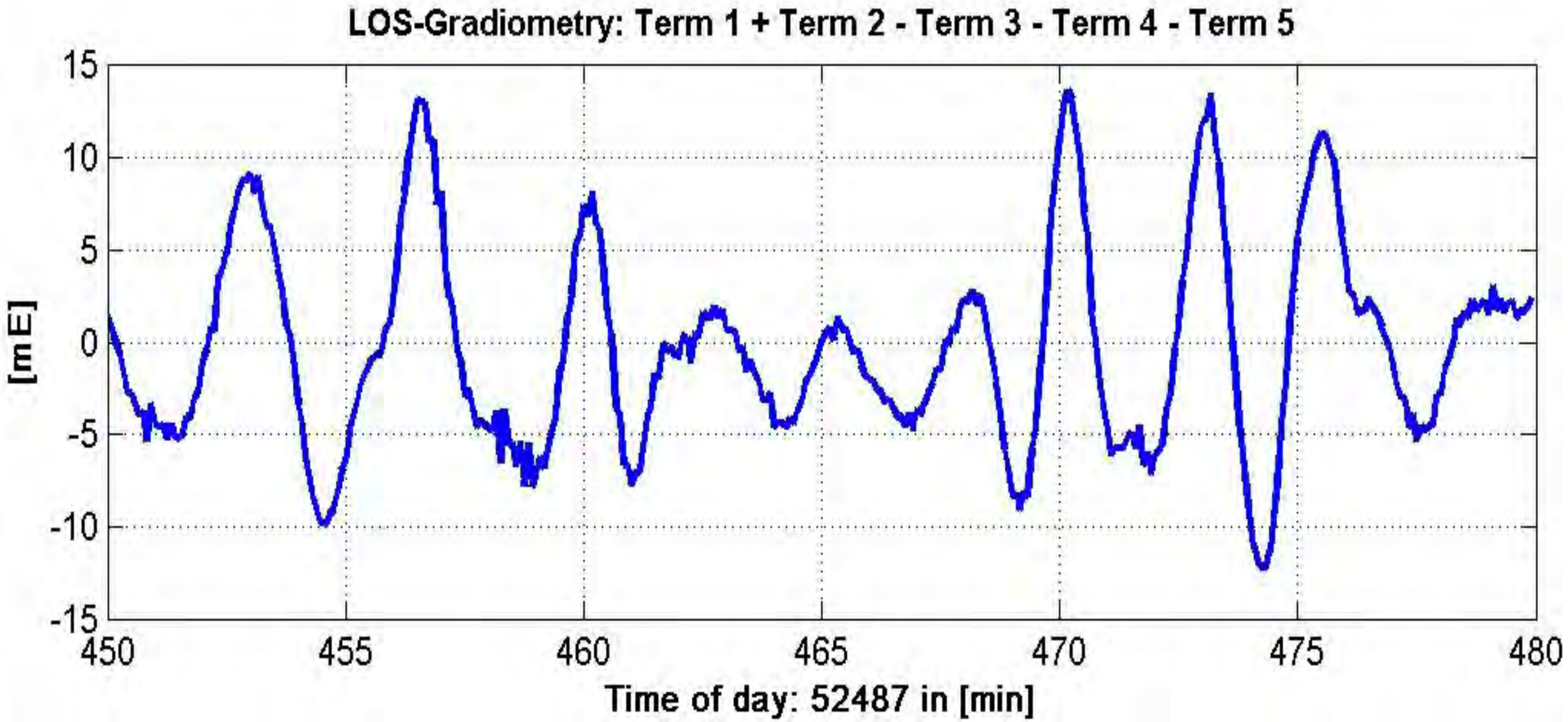
$$\ddot{\rho} = (\nabla V_2 - \nabla V_1)^T \mathbf{e}_{12} + \frac{1}{\rho}\left(||\dot{\mathbf{X}}_{12}||^2 - \dot{\rho}^2 \right)$$

