Comparison of genetic algorithm and descend direction algorithm for SST-data

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The gravity field of the Earth is usually represented in spherical harmonics, which causes problems in the regional improvement. Therefore the signal and the model are separated into a global and a local part by subtracting synthetic observations of a known gravity field from the measurement. The residual signal is analyzed by radial base functions with different scale factors, positions and shapes. The residual analysis could be achieved by a linear adjustment of the scale factors – fixing the shapes and the positions a priori or by solving a non-linear optimization problem for several parameters per base function. The latter method allows a reduction of the amount of base functions and stabilizes the solution. For the non-linear problem a local optimization is implemented, improving an initial guess of the parameters by an iterative trust region algorithm. A genetic algorithm is tested as global optimization method, which generates a population of possible solutions and improves them according to stochastic rules of evolution.

The residual signal is modeled by a superposition of radial base functions (RBF). Their potential can be described by a sum of radial base functions (RBF). Their potential can be described by a sum of radial base functions (RBF). Their potential can be described by a sum of radial base functions (RBF). Their potential can be described by a sum of radial base functions (RBF).

$$\psi_i(x, v) = \sigma_i \psi_i(x, r) = \sigma_i \phi_i(r) \psi_i(\eta)$$

The argument is depending on the spherical distance $\rho$ and the center $(\lambda_0, \phi_0)$ of the base function. Every potential function $\psi_i$ contains the following parameters $\psi_i$:

- $\sigma_i$: Scale factor of the base function
- $\phi_i$: Shape parameter, i.e. a sequence of real values (here: exponential model: $\sigma_n = \sigma_0^n$ for $n \leq N$)
- $\lambda_0, \phi_0$: Center of the base function

In this study, GRACE-like observations (range $\mu$ and range-rate $\nu$) are transformed into the line-of-sight (LOS) gradient [2, 3]. The functional model is given by

$$\sum_i \psi_i(x, v) \frac{\partial}{\partial \eta} [\nabla \psi_i(x, v)] = \sum_i \psi_i(x, v) \frac{\partial}{\partial \eta} [\nabla \psi_i(x, v)]$$

neglecting the non-gravitational and time dependent effects. Additionally to the observations, the equation contains the following elements:

- $\nabla \psi_i$: LOS-derivative of the gradient field at position $x_i$ or $x_f$
- $\lambda_{LOS}$: Relative velocity in the LOS-direction
- $\phi_{LOS}$: Unit vector in LOS-direction
- $\eta$: Radius to the barycenter of the two satellites
- $\lambda$: Semimajor axes of the Keplerian ellipse (of the barycenter)
- $\phi$: Argument of latitude of the barycenter

2. Non-linear optimization

Non-linear optimization problems can be solved by global or local methods. In many cases, the squared sum of the residuals

$$\sum_{t} (\epsilon(t) - f(x(t)))^2$$

is used as objective function of the minimization.

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Conditions could be realized by a penalty term in the objective function or by modifications of the algorithm.

2. Trust region algorithm

Local methods require initial values for all parameters and minimize the objective function in a descent direction. The trust region method is based on the least squares adjustment of a linearized model, but it enables a restriction of each parameter into an interval. The analysis of SST-data in this project consists in the following steps (Figure 1):

1. Initial positions of the RBF are determined by image processing of the interpolated data in the orbit (extrema of the signal).
2. An initial shape parameter and its mathematical model is chosen.
3. The scale factors are estimated by a linear adjustment.
4. All parameters are optimized by a trust region algorithm.
5. Non accepted base functions are removed.
6. The scale factors of the remaining RBF are estimated.

Solutions mutate by adding random numbers to the parameters. The individuals for mutation and combination are chosen arbitrarily from the previous generation, but better solutions are preferred by a weighting in the stochastic process of selection.

To achieve a better fit between the observation and the model, a hybrid genetic algorithm is developed. The non-linear parameters $(\sigma_i, \lambda_0, \phi_0)$ are determined by the genetic algorithm, while the scale factors $\psi_i$ are estimated by linear adjustment. The condition number of the normal equation is used for a penalty term

$$\sum_{i} (\epsilon_i - f_i) = \sum_{i} (\epsilon_i - f_i)$$

2.2 Genetic algorithm

Genetic algorithms (GA) are a simple and general method for global optimization. They simulate natural evolution for a set (= generation) of possible solutions (= individuals/parents) [1]. The solutions are sorted according to a scalar fitness, which is determined by the objective function. The next generation is produced from the previous one by the following rules:

- The best solutions remain in the next generation.
- Previous solutions are combined by interchanging values at random positions of the vector.

4. Results and outlook

In the experiment, the radial base functions are developed up to the degree $N = 120$. The genetic algorithm is tested with 5, 10, 15, 20 and 25 radial base functions, using 50 individuals and 10 generations. This choice requires more than 5000 evaluations of the objective function, which increases the calculating time up to 1 - 2 h.

Both methods are able to approximate the signal in the orbit, but the result of the local optimization is much faster and provides a better fit (cf. Figure 3 and Table 1). The solutions and the calculating time of the genetic algorithm depend on the choices of the user, like the number of base functions, the population size or the number of generations. Local optimization should be preferred, if an initial guess of the parameters and the derivatives of the observation are available (cf. Table 2).