What is Required to Recover the Time-variable Gravitational Field Using Satellite Gradiometry?

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The ESA mission **GOCE** has been instrumental in mapping the Earth static gravitational field with an unprecedented precision and homogeneous spatial resolution. One reason for this success was the performance of the onboard ultrasensitive electrostatic gradiometer. Today, the development of new technologies based on optical or cold-atom interferometry opens the way to even **more sensitive space inertial sensors**. Such sensors could be the core of future space gradiometers capable of mapping the time-variable gravitational field, offering an alternative solution to GRACE-like missions.

Here, we derive and evaluate a set of requirements for the different measured quantities involved in gravitational field recovery in order to fulfil this objective. Since such requirements depend on the orbit choice, we present the results for a low (303 km) and a higher altitude (361 km) polar and circular orbit with a repeat cycle of 29 solar days.

Gravitational gradiometry metrology

We note **V** the GGT (gravitational gradient tensor) expressed in Eötvös unit (E), with 1 E = 10^{-9} s^{-2}.
In a non-inertial frame like the Gradiometer Reference Frame R_{GRF} (moving with the spacecraft), the GGT is determined by measuring the acceleration gradient tensor $\boldsymbol{\Gamma}$ from which the GGT is extracted. For the diagonal gradients we have

$$V_{xx} = \Gamma_{xx} + \omega_y^2 + \omega_z^2$$
$$V_{yy} = \Gamma_{yy} + \omega_x^2 + \omega_z^2$$
$$V_{zz} = \Gamma_{zz} + \omega_x^2 + \omega_y^2$$

where $(\omega_x, \omega_y, \omega_z)^t$ is the angular velocity vector of R_{GRF} with respect to the inertial frame.
In a linear approximation neglecting the sensor's scale factors and the uncalibrated cross-talks between axes, the error degrading the estimated gradient $\hat{V}_{xx}$ is given by

$$\hat{V}_{xx} = V_{xx} + n_{xx} - 2\omega_y n_{\omega y} - 2\omega_z n_{\omega z} + 2d\theta_z V_{xy} + 2d\theta_y V_{xz} + \nabla V_{xx}.\,dr + dealerr + A_{cm}\frac{a_x}{L}$$

- gradiometer noise (n_{xx})
- contribution of the error $n_{\omega i}$ degrading ω_i in the correction of the centrifugal terms
- contribution of the error $d\theta_i$ on the determination of the attitude of R_{GRF}
- contribution of the satellite position error dr
- dealiasing solutions error
- non-zero common-mode rejection ratio (CMRR)

Similar expressions apply to V_{yy} and V_{zz}.

Calibration of the gradiometer

Most concepts of a gravitational gradiometer are based on the principle of matched pairs of accelerometers (Fig. 2). For instance, Γ_{xx} is approximated by:

$$\Gamma_{xx} = \frac{a_x(A1) - a_x(A4)}{L} + o(L)$$

where L is the distance between the accelerometers A1 and A4 and a_x is the x-component of the mesured acceleration. In a simplified situation we measure

$$\tilde{a}_x(A1) = s_x(A1)\, a_x(A1) + n(A1)$$

where s_x is the scale factor and n is the noise of the accelerometer. Scale factors of matched accelerometers must be equalized as much as possible to limit the projection of the common-mode acceleration on the differential mode.

Internal calibration ➡ critical element in the detection of the time-variable gravity signal.
The calibration method developed by Siemes (2012) for GOCE data is used to evaluate the estimation of the scale factors of the optical accelerometers. This calibration method is based on certain conditions for accelerometer data and star sensor data.