$$\frac{\partial^2 V}{\partial y^2} = \mathbf{e}_{12}^T \mathbf{G} \mathbf{e}_{12} = \frac{\ddot{\rho}}{\rho} + \frac{\dot{\rho}^2}{\rho^2} - \frac{||\dot{\mathbf{X}}||^2}{\rho^2} - \frac{1}{\rho} \sum_i \mathbf{g}_i^T \mathbf{e}_{12}$$

$$- \frac{1}{\rho} \left(\nabla V_2^0 - \nabla V_1^0 \right)^T \mathbf{e}_{12}$$

Example: 30 min arc $K^0_{lm} = EGM96; L_{max} = 100$

$$\frac{\partial^2 V}{\partial y^2} = \frac{\ddot{\rho}}{\rho} + \frac{\dot{\rho}^2}{\rho^2} - \frac{||\dot{\mathbf{X}}||^2}{\rho^2} - \frac{1}{\rho}\sum_i \mathbf{g}_i^T \mathbf{e}_{12} - \frac{1}{\rho}\left(\nabla V_2^0 - \nabla V_1^0\right)^T \mathbf{e}_{12}$$



Local radial base functions

Modelling the (residual) signal by superposition of localizing radial base functions

$$\delta V(\lambda, \theta, r) = \frac{GM}{R} \sum_{b=1}^{B} \eta_b \Psi(\sigma_b, \varpi_b, r)$$

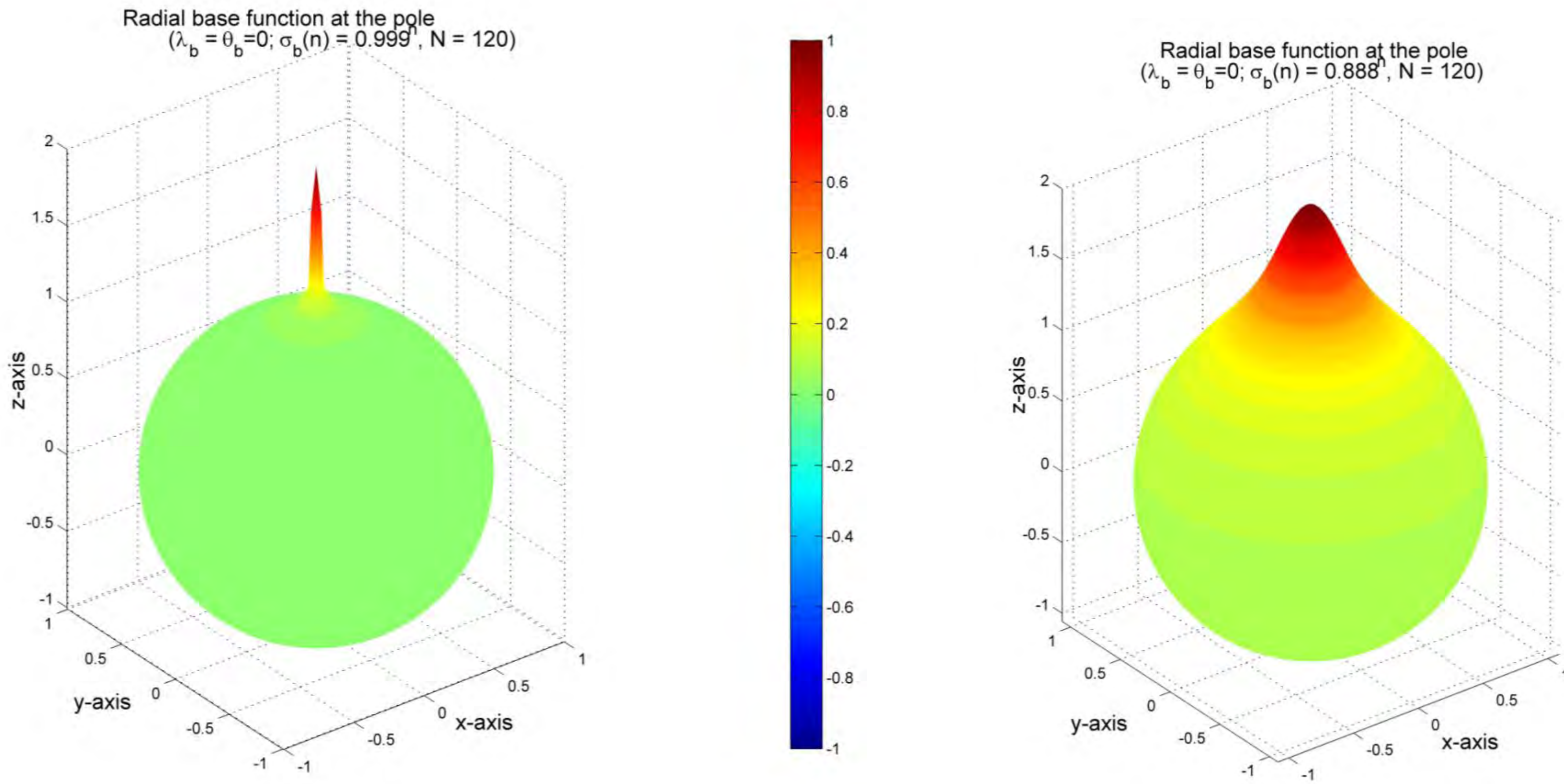
$$= \frac{GM}{R} \sum_{b=1}^{B} \eta_b \sum_{n=1}^{N} \left(\frac{R}{r}\right)^{n+1} \sigma_b(n) P_n(\cos \varpi_b)$$

