REeal data AnaLysis GOCE

Gravity field determination from GOCE

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BMBF Geotechnologien Statusseminar:
„Erfassung des Systems Erde aus dem Weltraum III“

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   • Satellite-to-Satellite Tracking (SST)
   • Satellite-gravity-gradiometry (SGG)

3 First gravity field solutions derived from GOCE

4 Summary and Outlook
Sensors for gravity field determination

The whole satellite is the sensor!

But three main sensor observations to be scientifically processed for gravity field determination:

GPS tracking (SST)  Gradiometry (SGG)  star tracker (STR)
From observations to the gravity field

\[ V(r, \theta, \lambda) = \frac{GM}{a} \sum_{l=0}^{l_{max}} \left( \frac{a}{r} \right)^{l+1} \sum_{m=0}^{l} (c_{lm} \cos(m\lambda) + s_{lm} \sin(m\lambda)) P_{lm}(\cos\theta), \quad (1) \]

- Satellite orbits
- Geolocated gravity gradients
- Gradiometer orientation

+ Orbit error information
+ Gradient error information
+ Accuracies
SST observations

Satellite-to-Satellite tracking in high low mode:

The Instrument:

The Principle:

Original measurements:
- GPS Code observations
- GPS Phase observations

Pseudo observations for GFR:
- kinematic satellite positions (ITRF, x, y, z), σ ≈ 20mm
- covariance matrix (GPS geometry)
- source a) HPF/EGG-C: SST_PSO
- source b) orbit determination in REAL GOCE (IGG-APMG)

Approaches within REAL GOCE: integral equation approach, acceleration approach, energy balance approach

Sensitivity: long wavelength part, s/h degree 2 to 100

Contribution: within a combined GOCE-only model d/o 2-25

Long wavelength GOCE models will never be competitive to GRACE!

\[ \ddot{r} = \frac{1}{m} F = g(c_{lm}, s_{lm}) + \frac{1}{m} \sum_{i} F_{i} \]
GOCE SST real data performance

2 months compared to GRACE/CHAMP (7 years):

solid: degree variance from difference to ITG-Grace2010s, dashed: degree variance from formal errors
Satellite-gravity-gradiometry:

The Instrument:

The Principle:

Original measurements:
- accelerations of the six accelerometers

Pseudo observations for GFR:
- 6 differential accelerations
- high temporal auto correlations in the gradients
- as calibrated gravity gradients $V_{ij}$
- source: ESA HPF/EGG-C: EGG_NOM

- REAL GOCE approaches: in situ time-wise approach, in situ short arc approach, in situ invariant approach
- Sensitivity: short wavelength part
- Contribution: within a combined GOCE-only model d/o 20-220 (250-270 future?)
Stochastic Characteristics of SGG Data

Estimated SGG noise as time series:

- measured mean & GRS reduced $V_{zz}$ gradient
- computed mean & GRS reduced $V_{zz}$ gradient (EIGEN5C)
- estimated mean reduced $V_{zz}$ noise (measured-computed)
- signal and noise have the same order of magnitude

Spectral characteristics of SGG noise:

- flat spectra only within the MBW
- two gradient components with very high noise levels ($V_{xy}, V_{yz}$)
- sharp peaks at $k$ per revolution ($k = 1, 2, \ldots$)
- noise level MBW: $8mE/\sqrt{Hz}$ for $V_{xx}, V_{yy}$ & $12mE/\sqrt{Hz}$ for $V_{xz}, V_{zz}$
- worse $V_{zz}$ performance not understood
Time-Wise Approach (IGG-TG)

gap-less part of independent gradients as equidistant time series:

optimized digital ARMA filter cascades to decorrelate gradient observation equations:

Observation equations:

1. potential in EFRF cf. (1)
2. $2^{nd}$ derivative according to $\lambda, \theta$
3. rotation to GRF (earth rotation, error-free STR) $(1 + v = Ax)_{GRF}$
4. application of decorrelation filters to obs. eqs. $(Fl + Fv = FAx)_{GRF}$
5. Final obs. eqs.: $(\bar{l} + \bar{v} = \bar{A}x)_{GRF}$, $Q_\Pi = I$

Solution process:

1. Least-Squares solution
2. using tailored parallel iterative solver (PCGMA, conjugate gradients)
3. combination of SGG with SST NEQ
4. stabilisation using Kaula’s rule
5. optimal weighting using VCE
6. iterative refinement of filter cascades
Short-Arc Approach (IGG-APMG)

analysis of short arcs (length of \( \approx 15 \) min):

Observation equations per short arc:

1. short arcs assumed to be uncorrelated
2. long wavelength error modeled with a bias parameter
3. \( 2^{nd} \) derivative of (1) according to \( x, y, z \)
4. rotation with STR data to GRF

full variance covariance information per short arc:

Solution process:

1. Least-Squares solution
2. assembling of arc-wise full NEQ
3. stabilisation using priori information
4. optimal weighting using VCE
5. local refinement with local base functions
Invariants Approach (GIS)

rotation invariants of gravity gradients
combination of tensor elements:

- Observation equations (STR independent):
  1. \( I_2 = \frac{1}{2} (V_{xx} + V_{yy} + V_{zz}) - V_{xy} - V_{xz} - V_{yz} \)
  2. Variance propagation of GG error model to invariants
  3. neglecting correlations among GGs
  4. \( \Sigma I_2 I_2 = \sum_{i \in x, y, z} \sum_{j \in x, y, z} J_{ij} \Sigma V_{ij} V_{ij} J_{ij}^T \)
  5. inserting \( \Sigma V_{ij} V_{ij} = (F_{V_{ij}}^T F_{V_{ij}})^{-1} \)

Invariant \( I_2 \) on 02-Nov-2009
main diagonal GGs only:

Solution process:
1. Least-Squares solution
2. Tailored parallel processing scheme
3. High performance computing with OpenMP and MPI
4. Stabilisation with a priori information
5. Optimal regularization
GOCE SGG real data performance

GOCE SGG performance (1-2 months) compared to EIGEN5C:

- **EIGEN5C**
- 1 month SGG (IGG–APMG)
- 2 months SGG (IGG–TG)
- ITG–Grace2010s

**solid**: degree variance from difference to EIGEN5C, **dashed**: degree variance from formal errors

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In general, there are three independent methods to determine the gravity field model from GOCE within REAL GOCE:

**SST**

<table>
<thead>
<tr>
<th>Method</th>
<th>General</th>
<th>Functional Model</th>
<th>Stochastic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>IGG-TG</td>
<td>preprocessed in HPF</td>
<td>energy balance</td>
<td>geometry of GPS constellation</td>
</tr>
<tr>
<td>IGG-APMG</td>
<td>short arcs</td>
<td>integral equation</td>
<td>geometry of GPS constellation, refinement with emp. covariance function</td>
</tr>
<tr>
<td>GIS</td>
<td>acceleration approach</td>
<td>numerical differentiation of kinematic GOCE orbit</td>
<td>variance-covariance information of GOCE orbit (GPS constellation)</td>
</tr>
</tbody>
</table>

**SGG**

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<tr>
<td>IGG-TG</td>
<td>time series</td>
<td>$2^{nd}$ derivative of potential in GRF for $V_{xx}$, $V_{yy}$, $V_{zz}$</td>
<td>ARMA filter cascades, decorrelation of continuous, gap-less time series</td>
</tr>
<tr>
<td>IGG-APMG</td>
<td>short arcs</td>
<td>$2^{nd}$ derivative of potential in GRF for $V_{xx}$, $V_{yy}$, $V_{zz}$</td>
<td>empirical covariance matrix, empirical parameters/arc, independence of arcs</td>
</tr>
<tr>
<td>GIS</td>
<td>time series</td>
<td>rotational invariants of the gradient tensor</td>
<td>propagation of gradient error model to invariants</td>
</tr>
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</table>
Cooperations within the gravity field processing groups:

- IGG-TG → GIS: Stochastic model for gradients (for error propagation to invariants)
- IGG-APMG → IGG-TG: alternative SST normal equation matrix for combination
- GIS → IGG-TG, IGG-APMG: STR independent solution
- IGG-TG ↔ IGG-APMG: stochastic model analysis, filter vs. covariance function
- ALL → ALL: outlier information, quality information on gradients
- ALL → ALL: mutual validation of results
GOCE-only model performance

GOCE performance (1-2 months) compared to EIGEN5C:

solid: degree variance from difference to EIGEN5C, dashed: degree variance from formal errors
GOCE-only model performance

Geoid heights compared to ITG-Grace2010s (d/o 150):

<table>
<thead>
<tr>
<th>sector [°]</th>
<th>min [m]</th>
<th>max [m]</th>
<th>mean [m]</th>
<th>rms [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>±83.5</td>
<td>-0.318</td>
<td>0.325</td>
<td>0.002</td>
<td>0.051</td>
</tr>
<tr>
<td>±90.0</td>
<td>-6.425</td>
<td>2.708</td>
<td>-0.060</td>
<td>0.544</td>
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</tbody>
</table>

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GOCE-only model performance

Geoid heights compared to EIGEN5C (d/o 200):

<table>
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<th>sector [°]</th>
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<th>max [m]</th>
<th>mean [m]</th>
<th>rms [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>±83.5</td>
<td>-3.301</td>
<td>4.011</td>
<td>0.002</td>
<td>0.254</td>
</tr>
<tr>
<td>±90.0</td>
<td>-6.930</td>
<td>4.011</td>
<td>-0.062</td>
<td>0.618</td>
</tr>
</tbody>
</table>
Gain of GOCE

Gain of the current GOCE models:

With only two months of data:
- consistent set of spherical harmonic coefficients d/o 2-224
- GRACE improvement d/o 140+
- Improvement of combined models in regions w. low-quality terrestrial data

Lot's of effort in stochastic modeling:
- High quality variance/covariance matrix
- Good reflection of errors by the VCM
- Formal errors are meaningful

There will be huge improvements with more GOCE data!
Summary and Outlook

Summary:

► Processing of real data started within REAL GOCE
► First results from all three approaches
► Processed until now: November (and December) 2009 (data released by ESA)
► For details on the three approaches: See posters!
► All milestones which are due until today have been achieved
► Work to achieve other milestones is in progress

Outlook:

► Understand the new observation type in more detail
► Processing of a longer time span
► Regional refinements of global solution