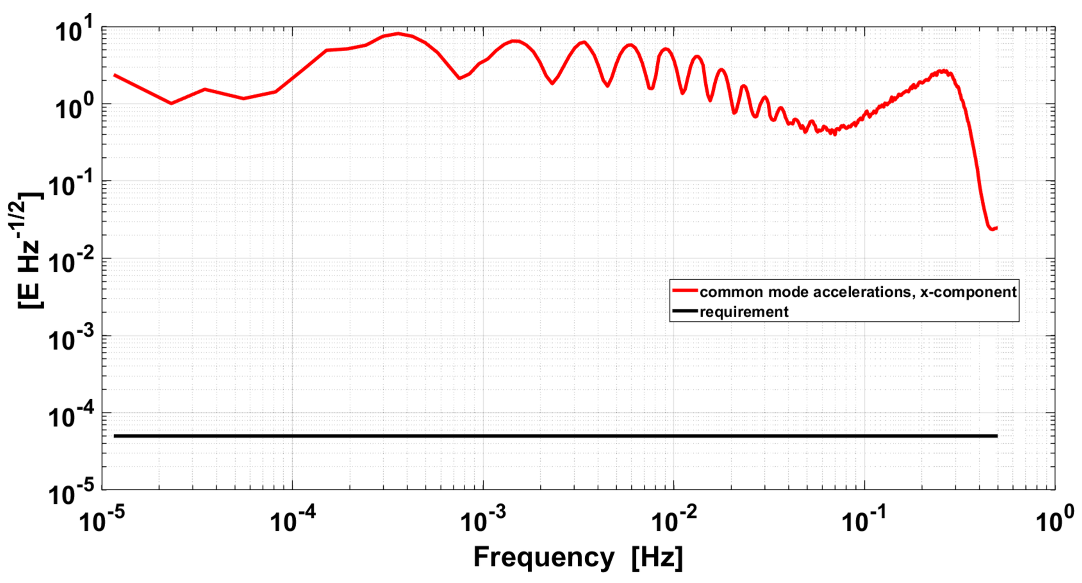

Fig. 1 Amplitude Spectral Density of the common mode accelerations and the required common mode rejection

The required calibration accuracy is estimated by the factor that shifts the common mode accelerations below the defined common mode rejection (Fig. 1 and Tab. 1).

Common Mode Rejection Ratio (CMRR): ratio of the common mode and differential mode signal ➡ evaluation of the calibration results

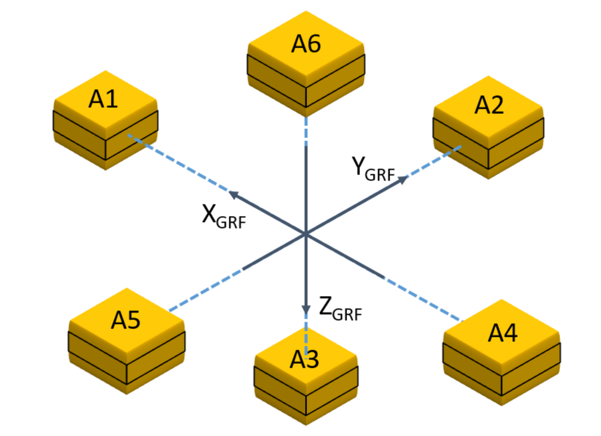

Fig. 2 Star configuration of the gradiometer, composed of 3 pairs of accelerometers.

Realistic satellite dynamics are included in the simulations using the High Performance Satellite Dynamics Simulator (Pelivan et al., 2012). The common mode accelerations for the x component are derived from GOCE data.
The calibration method is tested for 2 different altitudes: 361 km (orbit 1) and 303 km (orbit 2).
In a simplified version the measured accelerations are corrected by $\frac{s_i}{\hat{s}_i}$,
s_i: true scale factor
$\hat{s}_i$: estimated scale factor.

The CMRR for the corrected measurements is

$$\mathrm{CMRR} = \frac{\hat{s}_j\, s_i - s_j\, \hat{s}_i}{\hat{s}_j\, s_i + s_j\, \hat{s}_i}$$

for the accelerometer pairs $ij = 14, 25, 36$.
The CMRR of the estimated scale factors is given in Tab. 1.

Tab. 1 Common Mode Rejection Ratio (CMRR) for the x,y and z components for the corrected measurements ("achieved") and estimated requirement ("required")

	Altitude: 303 km		Altitude: 361 km	
	required	achieved	required	achieved
CMRR_x	$3.1\ 10^{-6}$	$3.3\ 10^{-8}$	$3.1\ 10^{-6}$	$1.5\ 10^{-7}$
CMRR_y	$8.6\ 10^{-9}$	$2.0\ 10^{-7}$	$3.8\ 10^{-8}$	$2.2\ 10^{-7}$
CMRR_z	$4.8\ 10^{-8}$	$6.6\ 10^{-8}$	$4.7\ 10^{-8}$	$2.9\ 10^{-7}$

References

Siemes, C.: GOCE gradiometer calibration and Level 1b data processing. ESA working paper EWP-2384. 2012.

Pelivan, I., Heidecker, A., Theil. S.: High Performance Satellite Dynamics and Control Simulation for Multi-Purpose Application. Journal of Aerospace Engineering, Sciences and Applications, 4, pp.119-130. 2012.