with:

η_b	scale factor
$\sigma_b(n)$	shape parameter
ϖ_b	spherical distance to the center of the base function
$P_n(\cos \varpi_b)$	Legendre polynomial
λ, θ, r	spherical coordinates of the point of interest
GM	gravitational constant
R	Earth radius

Properties

- localizing system of base functions
- isotropic = symmetric to the center point
- parameter $\sigma_b(n)$ defines shape



LOS-Gradient + Local Radial base Functions

General formula for the second derivative in flight direction (y) of the potential:

$$\frac{\partial^2 V}{\partial y^2} = \frac{1}{a^2}\frac{\partial^2 V}{\partial u^2} + \frac{1}{a}\frac{\partial V}{\partial r}$$

with:

	a	semi-major axis
	u	argument of latitude
	r	radius

Applying to the residual signal and the local base function:

$$\frac{\partial^2 (\delta V)}{\partial y^2} = \frac{GM}{R}\sum_{b=1}^{B} \eta_b \frac{\partial^2 \Psi(\sigma_b, \varpi_b, r)}{\partial y^2}$$

$$\begin{aligned}\frac{\partial^2 \Psi}{\partial y^2} &= \frac{1}{a}\sum_{n=0}^{N}\left(\frac{R}{r}\right)^{n+1}\sigma_b(n)\left[(n+1)\left(\frac{(n+1)\,e^2\sin^2 E}{1-e^2} - \frac{r e\cos E}{a(1-e^2)} - \frac{a}{r}\right)P_n(\zeta_b) - \right.\\ &\quad - \left.\left(2\frac{(n+1)\,e\sin E}{\sqrt{1-e^2}}\frac{\partial \zeta_b}{\partial u} - \zeta_b\right)\dot{P}_n(\zeta_b) + \left(\frac{\partial \zeta_b}{\partial u}\right)^2 \ddot{P}_n(\zeta_b)\right]\end{aligned}$$

