Comparison of genetic algorithm and descend direction algorithm for SST-data

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Abstract

The gravity field of the Earth is usually represented in spherical harmonics, which causes problems in the regional improvement. Therefore the signal and the model are separated into a global and a local part by subtracting synthetic observations of a known gravity field from the measurement. The residual signal is analyzed by radial base functions with different scale factors, positions and shapes.
The residual analysis could be achieved by a linear adjustment of the scale factors – fixing the shapes and the positions a priori – or by solving a non-linear optimization problem for several parameters per base function. The latter method allows a reduction of the amount of base functions and stabilizes the solution.
For the non-linear problem a local optimization is implemented, improving an initial guess of the parameters by an iterative trust region algorithm. A genetic algorithm is tested as global optimization method, which generates a population of possible solutions and improves them according to stochastic rules of evolution.

1. Modeling of SST-data

THE residual signal is modeled by a superposition of radial base functions (RBF). Their potential can be described by a sum of Legendre polynomials:

$$\Psi_b(\vec x_e,\psi_b) = \eta_b \frac{GM}{R}\sum_{n=n_0}^{\tilde N}\left(\frac{R}{r}\right)^{n+1}\sigma_b(n)P_n(\cos\varpi_b). \quad (1)$$

The argument is depending on the spherical distance ϖ_b between the calculating point $\vec x_e$ and the center (λ_b,ϑ_b) of the base function. Every potential function Ψ_b contains the following parameters ψ_b:

- η_b: scale factor of the base function,
- $\sigma_b(n)$: the shape parameter, i.e. a sequence of real values, (here: exponential model: $\sigma_b(n)=(\sigma_b)^n$ for $n_0 \le n \le \tilde N$)
- (λ_b,ϑ_b): center of the base function.

In this study, GRACE-like observations (range ρ and range-rate $\dot\rho$) are transformed into the line-of-sight (LOS) gradient [2, 3]. The functional model is given by

$$\overbrace{\frac{\ddot\rho}{\rho}+\frac{\dot\rho^2}{\rho^2}-\frac{\|\dot{\vec X}_{LOS}\|}{\rho^2}-\frac{\langle\nabla T_{ref}(\vec x_1)-\nabla T_{ref}(\vec x_2),\vec e_{LOS}\rangle}{\rho}}^{\text{observation:}\vec f} = \underbrace{\frac{1}{a}\frac{\partial\left\{\sum_{b=1}^{B}\Psi_b(\vec x_e,\psi_b)\right\}}{\partial r}+\frac{1}{a^2}\frac{\partial^2\left\{\sum_{b=1}^{B}\Psi_b(\vec x_e,\psi_b)\right\}}{\partial u^2}}_{\text{model/approximation:}\tilde f_B(\psi_1,\dots,\psi_B)}, \quad (2)$$

neglecting the non-gravitational and time depending effects. Additionally to the observations, the equation contains the following elements:

- $\nabla T_{ref}(\vec x_i)$: gradient of the reference field at position $\vec x_1$ or $\vec x_2$,
- $\dot{\vec X}_{LOS}$: relative velocity in the LOS-direction,
- $\vec e_{LOS}$: unit vector in LOS-direction,
- r : radius to the barycenter of the two satellites,
- a : semimajor axes of the Keplerian ellipse (of the barycenter),
- u : argument of latitude of the barycenter.

2. Non-linear optimization

NON-linear optimization problems can be solved by global or local methods. In many cases, the squared sum of the residuals

$$\vec v := \tilde f_B(\psi_1,\dots\psi_B) - \vec f \quad (3)$$

is used as objective function of the minimization:

$$\Xi(\psi_1,\dots,\psi_B) = \|\tilde f_B(\psi_1,\dots\psi_B)-\vec f\|^2 = \vec v^\top\vec v \to \min. \quad (4)$$

Conditions could be realized by a penalty term in the objective function or by modifications of the algorithm.

2.1 Trust region algorithm

Local methods require initial values for all parameters and minimize the objective function in a descend direction. The trust region method is based on the least squares adjustment of a linearized model, but it enables a restriction of each parameter into an interval. The analysis of SST-data in this project consists in the following steps (Figure 1):

1. Initial positions of the RBF are determined by image processing of the interpolated data in the orbit (extrema of the signal).
2. An initial shape parameter and its mathematical model is chosen.
3. The scale factors are estimated by a linear adjustment.
4. All parameters are optimized by a trust region algorithm.
5. Non accepted base functions are removed.
6. The scale factors of the remaining RBF are estimated.

