Canonical Correlation Analysis (CCA) of GRACE, hydrological and hydro-meteorological signals

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Geodetic Week, Oct. 2010
GRACE, Hydrology and Hydro-meteorology

\[ P - ET_a - R = \frac{dS_H}{dt} \leftrightarrow \frac{dM}{dt} \leftrightarrow \frac{dS_A}{dt} = -\nabla.Q - R \]

Hydrology

GRACE

Hydro-meteorology

![Graph showing time series data for different parameters over months from January 2003 to January 2007.](image-url)
GRACE, times series, monthly mean and residual

Residual contains:
- Error
- Physical variations
EOF analysis

\[ A = U \Sigma V^T \]

- \( U \) : Eigenvectors of matrix \( AA^T \)
- \( \Sigma \) : Eigenvalues of matrix \( A^T A \) or \( AA^T \)
- \( V \) : Eigenvectors of matrix \( A^T A \)
EOF analysis

\[ \frac{dM}{dt} = U_G \Sigma_G V_G^T \]

GRACE

\[ P = U_p \Sigma_p V_p^T \]

Precipitation

\[ \nabla \cdot Q = U_D \Sigma_D V_D^T \]

Vertically integrated moisture flux divergence

![Graph showing EOF analysis results]
Canonical Correlation Analysis (CCA)

\[ A = U_A \Sigma_A V_A^T \]
\[ B = U_B \Sigma_B V_B^T \]

\[ \Sigma_{AB} = U_A^T A_{AB}^T B V_{AB} \]

\[ R \Sigma_{AB} = (U_A^T U_B) S \]

- Measuring the linear relationship between two multi dimensional variables
- Finding two sets of basis vectors such that the correlation between the projections of the variables onto these basis vectors is maximized
- Determine Correlation Coefficients
- Correlation coefficients: Proportion of correlation between the canonical variates accounted for the particular variable
- Correlation coefficient represents unique contribution of each variable to relation
CCA of GRACE $\left(\frac{dM}{dt}\right)$ and Divergence $\nabla \cdot Q$

\[ \left(\frac{dM}{dt}\right)^T \nabla \cdot Q = U_{GD} \Sigma_{GD} V_{GD}^T \]
CCA of GRACE $\frac{dM}{dt}$ and Divergence $\nabla \cdot Q$

\[
\left( \frac{dM}{dt} \right)^T \nabla \cdot Q = U_{GD} \Sigma_{GD} V_{GD}^T
\]

Mode 2

Correlation coefficient : 88%
CCA of GRACE \( \left( \frac{dM}{dt} \right)^T \) and Precipitation \((P)\)

\[
P = U_{GP} \Sigma_{GP} V_{GP}^T
\]
CCA of GRACE \( \frac{dM}{dt} \) and Precipitation \( P \)

\[
\left( \frac{dM}{dt} \right)^T P = U_{GP} \Sigma_{GP} V_{GP}^T
\]

Mode 2

Correlation coefficient : 29%

Projection of \( U_{GP} \)

Projection of \( V_{GP} \)

Graphs showing the projection of \( U_{GP} \) and \( V_{GP} \) on a world map, with time series plots below for Jan 2004 to Jan 2008.
CCA on catchments based signals
CCA on catchments based – GRACE and hydro-meteorology

\[
\left( \frac{dM}{dt} \right)^T \nabla \cdot Q = U_G \Sigma_G V_G^T
\]
CCA on catchments based – GRACE and hydro-meteorology

\[ \frac{dM}{dt} \leftrightarrow -\nabla \cdot Q - R \quad \text{Residual} \]
CCA on catchments based – GRACE and precipitation

\[ \frac{dM}{dt}^T P = U_{GP} \Sigma_{GP} V_{GP}^T \]
CCA on catchments based – GRACE and Precipitation

\[
\frac{dM}{dt} \quad \leftrightarrow \quad P - R \quad \text{Residual}
\]
Summary and Outlook

Summary

• CCA was performed on GRACE, hydrological and hydro-meteorological signals
• Selecting the 75% of correlation of GRACE, hydrological and hydro-meteorological signals leads to:
  • Improvement in correlation of signals
  • Decreasing the RMS of residual
• The correlation of GRACE, hydrological and hydro-meteorological residuals does not show improvement or deterioration

Outlook

• Applying CCA on different climate regions
• Using other source of data sets like temperature
Thanks for your attention