Optimization

- Objective:
  Optimal fit of the model to the pseudo-observation

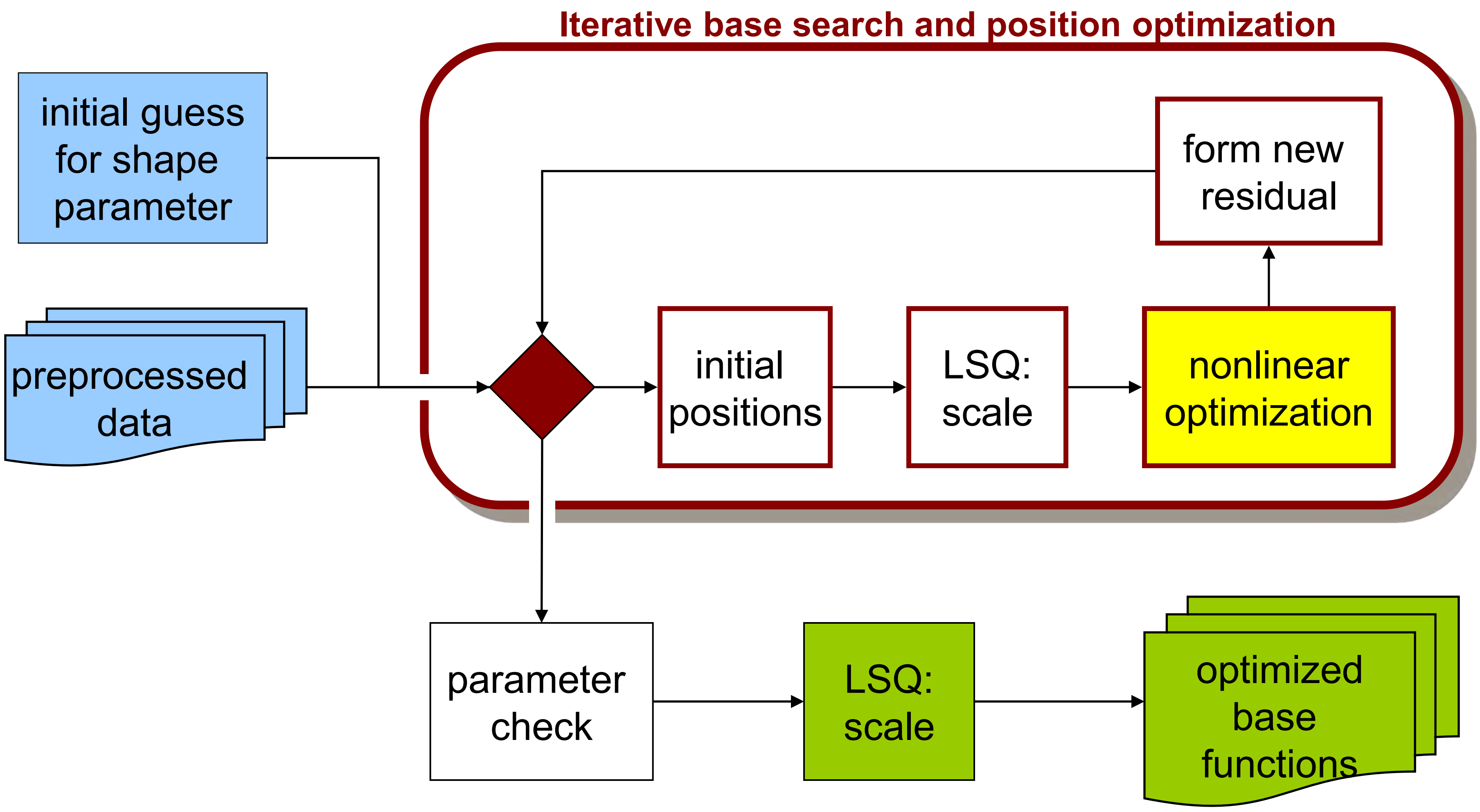
  Model: $$\mathbf{e}_{12}^T \mathbf{G} \mathbf{e}_{12} = \frac{GM}{R} \sum_{b=1}^{B} \eta_b \frac{\partial^2 \Psi(\sigma_b, \varpi_b, r)}{\partial y^2} + \varepsilon$$