Requirement on the observables

Our approach follows 2 steps: first, we verify the requirement for the total error degrading the gradients in the LNORF (step 1). Then, we allocate this error to the different error contributors (step 2). In this respect, we make the following assumptions:

- only V_{xx}, V_{yy} and V_{zz} sampled at 1 Hz during 29 days are considered.
- the input gravity field model is EIGEN6c4 up to d/o 360 for the static part and ESA updated ESM up to d/0 180 (April 2006) for the time-variable part
- the PSD of the gradiometer noise increases as f ^{4} after 0.01 Hz and as f $^{-2}$ below 10^{-4} Hz

Below, only the results concerning the lower altitude are presented.

Step 1: Noise requirement for the gradients after rotation to the local north-oriented frame (LNOF) for the low orbit

Simulations run for 3 different levels of total error: nominal (1), 5 x nominal (2) and 10 x nominal (3).

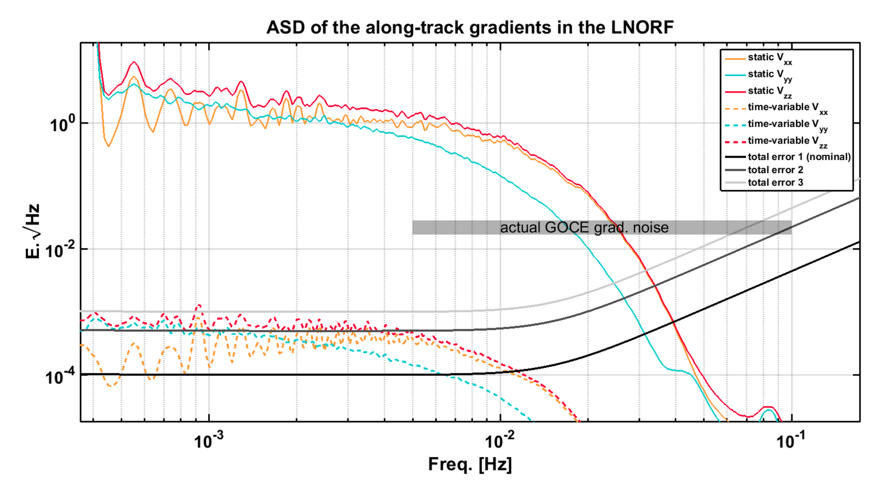

Fig. 3 Square-root of the PSDs of the static and variable part of the gravitational gradients along the orbit and requirement on the error degrading the final gradients used in the gravity field recovery.

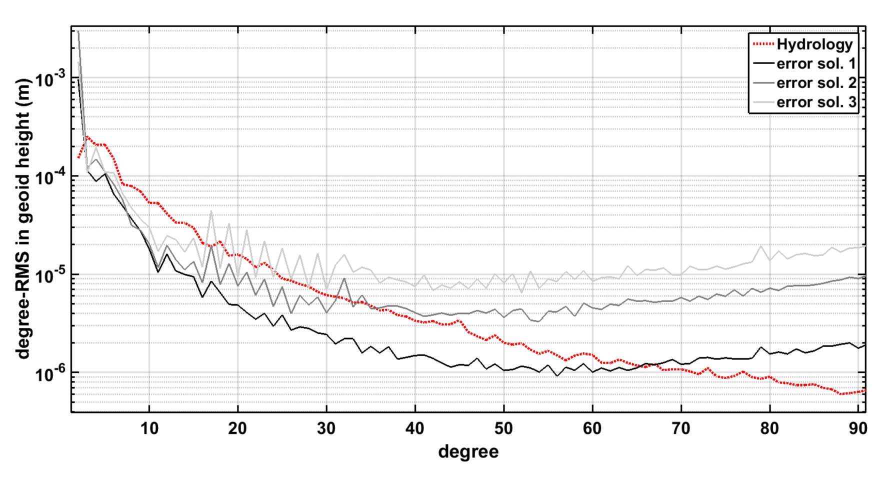

Fig. 4 Geoid degree-variance of the error of the simulated recovered gravity models for the 3 different total noises tested. For comparison, the average hydrological signal is also plotted.

