Recovery of residual GRACE-observations by radial base functions

MARKUS ANTONI, Wolfgang Keller, Matthias Weigelt

Geodetic Institute, University of Stuttgart

markus.antoni@gis.uni-stuttgart.de

Abstract

Gravity fields are mainly represented in spherical harmonics, which causes problems in the regional improvement. Therefore the signal and the model are separated into a global and a local part, by subtracting a synthetic observation of a reference field from the measurement. The residual signal is analyzed by radial base functions, each of them described by a scale factor, a shape parameter and a position. To avoid an over-parametrization a non-linear algorithm is used to optimize these parameters for a minimal number of base function.

1. Optimized radial base functions

THE potential of a radial base function in the (earth-fixed) orbit $\vec{x}_e$ can be described by a sum of Legendre polynomials [1]:

$$\Psi_b(\vec{x}_e, \psi_b) = \eta_b \frac{GM}{R} \sum_{n=0}^{\tilde{N}} \left(\frac{R}{r}\right)^{n+1} \sigma_b(n) P_n(\cos \varpi_b) \quad (1)$$

The argument ψ_b contains the parameter η_b (scale factor), $\sigma_b(n)$ (shape-parameter), and the position of the center (λ_b, ϑ_b). Latter one is hidden in the spherical distance ϖ_b between center and calculating point. For the shape parameter an exponential model $\sigma_b(n) = \sigma_b^n$ is chosen, so that the behavior is described by a single value per base function.

Two kinds of observations are modeled in the study:

- the energy integral, where the residual potential δT in the orbit is calculated by the superposition:

$$\delta T(\vec{x}_e, \psi_1, \psi_2, ..., \psi_B) = \sum_{b=1}^{B} \Psi_b(\vec{x}_e, \psi_b) \quad (2)$$

- the line-of-sight gradient (LOS), i.e. second derivate in flight direction of GRACE, which is generated by applying the differential operator

$$\frac{1}{a^2}\frac{\partial^2}{\partial u^2} + \frac{1}{a}\frac{\partial}{\partial r} \quad (3)$$

  to the field δT, using the argument of latitude u and the semi-major axis a of the Kepler-ellipse [2, 3].

Both kind of observations can be described pointwise as function of the unknown parameters ψ_b and in closed formulas, if the orbit integration is done in a previous step.

2. Methodology

IN order to optimize the parameters of the base functions, the following steps are necessary (cf. figure 1):

1. calculating a residual signal by subtracting a synthetic observation of the reference field
2. finding initial positions and shape parameters
3. linear adjustment to estimate proper scale factors
4. nonlinear Levenberg-Marquardt algorithm to optimize all parameters
5. comparing/subtracting the residual signal and the approximation
6. repetition of the steps $2-6$ until the approximation is "'good enough"'
7. removing non accepted base functions and linear adjustment of the scale factor of the remaining ones