  subject of the optimization: $\eta_b, \sigma_b, \lambda_b, \phi_b$

  $\rightarrow$ **non-linear least-squares problem**

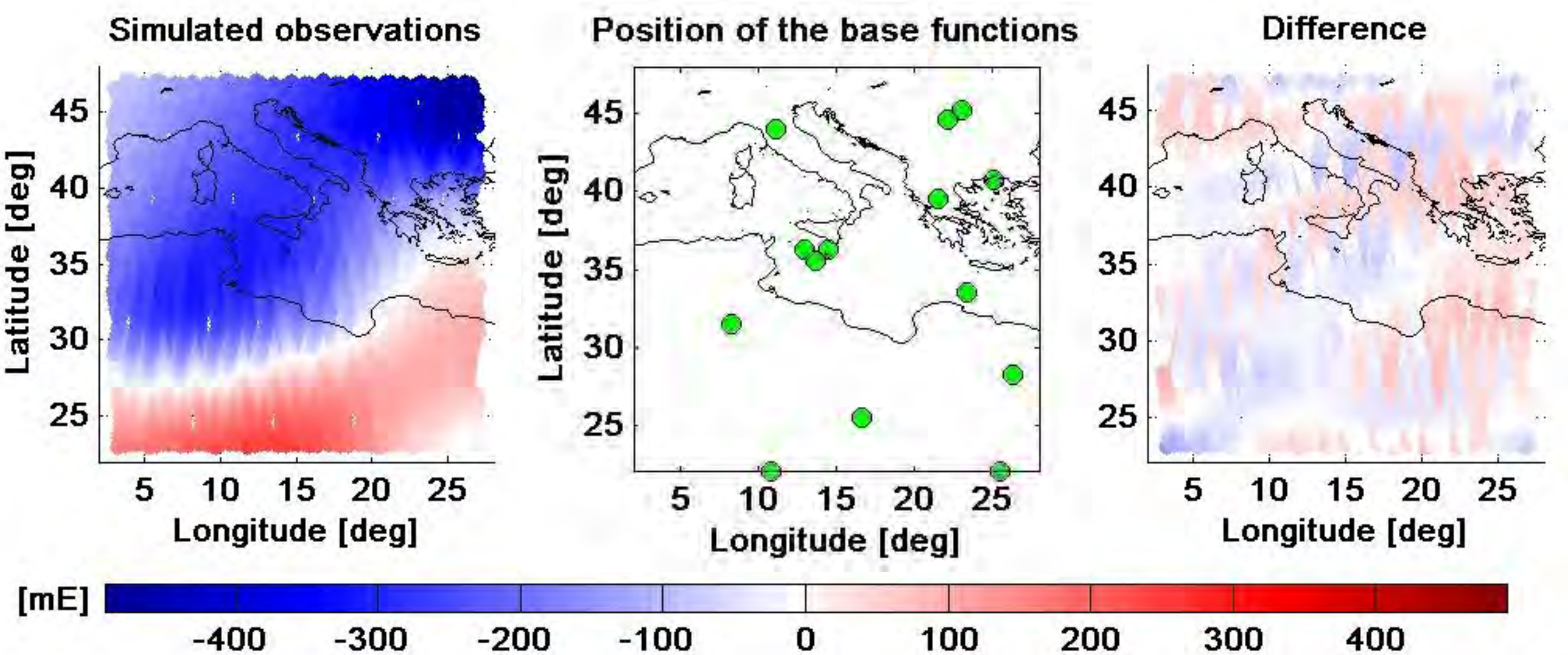
- Advantages:
  - small number of parameters
  - no instabilities

