Accounting for Leakage Effects in GRACE-Derived Mass Estimates

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Outline

- GRACE-derived mass changes
- Leakage effects
- Accounting for leakage by forward gravity modelling
- Simulation study 1: Greenland only
- Simulation study 2: Greenland / Arctic
- GRACE-derived ice mass changes over Greenland
- Conclusions
GRACE-derived mass changes

- GRACE “monthly” gravity field solutions
  \( c_{lm}, s_{lm} \)

- “monthly” residuals
  \( \Delta c_{lm}, \Delta s_{lm} \)

- “de-striping”
  \( \Delta c_{lm}^{\text{fil}}, \Delta s_{lm}^{\text{fil}} \)

- Spatial averaging
  \( W_i \Delta c_{lm}^{\text{fil}}, W_i \Delta s_{lm}^{\text{fil}} \)

- Equivalent Water Thickness
  \( EWT(\lambda, \varphi) \)

- Frequently used procedure
- Spherical harmonic representation of geopotential
- Application of filters
- Mass estimates via Newton’s integral expressed in spherical harmonics
- Spatial and spectral leakage
GRACE-derived mass changes

GRACE “monthly” gravity field solutions
\[ c_{lm}, s_{lm} \]

“monthly” residuals
\[ \Delta c_{lm}, \Delta s_{lm} \]

“de-striping”
\[ \Delta c_{lm}^{fil}, \Delta s_{lm}^{fil} \]

Spatial averaging
\[ W_i \Delta c_{lm}^{fil}, W_i \Delta s_{lm}^{fil} \]

Equivalent Water Thickness
\[ EWT(\lambda, \varphi) \]

Short-term geopotential variations in terms of spherical harmonic coefficients (SHC)

Data used:
- CSR, GFZ, JPL
- RL04 GRACE-only
- up to degree/order 60
- August 2002 to July 2008 (6 years)
**GRACE-derived mass changes**

- **GRACE “monthly” gravity field solutions**
  \[ c_{lm}, s_{lm} \]

- **“monthly” residuals**
  \[ \Delta c_{lm}, \Delta s_{lm} \]

- **“de-striping”**
  \[ \Delta c^\text{fil}_{lm}, \Delta s^\text{fil}_{lm} \]

- **Spatial averaging**
  \[ W_i \Delta c^\text{fil}_{lm}, W_i \Delta s^\text{fil}_{lm} \]

- **Equivalent Water Thickness**
  \[ EWT(\lambda, \varphi) \]

Residuals relative to the 6-years average:
\[ \Delta c_{lm} = c_{lm} - c_{lm}^{\text{mean}} \]
\[ \Delta s_{lm} = s_{lm} - s_{lm}^{\text{mean}} \]

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GRACE “monthly” gravity field solutions

\[ c_{lm}, s_{lm} \]

“monthly” residuals

\[ \Delta c_{lm}, \Delta s_{lm} \]

“de-striping”

\[ \Delta c_{lm}^{fil}, \Delta s_{lm}^{fil} \]

Removal of correlated errors in the spectral domain (Swenson & Wahr 2006):

spatial averaging

\[ W_l \Delta c_{lm}^{fil}, W_l \Delta s_{lm}^{fil} \]

Equivalent Water Thickness

\[ EWT(\lambda, \varphi) \]
GRACE-derived mass changes

**GRACE “monthly” gravity field solutions**

\[ c_{lm}, s_{lm} \]

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\[ \Delta c_{lm}, \Delta s_{lm} \]

**“de-striping”**

\[ \Delta c_{lm}^{\text{fil}}, \Delta s_{lm}^{\text{fil}} \]

**spatial averaging**

\[ W_i \Delta c_{lm}^{\text{fil}}, W_i \Delta s_{lm}^{\text{fil}} \]

Equivalent Water Thickness

\[ EWT(\lambda, \varphi) \]

- Gaussian smoothing applied with smoothing radius \( R = 500 \text{ km} \)
- Damping errors of high-degree SHC via weight factor \( W_i (\text{Jekeli 1981}) \)
- Removes most of the power beyond degree/order 30-40 (for \( R = 500 \text{ km} \))

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Surface mass changes (Wahr et al. 1998):

Equivalent Water Thickness

\[ EWT(\lambda, \varphi) = \]
\[
\frac{2\pi a \rho_{\text{ave}}}{3 \rho_w} \sum_{l=0}^{L} \frac{2l+1}{1+k_l} W_l \sum_{m=0}^{l} P_{lm} (\sin \varphi) \times
\]
\[
(\Delta c_{lm}^{\text{fil}} \cos m\lambda + \Delta s_{lm}^{\text{fil}} \sin m\lambda)
\]

Gravity inversion assumes surface mass changes only
GRACE-derived mass changes

GRACE-derived secular equivalent water thickness (EWT) variations
August 2002 to July 2008 (CSR)

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Leakage effects

Leakage problem:
- restricted spectral resolution of gravitational field estimates ($L=60$)
- spatial averaging (smoothing)
- Newton’s law: potential signal spreads with $1/r$

Leakage-out effects:
Signal leaks out of the region of interest (e.g., Greenland – globe)

