Self-adaptive choice of a system of localizing base function for regional gravity field recovery

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Abstract
For regional gravity field recovery by spherical wavelets, the base functions are usually placed on a regular grid. In most cases this leads to an overparametrization and an unstable system. On the other hand a simultaneous optimization of both the locations and the shape-parameters of the wavelet base yields a nonlinear optimization problem with a large number of local minima. The paper aims at a demonstration how this optimization problem can be solved by advanced search algorithms as e.g. genetic algorithms.

1. Definition of the Problem
A GRACE scenario for regional gravity recovery was chosen in the following way:
• The gravitational potential \( V \) was simulated by the potential of three buried masses \( \delta V \) on top of EGM96.
• A one-week GRACE orbit was integrated in the field.
• Along all arcs crossing the region of interest simulated range-rates \( \rho \) were computed from positions and velocities of the orbit integration.
• The range-rates \( \rho \) were transformed into along track gravity gradients \( \delta V_{\text{off}} \) at the midpoints between the two satellites.
• Residual gravity gradients \( \delta V_{\text{res}} \) were computed subtracting the EGM96 gravity gradients \( V_{\text{EGM}} \) from the synthetic gravity gradients \( V_{\text{syn}} \).

The goal of the investigations was the recovery of the buried masses potential \( \delta V \) from the residual gravity gradients over the region.

2. Optimization Problem
As base functions spherical wavelets were chosen
\[
\psi(\omega, \sigma, g) = \sum_{m=0}^{n} \sigma^{m} p_{n}(\omega, g) \theta_{m}^{(g)}(y)
\]
with \( \sigma \) being the shape parameter and \( \theta_{m}^{(g)} \in \text{SO}(3) \) the location of the wavelet on the sphere. The spherical wavelet produces along the orbit \( x \) the following residual along-track gravity gradient
\[
\frac{\partial^{2} \psi(x, \sigma, g)}{\partial y^{2}} = \sum_{m=0}^{n} \sigma^{m} \left[ \frac{1}{\text{Ria}(x)} \right]^{n+2} \sum_{n=0}^{n} \sum_{m=0}^{n} \sigma^{m} \Delta_{mm}(g) \nabla_{nm}(g) \theta_{m}^{(g)}
\]
with \( g \) being the orbital rotation.
\[
g = R_{y}(\pi - \omega) \quad \text{and} \quad R_{y}(-1)R_{y}(\pi) \quad \text{and} \quad R_{y}(\pi + \omega + M)
\]
Each measured residual gravity gradient \( \delta V_{\text{off}}(t) \) is to be represented as a linear combination of the spherical wavelets
\[
\delta V_{\text{off}}(t) = \sum_{j=1}^{N} c_{j} \frac{\partial^{2} \psi(x, \sigma, g)}{\partial y^{2}}
\]
For every choice of the free parameters \( c_{j}, g_{j}, \sigma \) a certain misfit between observation and linear combination will be generated
\[
v_{i} = \delta V_{\text{off}}(t_{i}) - \sum_{j=1}^{N} c_{j} \frac{\partial^{2} \psi(x, \sigma, g)}{\partial y^{2}}
\]
The optimal choice of the free parameters is those, which minimizes the sum of the squares of the misfits
\[
\Phi(c_{j}, g_{j}, \sigma) = \sum_{i=1}^{T} v_{i}^{2} \rightarrow \text{min}
\]
Since this optimization problem is non-linear, only serach methods can be applied to find the global minimum. One possible search methods are genetic algorithms.

3. Genetic Algorithms
Genetic algorithms simulate the natural process of evolution in order to find the global minimum of a function. It consists of 4 steps:
1. Generation of an initial population: A number of points in the parameter space is picked randomly. Each point is considered an individual and the set of all points is called the population.
2. Evaluation: The individuals are ranked according to their target-function values.
3. Selection: A number of individuals is selected for mating. The higher its ranking the higher is the probability of an individual to be selected for mating.
4. Off-springs: a number \( k \) of off-springs is generated by swapping the lower parts of the mantissa in the parameter values of the individuals selected for mating. The \( k \) individuals with the lowest ranking are eliminated from the population.
5. The algorithm repeats with a evaluation of the new generation.
For an sufficiently large initial population and for sufficiently many evolution steps the population will concentrate around the global minimum.

4. Numerical Experiments
First in a traditional way 196 spherical wavelets were placed around the ground tracks on a regular grid. Therefore the only parameters to be determined are the weights \( c_{j} \) in the linear combination. This is a convex problem and can be solved in the usual way. The grid and the results are shown in figure 1(b).

The recovered solution is practically in no relation to the original field. This is due to the high condition number \( 1/\text{det} \) of the corresponding normal equation matrix.
If only 5 base functions, including there locations and their shape-parameters are chosen by a genetic algorithm the result as shown in figure 2(b) is much better.
Visually, the recovered solution cannot be distinguished from the given field. Hence, with a much lower number of base functions the genetic algorithm produced much better result than the traditional regular-grid technique.