Optimization workflow



Closed loop - simulation

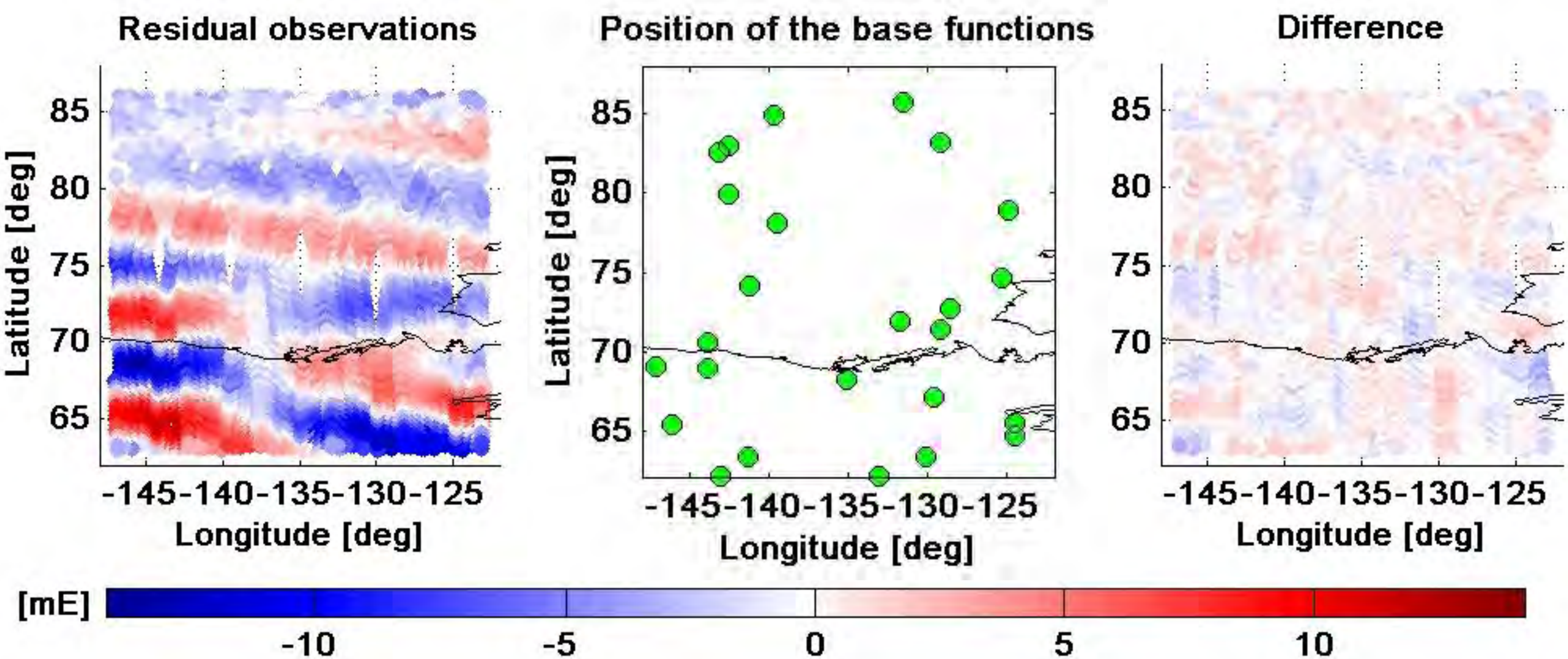
- LOS pseudo-measurements from 20 hidden base functions
- 21 base functions in use
- 98.8% of the simulated signal recovered