Leakage-in effects:
Signal leaks in the region of interest (e.g., Canada – Greenland)
Leakage effects – A simple example

A simple example:

- Synthetic EWT change:
  - Disc at $\lambda = 180^\circ$, $\varphi = 0^\circ$
  - Radius = 10°, EWT = 0.1 m
- Spatial and spectral leakage

Restore synthetic EWT change:

- Express potential change in spherical harmonics and apply GRACE mass estimation procedure without “de-striping”
- Synthetic EWT change can completely be restored when using a high spherical harmonic degree (e.g. $N = 1200$)
  - no spatial smoothing / - no “de-striping”
Leakage effects – A simple example

**Spectral leakage:**
Low degree: N=60, no spatial smoothing

EWT change:
- 94.3 % of mass recovered
- 5.7 % leakage (spectral)

**Spatial leakage:**
Low degree: N=60, Gaussian smoothing (R=500 km)

EWT change:
- 70.8 % of mass recovered
- 29.2 % leakage
  - 23.5% spatial, 5.7% spectral
Accounting for leakage by forward gravity modelling

4-Step procedure:

- Based on potential forward modelling
- Steps 2 & 3 are intermediate to obtain more realistic input
- Leakage expressed by a single scale factor
- Spatial distribution taken from original result
- No GIA correction required

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Step 1: Extract initial mass information over land area only: $M_1$

Step 2: Extract initial mass information inside the extended area: $M_2$

Step 3: Intermediate amplification factor $IAF = \frac{M_2}{M_1}$

- Restore the mass-change $M_2$ back onto the land area:
  - $M_1^* = IAF \cdot M_1$

Step 4: Potential forward modeling from $M_1^*$

- Derive mass-change inside the extended area: $M_{2,syn}$

- Total mass $M_{2,syn}^* = M_2 \left(1 + \left[1 - \frac{M_{2,syn}}{M_2}\right]\right)$

- Signal loss (%) = $100\left(1 - \frac{M_{2,syn}}{M_2}\right)$

- Final amplification factor: $FAF = \frac{M_{2,syn}^*}{M_0}$

- Restore the total mass-change $M_{2,syn}^*$ back onto the land area:
  - $M_{1,syn} = FAF \cdot M_0$

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Baur, Kuhn & Featherstone JGR (2009)
Simulation study 1: Greenland only

Potential forward modelling

Study 1: Greenland, mass-change (input): -500 km$^3$
no disturbing signals considered
Simulation study 1: Greenland only

Potential forward modelling

Study 1: Greenland
no disturbing signals considered
Mass-change (input): -500 km$^3$

Mass-change estimates:

<table>
<thead>
<tr>
<th>Region</th>
<th>$M_2$ (km$^3$)</th>
<th>IAF</th>
<th>Signal loss (%)</th>
<th>$M_{1,syn}$ (km$^3$)</th>
<th>FAF</th>
<th>Error (%)</th>
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</thead>
<tbody>
<tr>
<td>Land area</td>
<td>-305.2</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>EA –0.07</td>
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Simulation study 1: Greenland only

Potential forward modelling

Study 1: Greenland
no disturbing signals considered
Mass-change (input): -500 km³

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<td>1.62</td>
<td>1.27</td>
</tr>
<tr>
<td>EA -0.05</td>
<td>-453.0</td>
<td>1.48</td>
<td>9.2</td>
<td>-464.4</td>
<td>1.62</td>
<td>1.11</td>
</tr>
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Simulation study 1: Greenland only

**Potential forward modelling**

Study 1: Greenland  
no disturbing signals considered  
Mass-change (input): -500 km$^3$

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<td>-464.4</td>
<td>1.62</td>
<td>1.11</td>
</tr>
<tr>
<td>EA –0.04</td>
<td>-477.9</td>
<td>1.57</td>
<td>3.7</td>
<td>-495.7</td>
<td>1.62</td>
<td>0.87</td>
</tr>
</tbody>
</table>

→ Little dependency on size of extended area (EA)

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Potential forward modelling

Study 2: Greenland, mass-change (input): -500 km³
disturbing signals considered

Disturbing signals:
- Canada
  +850 km³
- Fennoscandia
  +100 km³
- Alaska
  -300 km³
Simulation study 2: Greenland / Arctic

Potential forward modelling

Study 2: Greenland
disturbing signals considered
Mass-change (input): -500 km$^3$

Mass-change estimates for Greenland:

<table>
<thead>
<tr>
<th>Region</th>
<th>$M_2$ (km$^3$)</th>
<th>IAF</th>
<th>$M_{1,syn}$ (km$^3$)</th>
<th>Leak-in/out (%)</th>
<th>FAF</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land area</td>
<td>-299.6</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>EA -0.07</td>
<td>-392.2</td>
<td>1.31</td>
<td>-480.3</td>
<td>1.8 / 22.5</td>
<td>1.60</td>
<td>3.94</td>
</tr>
<tr>
<td>EA -0.06</td>
<td>-416.4</td>
<td>1.39</td>
<td>-480.8</td>
<td>2.1 / 15.5</td>
<td>1.60</td>
<td>3.84</td>
</tr>
<tr>
<td>EA -0.05</td>
<td>-440.0</td>
<td>1.47</td>
<td>-480.0</td>
<td>2.5 / 9.1</td>
<td>1.60</td>
<td>4.00</td>
</tr>
<tr>
<td>EA -0.04</td>
<td>-462.3</td>
<td>1.54</td>
<td>-479.2</td>
<td>3.0 / 3.7</td>
<td>1.60</td>
<td>4.15</td>
</tr>
</tbody>
</table>