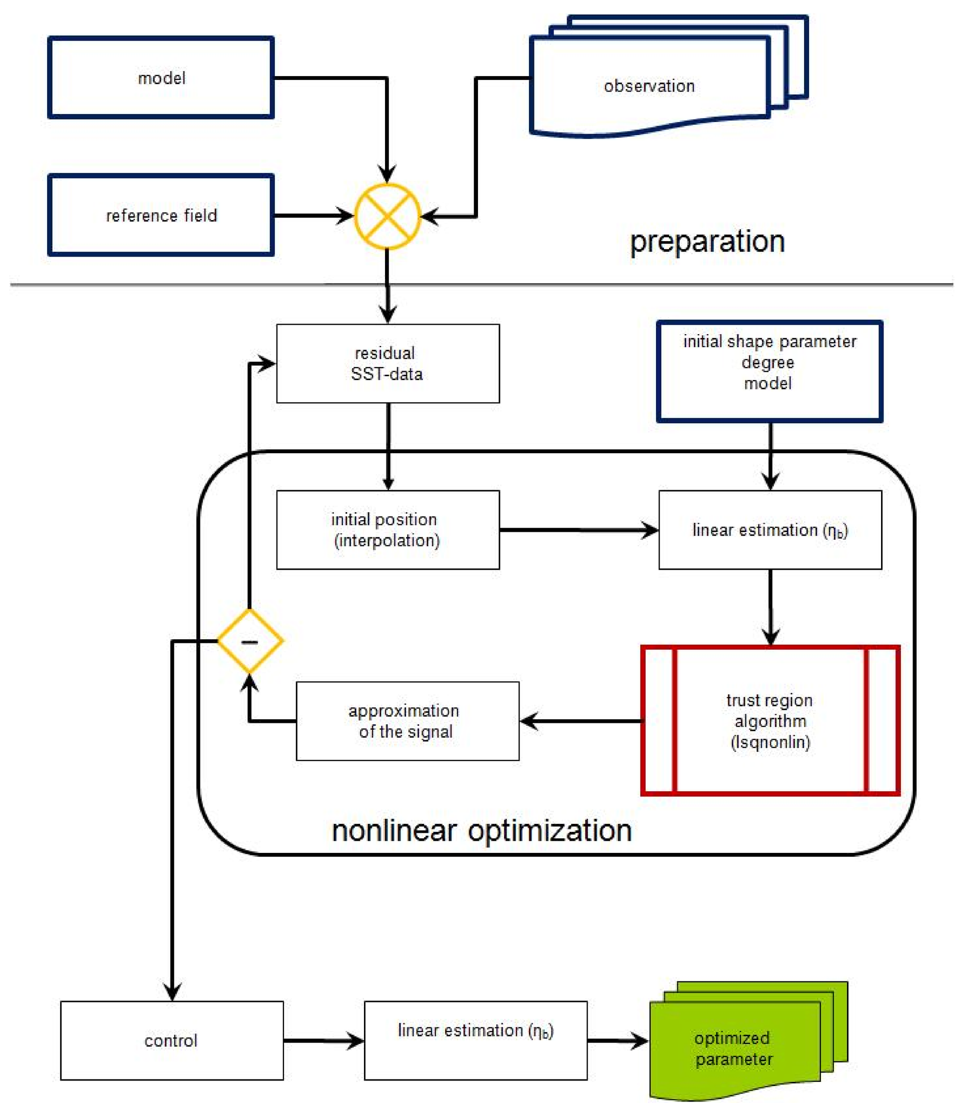

Figure 1: *Fitting residual SST-data in the orbit*

2.2 Genetic algorithm

Genetic algorithms (GA) are a simple and general method for global optimization. They simulate natural evolution for a set (= generation) of possible solutions (= individuals/parents) [1]. The solutions are sorted according to a scalar fitness, which is determined by the objective function. The next generation is produced from the previous one by the following rules:

→ The best solutions remain in the next generation.

→ Previous solutions are combined by interchanging values at random positions of the vector.

→ Solutions mutate by adding random numbers to the parameters.

The individuals for mutation and combination are chosen arbitrarily from the previous generation, but better solutions are preferred by a weighting in the stochastic process of selection.