First results for GRACE: August 2002

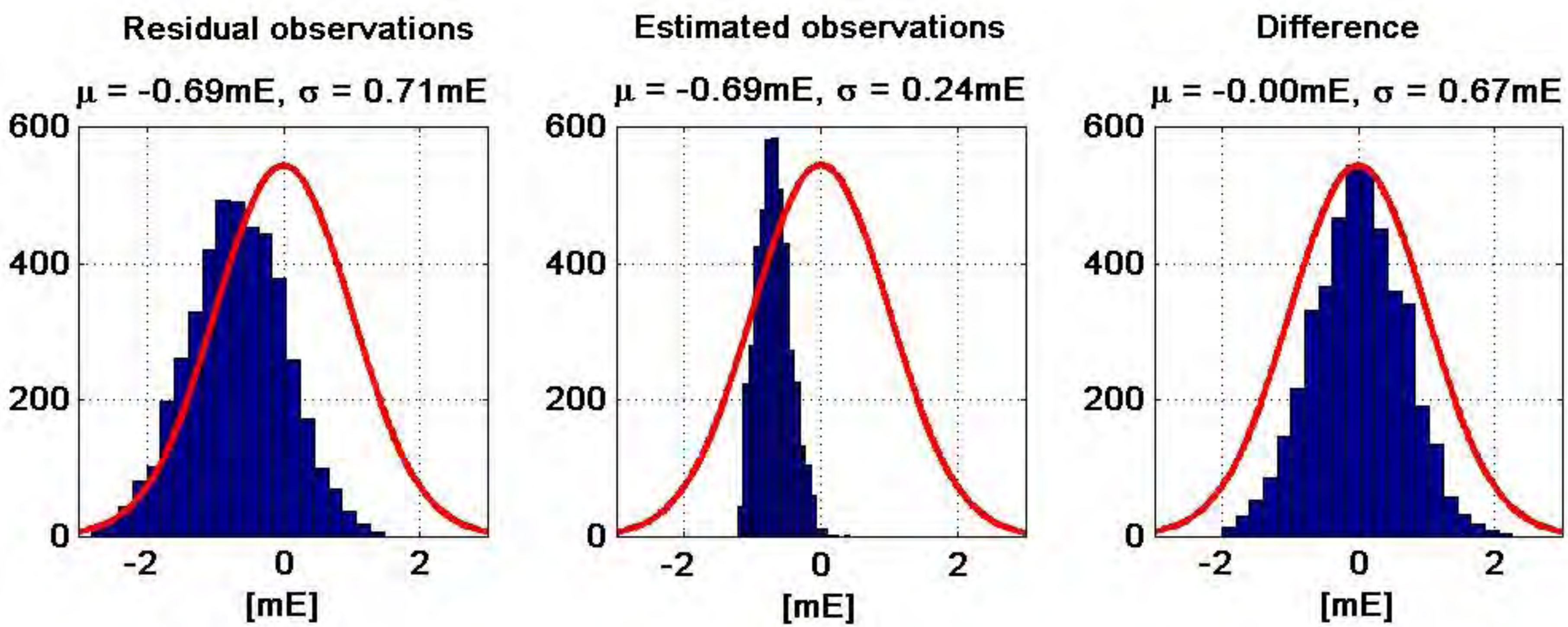
- Reference field: GGM02s, Lmax = 50
- 254 arcs and 28 base functions in use
- 97.3 % of the input signal recovered

	before	after
Max	11.70mE	3.67mE
Min	-14.12mE	-5.66mE
RMS	3.97mE	0.96mE



GRACE: August 2002

- Reference field: GFZ 08 2002 Rev. 4, L_{max} = 120
- 254 arcs and 31 base functions in use
- 34.3 % of the input signal recovered



Conclusions

- K-band observation contains residual signal
  - not recoverd by a global spherical harmonic analysis
  - GSHS is suboptimal in local areas
- residual signal after the estimation shows statistic characteristics of a normal distribution
- framework for further studies

Thank you

Regional gravity recovery from GRACE using position optimized radial base functions

M. Weigelt, M. Antoni, W. Keller

weigelt@gis.uni-stuttgart.de

GGEO 2008 Symposium
Session 2
23th June, 2008