→ Little dependency on size of extended area (EA)
GRACE-derived ice mass changes over Greenland

GRACE analysis – real data
August 2002 to July 2008
Baur, Kuhn & Featherstone JGR (2009)

Disturbing signals:
• Canadian Shield
• Alaska
• Fennoscandia

Extended area
Leakage effects
# GRACE-derived ice mass changes over Greenland

<table>
<thead>
<tr>
<th>Processing Centre</th>
<th>Radius R (km)</th>
<th>Land only (km³)</th>
<th>IAF</th>
<th>Leak-in/out (%)</th>
<th>FAF</th>
<th>Total Change (km³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSR</td>
<td>300</td>
<td>-781.3</td>
<td>1.94</td>
<td>1.1 / 47.1</td>
<td>1.85</td>
<td>-1447.0 ± 192.6</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>-751.3</td>
<td>1.92</td>
<td>1.2 / 49.0</td>
<td>1.92</td>
<td>-1440.5 ± 176.0</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>-713.6</td>
<td>1.91</td>
<td>1.3 / 52.1</td>
<td>2.03</td>
<td>-1449.3 ± 169.8</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>-670.3</td>
<td>1.90</td>
<td>1.5 / 55.9</td>
<td>2.19</td>
<td>-1469.9 ± 170.2</td>
</tr>
<tr>
<td>GFZ</td>
<td>300</td>
<td>-645.2</td>
<td>1.91</td>
<td>1.5 / 46.6</td>
<td>1.82</td>
<td>-1174.8 ± 296.1</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>-612.4</td>
<td>1.89</td>
<td>1.7 / 48.6</td>
<td>1.88</td>
<td>-1153.2 ± 196.2</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>-573.9</td>
<td>1.87</td>
<td>2.0 / 51.9</td>
<td>1.99</td>
<td>-1143.9 ± 169.4</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>-531.3</td>
<td>1.86</td>
<td>2.4 / 56.0</td>
<td>2.16</td>
<td>-1145.2 ± 166.2</td>
</tr>
<tr>
<td>JPL</td>
<td>300</td>
<td>-265.7</td>
<td>2.31</td>
<td>2.6 / 57.1</td>
<td>2.20</td>
<td>-584.0 ± 275.4</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>-264.3</td>
<td>2.17</td>
<td>3.0 / 56.6</td>
<td>2.16</td>
<td>-569.6 ± 168.8</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>-257.7</td>
<td>2.05</td>
<td>3.5 / 57.7</td>
<td>2.18</td>
<td>-561.9 ± 131.5</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>-245.6</td>
<td>1.97</td>
<td>4.3 / 60.4</td>
<td>2.28</td>
<td>-559.8 ± 116.0</td>
</tr>
</tbody>
</table>

- Little dependency on smoothing radius
- Huge dependency on processing centre
GRACE-derived ice mass changes over Greenland

GRACE-derived **annual ice-volume variations** (km³/yr) over Greenland, **no GIA correction applied**

<table>
<thead>
<tr>
<th>Study</th>
<th>Period</th>
<th>Change-rate without GIA</th>
<th>Change-rate with GIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>This study (CSR)</td>
<td>08/2002 – 07/2008</td>
<td>-242 ± 14</td>
<td>---</td>
</tr>
<tr>
<td>This study (JPL)</td>
<td>08/2002 – 07/2008</td>
<td>-194 ± 24</td>
<td>---</td>
</tr>
<tr>
<td>This study (GFZ)</td>
<td>08/2002 – 07/2008</td>
<td>-96 ± 23</td>
<td>---</td>
</tr>
<tr>
<td>Ramillien et al. 2006</td>
<td>07/2002 – 03/2005</td>
<td>-130 ± 11</td>
<td>-141 ± 16</td>
</tr>
</tbody>
</table>

- different periods investigated
- different methods applied
- rigorous consideration of “leakage-out” effects
- rigorous consideration of “leakage-in” effects
Conclusions

- Leakage effects have to be considered
  - over 50% of final signal for Greenland
- Leakage effects are largely originated by spatial smoothing
- 4-steps procedure successfully implemented for Greenland
  - Can separate between leakage out and leakage in effects
  - Leakage out effects dominate for Greenland
  - Leakage in effects are dominated by the Canadian Shield
  - Insensitive with respect to size of extended area and magnitude of Gaussian smoothing radius (e.g. 300 – 600 km)
- Method requires information on the spatial extend of mass changes (e.g. land vs. ocean)
- Method is applicable to any region of interest (cryosphere, river basins, GIA areas, ...)
- Huge differences between different processing centres
- Effect of “de-striping” on mass estimates?