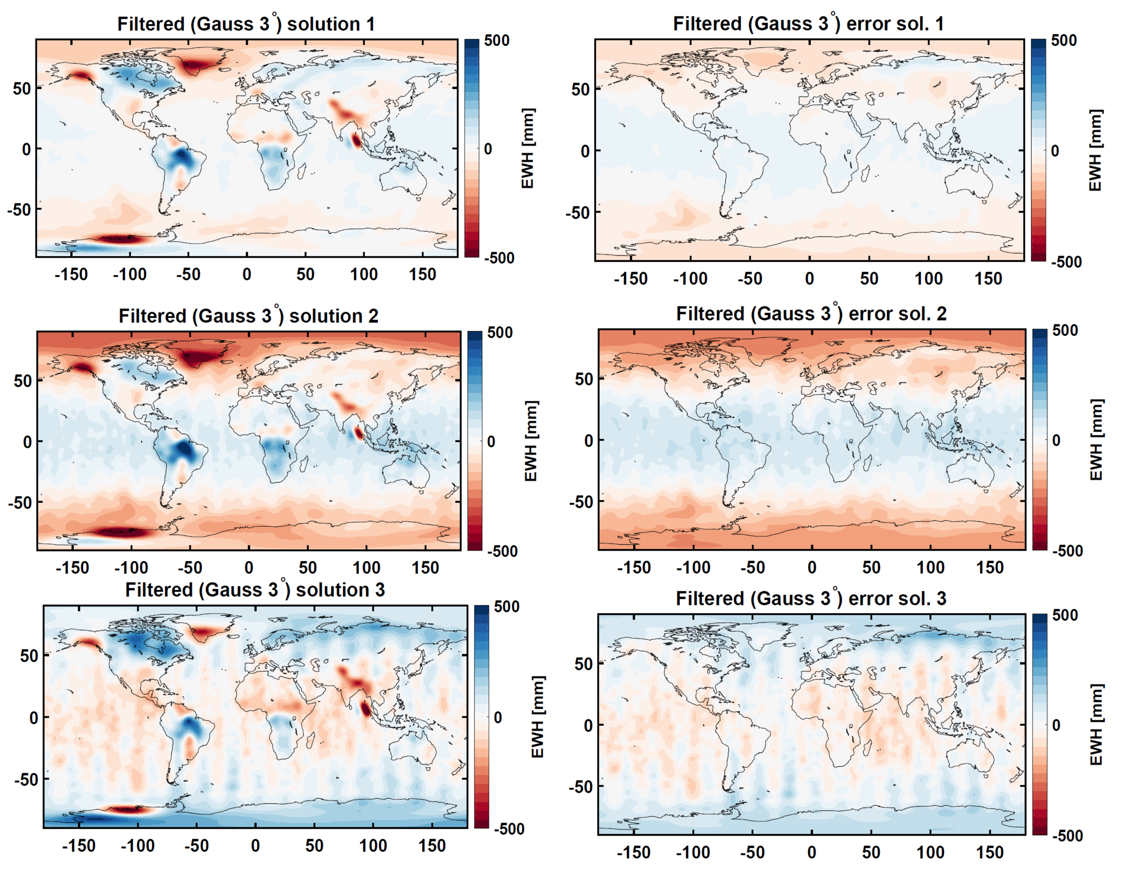

Fig. 5 Recovered time-variable gravity fields for the 3 different levels of total noise (left-hand column). Their difference with the error-free model averaged over the 29 days is displayed on the right-hand column. All fields are expressed in equivalent water height and filtered with a 3° wide Gaussian filter.

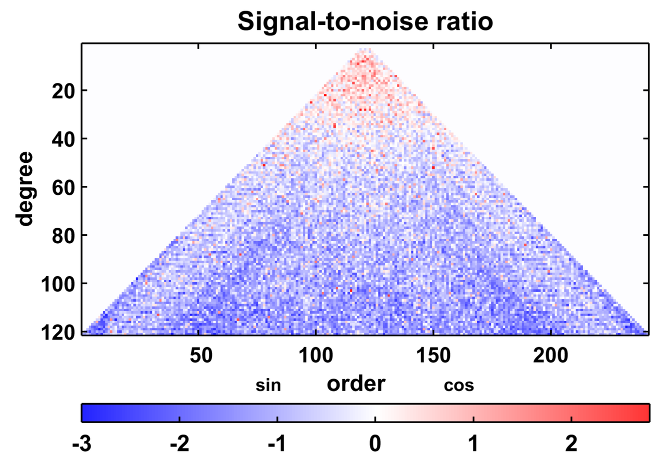

Fig. 6 SNR of the time-variable gravitational signal for the nominal requirement. The SNR is calculated as:

$$\mathrm{SNR(l,m)} = \log_{10}\left(\left|\frac{median(S|C_{l,m}^{t-var}(k))_k}{err(S|C_{l,m})}\right|\right)$$

For the higher altitude (361 km), only noise levels (1) and (2) enable to detect the time-variable gravity field.

Step 2: Noise requirements for the contributors

We now split the nominal total noise into the different contributors.