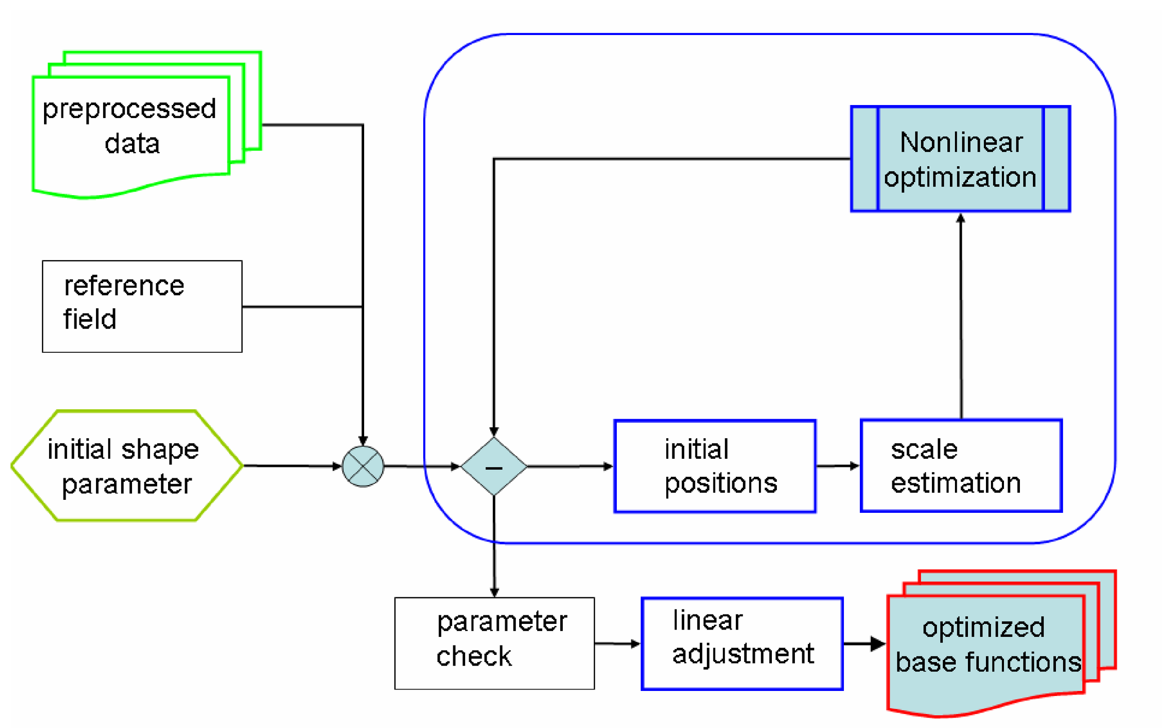
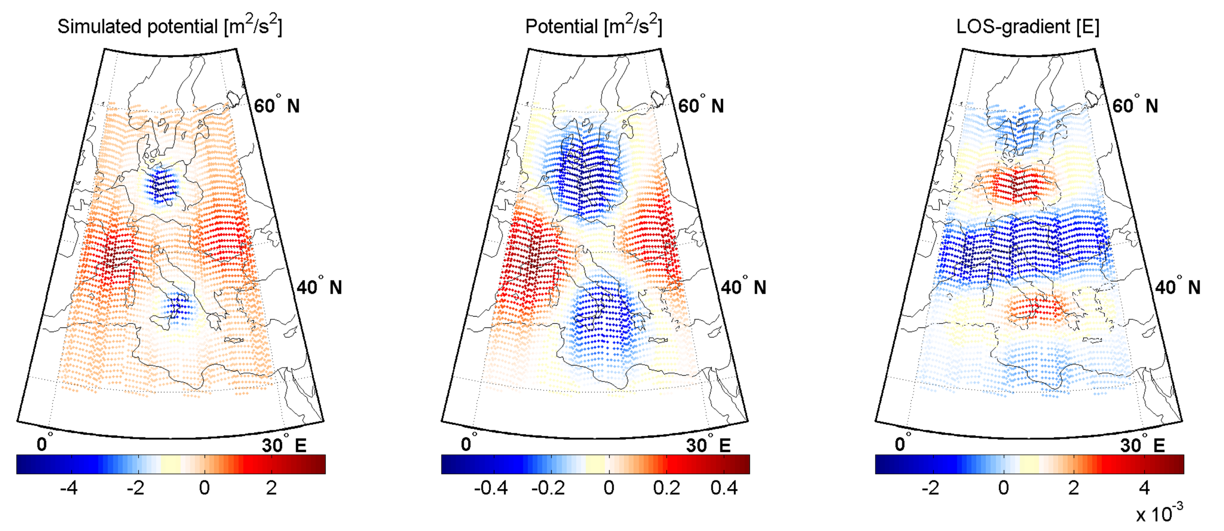


Figure 1: *Workflow of the nonlinear optimization*

For pointwise observations the position of additional radial base functions should be evident in the orbit as well. In several simulations it turned out that the best localization is close to the maxima and minima of the residual signal. Therefore the data are interpolated and smoothed in the orbit to find the initial positions of a small number of base functions.

3. Simulated observations

THE approach is tested by a GRACE-like scenario, using the EGM96 up to degree $N = 150$ as reference field. Before the orbit integration some additional radial base functions added to the field in terms of spherical-harmonic coefficients, to generate a disturbed signal.



(a) potential on the ground (b) potential in the orbit (c) LOS-gradient in the orbit

Figure 2: *Residual signal caused by 4 base functions*

Figure 2(a) illustrates the residual potential on the ground caused by four additional base functions, while the effects in the orbit are shown in the figures 2(b) and 2(c).