To achieve a better fit between the observation and the model, a hybrid genetic algorithm is developed. The non-linear parameters $\{\sigma_b,\lambda_b,\vartheta_b\}$ are determined by the genetic algorithm, while the scale factors η_b are estimated by linear adjustment. The condition number κ of the normal equation is used for a penalty term

$$\Xi(\psi_1,\dots,\psi_B) = \vec v^\top\vec v + \begin{cases}\log\kappa(\mathbf{A}^\top\mathbf{A})\times\frac{\|\vec f\|^2}{100} & \text{if } \kappa\ge 10^8\\ 0 & \text{else,}\end{cases} \quad (5)$$

to avoid an ill-posed system.

3. Simulated observations

THE approach is tested in a GRACE-like scenario, using the EGM96 up to degree $N=120$ as reference field. In the orbit integration 5 radial base functions are added in terms of spherical-harmonic coefficients to generate the disturbed gravity field. A region of interest with 4080 observations is selected from the signal. Figure 2 shows the potential and the LOS-gradient in the orbit after subtracting the effects of the reference field at the integrated positions.

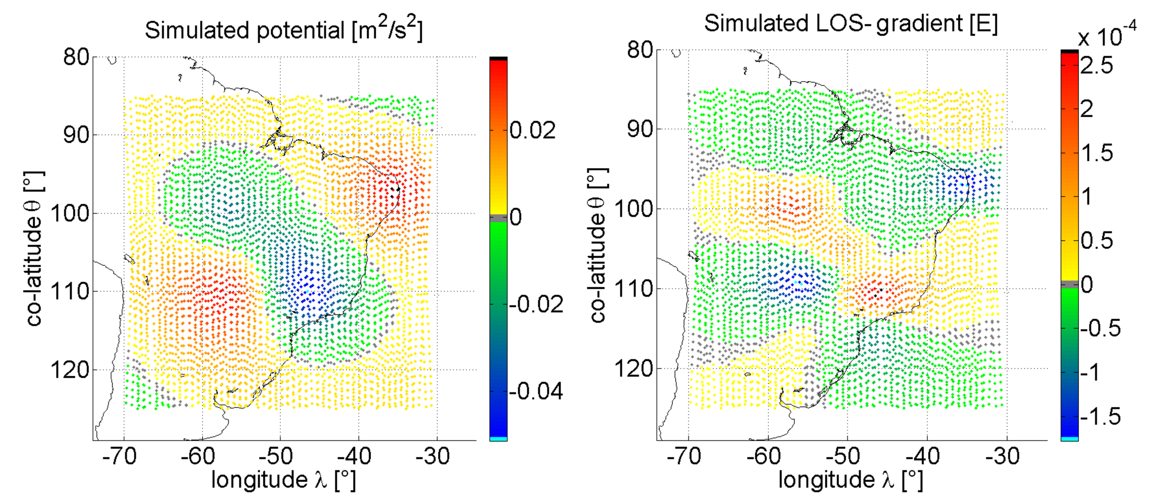

Figure 2: *Disturbing potential and LOS-gradient in the orbit*

4. Results and outlook

IN the experiment, the radial base functions are developed up to the degree $\tilde N=120$. The genetic algorithm is tested with 5, 10, 15, 20 and 25 radial base functions, using 50 individuals and 100 generations. This choice requires more than 5000 evaluations of the objective function, which increases the calculating time up to $1-4.2$ h.

The trust region method detects 18 base functions located at the extrema of the interpolated signal. After 45 iterations and around 5 min the algorithm is stopped, as the relative improvement of the objective function is less than the default threshold of 10^{-6}.

	MAX [mE]	MIN [mE]	STD [mE]	correlation [%]
trust region	0.0380	−0.0397	0.0079	98.9
GA (5 RBF)	0.2009	−0.1205	0.0302	81.5
GA (10 RBF)	0.0797	−0.0898	0.0163	95.0
GA (15 RBF)	0.0762	−0.0549	0.0139	96.4
GA (20 RBF)	0.1080	−0.0788	0.0150	95.8
GA (25 RBF)	0.0577	−0.0380	0.0091	98.5

Table 1: *statistic of the differences between the observation and the approximation (observation: MAX: 0.268 mE, MIN: -0.178 mE, STD: 0.0521 mE)*

Both methods are able to approximate the signal in the orbit, but the result of the local optimization is much faster and provides a better fit (cf. Figure 3 and Table 1). The solutions and the calculating time of the genetic algorithm depend on the choices of the user, like the number of base functions, the population size or the number of generations. Local optimization should be preferred, if an initial guess of the parameters and the derivatives of the observation are available (cf. Table 2).

