Vertical Crustal Motion Analysis Using the Canadian Precise Levelling Network for the Definition of a Kinematic Vertical Datum in Canada

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Contents

• Objectives
  – To analyse the Canadian Precise Levelling Network to determine the retrievability of vertical crustal motion information.
  – To determine the feasibility of defining a kinematic vertical datum.

• Motivation

• Theoretical aspects of kinematic vertical datum realisation

• Data processing

• Least squares adjustment, error analysis, and statistical testing

• Summary and Conclusions
Why levelling measurements?

- Oldest precise height measurement type available ≈100 years.
- GPS, SLR, VLBI, and SAR/InSAR measurements for crustal motion studies are available only for the past ≈20 years.
- Minimal change in accuracy of levelling instruments over the years.
- Crustal motion studies benefit from independent datasets for cross-validation.
- Last chance for an adjustment of this nature!
Study Area for the Analysis
Adjustment for the Realisation of Kinematic Vertical Datum of a Levelling Network

Kinematic height model

\[ H_i(t_k) = H_i(t_0) + v_i(t_k - t_0) \]

Levelling observation equation

\[ \Delta H_{ij}(t_k) = H_j(t_k) - H_i(t_k) \]

Applying the kinematic height model to the observation equation

\[ \Delta H_{ij}(t_k) = H_j(t_0) - H_i(t_0) + v_j(t_k - t_0) - v_i(t_k - t_0) \]
Simplifying the equations and writing in matrix form

\[ \Delta H(t) = [A^H | T A^H] \begin{bmatrix} H \\ v \end{bmatrix} + \epsilon \]

\[ \Delta H(t) = AH + \epsilon \]

Least squares adjustment problem for kinematic vertical datum realisation

\[ \min ||\Delta H(t) - AH||^2 \quad \text{subject to} \quad D^T H = c \]

Solution \[ \hat{H} = (A^T PA + DD^T)^{-1}(A^T P \Delta H + Dc) \]

\textbf{S-transform} – \((I - T(D_b^T T)^{-1} D_b^T)\), where \(T\) is a coefficient matrix and \(D_b\) is the datum matrix of datum \(b\).
Network Configurations at Various Stages of Data Processing

Reduction refers to the reduction of number of network points (in effect parameters) by combining observations.

<table>
<thead>
<tr>
<th>Network type</th>
<th>Observations</th>
<th>Parameters</th>
<th>Rank Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before reduction</td>
<td>3324</td>
<td>3422</td>
<td>560</td>
</tr>
<tr>
<td>After reduction</td>
<td>3730</td>
<td>2292</td>
<td>24</td>
</tr>
</tbody>
</table>

For defining an ideal minimum constraint kinematic vertical datum the rank deficiency of the design matrix must only be 2.
Before reduction

After reduction
Indicators of Vertical Crustal Motion Retrievability

a) Time interval between the epochs of observation in the relevelling lines

![Histogram of Time Interval between Relevellings (years)]

\[ Q_{\hat{H}} = \left( A^T P A \right)^{-1} \]
Results from the Kinematic Vertical Datum Realisation

- Final network for adjustment had 22 excess constraints

- *A priori* information for excess constraints:
  - velocities from postglacial rebound models
  - heights estimated by Geodetic Survey Division, Natural Resources Canada
  - zero values for all the excess velocity constraints

- Different weight matrices were used for the adjustment
  - Precision of the observations as weights
  - Unit weights
Datum points fixed for all the adjustments carried out
Velocity Estimates

EXCESS CONSTRAINTS – velocities; WEIGHTS – precision of observations
EXCESS CONSTRAINTS – heights; WEIGHTS – precision of observations
EXCESS CONSTRAINTS – all values assigned ‘0’; WEIGHTS – precision of observations
EXCESS CONSTRAINTS – heights; WEIGHTS – Unit weights
Local Redundancy Analysis

\[ r_i = 1 - \frac{\sigma^2_{\Delta \hat{H}}}{\sigma^2_{\Delta H}} \]

Figure 1: Local redundancy values of each of the observations in the network
CONSTRANTS – Heights; WEIGHTS – Precision of observations
Statistical Testing of Observations

Baarda’s data snooping

Values ‘> 1’ and ‘< −1’ are the erroneous observations
Pope’s $\tau$ test

Values ‘$> 1$’ and ‘$< -1$’ are the erroneous observations
Statistical Testing of Estimated Parameters

\[ T_i = \frac{\hat{v}_i}{\sqrt{\hat{\sigma}_0^2 Q_{\hat{v}_{ii}}}} \left\{ \begin{array}{ll} \geq 1 & \text{significant} \\ < 1 & \text{insignificant} \end{array} \right. \]

<table>
<thead>
<tr>
<th>Excess constraints</th>
<th>Weights</th>
<th>% of significant parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>Precision of observations</td>
<td>52.9</td>
</tr>
<tr>
<td>Height</td>
<td>Precision of observations</td>
<td>62.6</td>
</tr>
<tr>
<td>Velocity</td>
<td>Unity</td>
<td>30.0</td>
</tr>
<tr>
<td>Height</td>
<td>Unity</td>
<td>36.4</td>
</tr>
</tbody>
</table>

Percentage of the significant parameters (only velocities) for the different cases of estimation.
Geological and Geophysical Interpretation

- Velocity trend along north–south indicates *postglacial rebound* signal
- Area ridden by heavy faulting
- Shallow earthquake activity throughout the region
- Study area very complex geologically and geophysically!
EXCESS CONSTRAINTS – Heights; WEIGHTS – Precision of observation
Summary

- **S-transformation** of a kinematic vertical datum with linear kinematic height model

- Even after **reduction** data gaps in the network – excess constraints situation

- Velocities from postglacial rebound models and heights of points as excess constraints

- Local redundancy indicates a weak network and statistical tests indicate low data quality
Conclusions

- Due to data gaps an ideal kinematic vertical datum cannot be defined
- Workable kinematic vertical datum – implemented using height excess constraints
- Height excess constraints provide a set of velocities that are independently interpretable when compared to velocity excess constraints
Acknowledgements

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