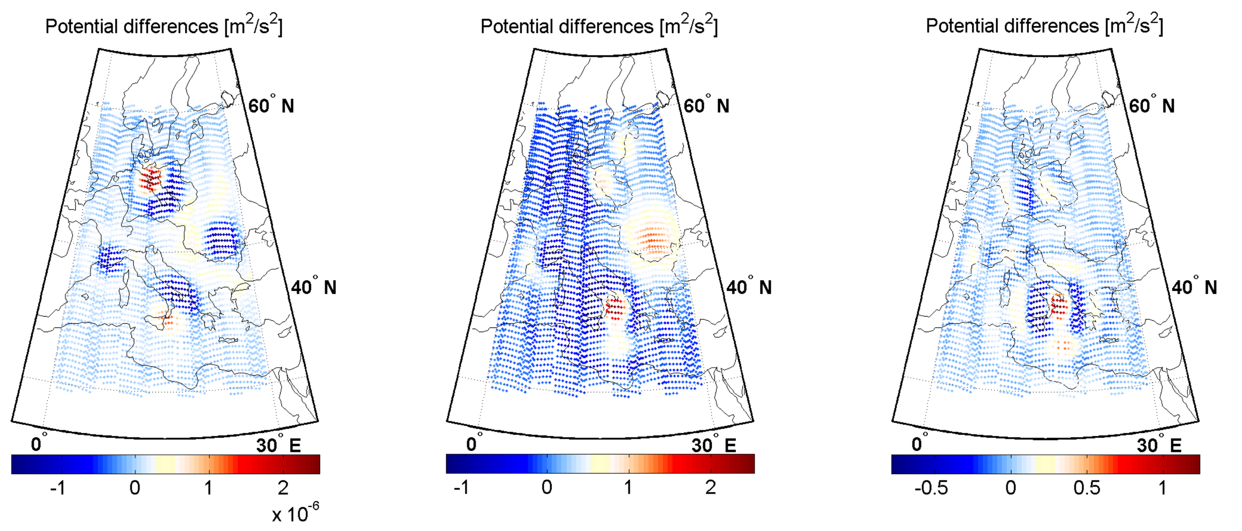
4. Results and outlook

AFTER the optimization the parameters ψ_b can be used to calculate an approximation of the observation in the orbit and the potential on the ground. By comparing the signal and the approximation in orbit, the best solution is achieved by the energy integral (cf. table 1). One reason is the clearly separated signal of the disturbing radial base functions in the orbit, so that the initial values are very good. The solution of the LOS-gradient suffers from the insensitiveness across the flight direction and an unsolved ambiguity. Nevertheless the correlation coefficient in the orbit is already 0.97 and can be improved by using a second iteration.

	Potential $[\frac{m^2}{s^2}]$	LOS-gradient (1.step)[E]	LOS-gradient (2.steps)[E]
max(signal)	4.89×10^{-1}	5.11×10^{-3}	5.11×10^{-3}
min(signal)	-5.84×10^{-1}	-3.66×10^{-3}	-3.66×10^{-3}
mean(signal)	-3.46×10^{-2}	8.89×10^{-5}	8.89×10^{-5}
std(signal)	2.03×10^{-1}	1.28×10^{-3}	1.28×10^{-3}
max(diff.)	5.87×10^{-7}	9.07×10^{-4}	1.81×10^{-4}
min(diff.)	-7.10×10^{-7}	-1.21×10^{-3}	-1.88×10^{-4}
mean(diff.)	4.85×10^{-10}	-1.51×10^{-5}	-3.22×10^{-7}
std(diff.)	1.86×10^{-7}	2.87×10^{-4}	3.30×10^{-5}
correlation	1.0000	0.9746	0.9997

Table 1: *Analysis of the signal and the difference (diff.) to the approximation in the orbit*

The difference between the estimated and the simulated potential on the ground is shown in figure 3 and analyzed in table 2 in more details.



(a) energy integral (b) LOS-gradient 1. step (c) LOS-gradient 2. step

Figure 3: *Difference between the simulated and the estimated potential on the ground*

	Potential	LOS-gradient (1.step)	LOS-gradient (2.steps)
max(diff.)	2.49×10^{-6}	2.52×10^{0}	1.23×10^{0}
min(diff.)	-1.68×10^{-6}	-1.23×10^{0}	-7.84×10^{-1}
mean(diff.)	-7.06×10^{-9}	4.02×10^{-2}	3.62×10^{-3}
std(diff.)	3.52×10^{-7}	3.70×10^{-1}	1.26×10^{-1}
correlation	1.0000	0.9272	0.9912

Table 2: *Difference between the estimated and the simulated potential on the ground, compared to the real field (min $= -5.7$, max $= 3.6$, std $= 9.4 \times 10^{-1}$ and mean $= -4.9 \times 10^{-2}$)*

The calculating time is less then 3 minutes for all optimizations and can be neglected compared to effort of the orbit integration. Another remarkable point is the small number of base functions for the approximation and the distances ρ between the positions of the simulated and the estimated base functions (cf. table 3).

	Potential	LOS-gradient (1.step)	LOS-gradient (2.steps)
calc. time	30 s	38 s	153 s
iterations	8	9	9 + 16
base functions	4	5	20
distance ρ [°]	$0.00 - 0.14$	$0.30 - 8.83$	$0.08 - 0.47$

Table 3: *Performance for the example of 3750 data points and a maximal degree of $\tilde{N} = 100$*

References

[1] A. Eicker: *Gravity Field Refinement by Radial Basis Functions from In-situ Satellite Data*, PhD thesis, Rheinische Friedrich-Wilhelms-Universitt Bonn, 2008

[2] R. Koop: *Global gravity modelling using satellite gravity gradiometry*, Technical report 38, Delft, 1993

[3] M. Weigelt, M. Antoni and W. Keller: *Regional gravity field recovery from GRACE using position optimized radial base functions* Proceedings of the GGEO 2008 (submitted)