- error on position in LNOF (requirement: $\mathrm{std}(\nabla V_{ii}.\,\boldsymbol{dr}) < 2\ 10^{-5}E$):

	V_{xx}	V_{yy}	V_{zz}
X [cm]	11	2.2	1.8
Y [cm]	2.2	11	1.8
Z [cm]	2.2	2.2	1.6

Tab. 2 Required standard deviation on the position error for the determination of the diagonal gravity gradients

- Error on the determination of ω_i (requirement on contribution: $<5\ 10^{-5}E\sqrt{\mathrm{Hz}}$):

	Altitude: 303 km			Altitude: 361 km		
	Noise (1)	Noise (2)	Noise (3)	Noise (1)	Noise (2)	Noise (3)
$n_{\omega x}$	$1.7\ 10^{-9}$	$8.5\ 10^{-9}$	$1.7\ 10^{-8}$	$2.5\ 10^{-9}$	$1.25\ 10^{-8}$	$2.5\ 10^{-8}$
$n_{\omega y}$	$1.3\ 10^{-11}$	$6.5\ 10^{-11}$	$1.3\ 10^{-10}$	$1.25\ 10^{-11}$	$6.25\ 10^{-11}$	$1.25\ 10^{-10}$
$n_{\omega z}$	$1.5\ 10^{-8}$	$7.5\ 10^{-8}$	$1.5\ 10^{-7}$	$6.2\ 10^{-7}$	$3.1\ 10^{-6}$	$6.2\ 10^{-6}$

Tab. 3 Required $\sqrt{\mathrm{PSD}}$ of the noise degrading the estimated angular velocity. A white noise is assumed. Unit: rad/s/$\sqrt{\mathrm{Hz}}$

- To limit the attitude error impact to 2x10^{-5} E /√Hz, the related Euler angles for a (XYZ) rotation must be determined with a noise below (1.78, 0.1, 1.78) arcsec/√Hz, resp.
- The de-aliasing solution error comes from the ESA ESM model and is set to its nominal level (see Fig. 7).

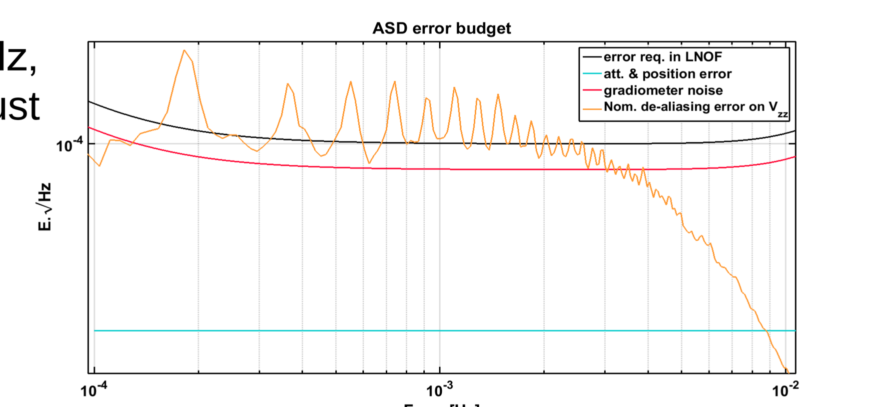

Fig. 7 Square-root of the PSDs of the error contributor in step 2.

Discussion & outlook

- For the low orbit, an error level of 0.1 mE/√Hz in the measurement bandwidth for the gradients in the LNOF is sufficient to recover the time-variable gravity field up to ≈ d/o 40 with a positive SNR. A level 1 mE/√Hz does not improve the resolution and precision of the recovered solution compared to GRACE.
- The requirement of 0.1 arcsec/√Hz for the knowledge of the rotation angle w.r.t the cross-track axis of the satellite is very challenging contrary to the one on the 2 others rotations which already was met in the case of GOCE. If not achievable, then only V_{yy} could be determined in the LNOF with the specified precision.
- The radial position of the satellite must be known with a 1-cm accuracy if V_{zz} is to be determined with the specified precision, which is not achievable for the moment.
- The required noise level degrading the measured angular velocity is beyond the performance of state-of-the-art space gyrometer, except for the case of the measurement of Vyy on the higher orbit and with noise (2) and (3).
- The error on the de-aliasing solutions must be reduced for frequencies smaller than 2x10^{-3} Hz.
- The requirement of the CMRR is achieved in along-track due to the drag free system. For cross-track and radial direction improvements are necessary, either in the calibration method or by employing some the drag free and attitude control.