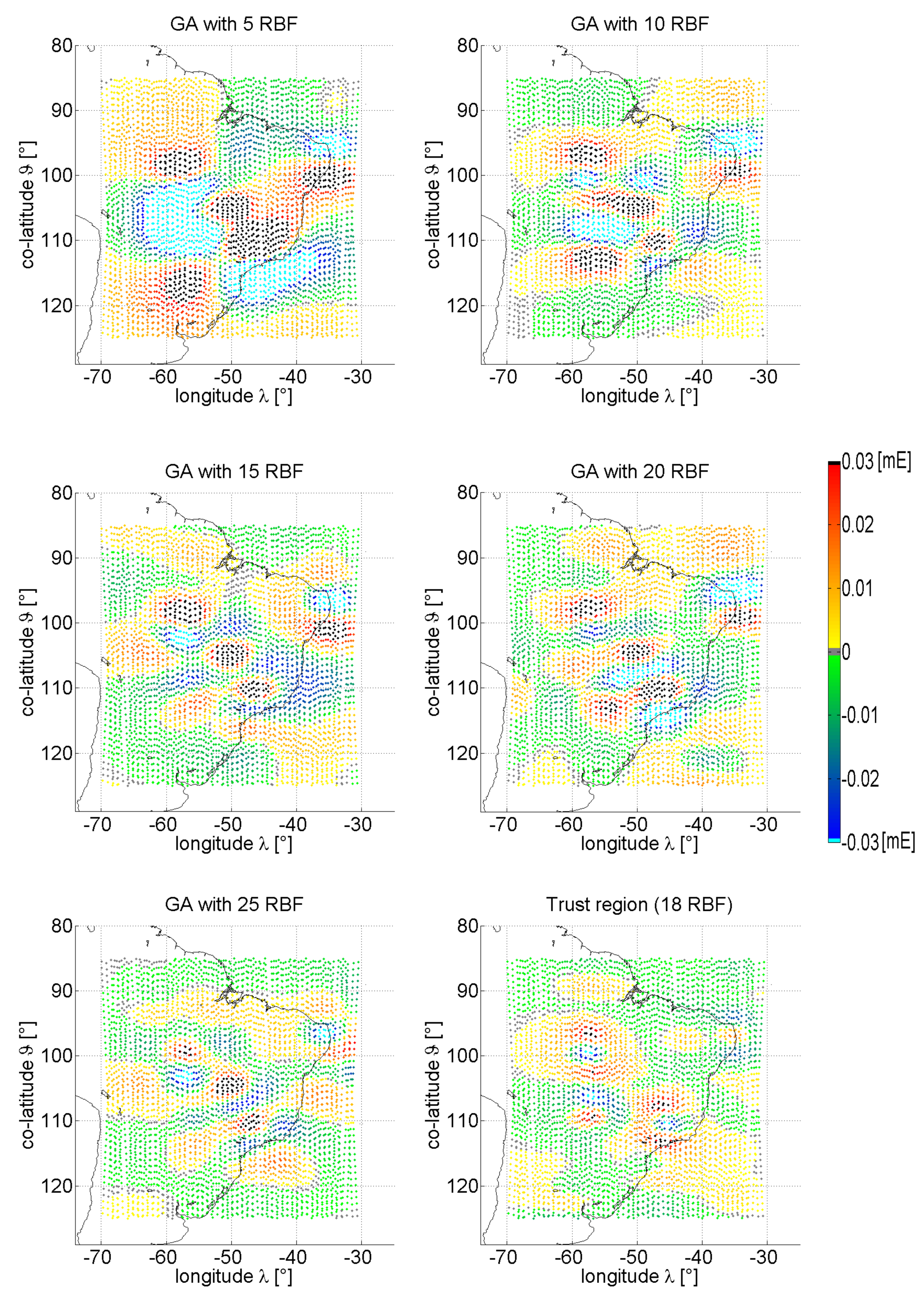

Figure 3: *Final error between the model and the signal in [mE] (The common colorbar is cut-off at ±0.03 [mE])*

	trust region	genetic algorithm
initial values	necessary	not necessary
derivatives	necessary	not necessary
number of RBF	"on the fly"	a priori choice
number of evaluations	iteration	iteration × population size
calculating time	5 min	1 – 4.2 h
repetition possible	yes	no (stochastic processes)

Table 2: *Local optimization (trust region) vs. global optimization (genetic algotithm)*

References

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