Assessing Greenland ice mass loss by means of point-mass modelling: methodology and results

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Motivation

satellite observations → point mass modelling → mass variations

Forsberg R, Reeh N (2007)
Mass change of the Greenland ice sheet from GRACE,
Outline

- Point-mass modelling methodology
- Parameter estimation
  - least-squares adjustment
  - genetic algorithms
- Simulation studies
- GRACE results
- Conclusions & outlook
Methodology

GRACE-derived gravitational disturbances (radial direction)

\[ \delta g = -\frac{GM}{r^2} \sum_{l=0}^{L} (l+1)(1+k_l) \left( \frac{a}{r} \right)^l \sum_{m=0}^{l} \bar{P}_{lm}(\sin \phi)(\Delta \bar{c}_{lm} \cos m\lambda + \Delta \bar{s}_{lm} \sin m\lambda) \]

\( \Delta \bar{c}_{lm}, \Delta \bar{s}_{lm} \) ... (secular) changes of spherical harmonic coefficients over the period of investigation (no filtering/smoothing applied)

\[ \Delta \bar{c}_{7,5}, \Delta \bar{s}_{7,5} \]

gravitational disturbances in satellite altitude: \( h = 500 \text{ km} \iff r = a + h \)
Methodology

GRACE-derived gravitational disturbances (radial direction)

recovering area delineation

the spatially limited recovering area avoids misinterpretation of disturbing signals (residual leakage effects remain)
Methodology

Point-mass modelling (radial direction)

\[ r = a + 500 \text{ km} \]

\[ \cos \psi_{i,j} = \sin \phi_i \sin \phi_j \]

\[ + \cos \phi_i \cos \phi_j \cos (\lambda_i - \lambda_j) \]

\[ \delta g_{i,j}^{\text{SP}} = \frac{G \delta m_j}{l_{i,j}^2} \]

\[ l_{i,j} = (a^2 + r^2 - 2ar \cos \psi_{i,j})^{1/2} \]

\[ \cos \alpha = \frac{r - a \cos \psi_{i,j}}{l_{i,j}} \]

\[ \delta g_{i,j} = \frac{G \delta m_j}{l_{i,j}^2} \cos \alpha \]
Methodology

**Functional model formulation**

modelling of observed GRACE gravitational disturbances in satellite altitude by means of unknown point-mass variations on the earth’s surface:

\[
\delta g_i = -\frac{GM}{r^2} \sum_{l=0}^{L} (l+1)(1+k_l) \left( \frac{a}{r} \right)^l \sum_{m=0}^{l} \tilde{P}_{lm} (\sin \varphi_i) (\Delta \bar{c}_{lm} \cos m\lambda_i + \Delta \bar{s}_{lm} \sin m\lambda_i)
\]

\[
= G \sum_{j=1}^{u} \frac{r - a \cos \psi_{i,j}}{(a^2 + r^2 - 2ar \cos \psi_{i,j})^{3/2}} \delta m_j, \quad i = 1 \ldots n
\]

\[
y \ (n\times1) \quad \text{observations (gravitational disturbances)}
\]

\[
A \ (n\times u) \quad \text{design matrix}
\]

\[
x \ (u\times1) \quad \text{unknown parameters (mass changes)}
\]
Methodology

functional model
\[ Ax = y + e \]

least-squares adjustment (LSA)
\[ \hat{x} = (A^T A + \lambda I)^{-1} A^T y \]

determination of the regularization parameter by (heuristic) selection rule
L-curve criterion:
optimal parameter selected from a set of pre-defined values

determination of the regularization parameter by global optimization (non-linear problem)
successive generations “produce” the optimal solution

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Genetic algorithms

process chain:
1. generate initial population (random values)
2. evaluate cost function of each individual
3. select “best” (fittest) individuals
4. produce new individuals
5. mutate offspring
6. replace individuals, generate new population
7. evaluate truncation criterion
Genetic algorithms

Example

\[ z = x \sin(4x) + 1.1y \sin(2y) = \min_{(x,y)} \]

global minimum?
Genetic algorithms

Example

\[ z = x \sin(4x) + 1.1y \sin(2y) = \min_{(x,y)} \]

global minimum?

analytical solution  numerical solution
\[ x = 9.039 \quad x = 9.044 \]
\[ y = 8.668 \quad y = 8.657 \]
\[ z = -18.555 \quad z = -18.550 \]

minimal costs/best fitness
Study 1a: noise-free simulation

- #simulated point masses: 71
- total variation: $-1000 \text{km}^3$
- #observations: 1338
- #modelled point masses: 234
Simulation studies

Study 1a: noise-free simulation

LSA solution
-999.5 km³ $\lambda = 7.0 \cdot 10^{-44}$
error: 0.05%

GA solution
-1002.3 km³ $\lambda = 7.9 \cdot 10^{-45}$
error: 0.23%
Simulation studies

Study 1b: impact of GRACE errors

anisotropic covariance function of gravitational disturbances in satellite altitude (derived from full SHC variance-covariance matrix of August 2003, provided by GFZ Potsdam)

LSA solution
-1004.8 km$^3$ \( \lambda = 1.0 \cdot 10^{-42} \)
error: 0.48%

GA solution
-1009.2 km$^3$ \( \lambda = 5.5 \cdot 10^{-44} \)
error: 0.92%
Simulation studies

Study 1c: impact of leakage signals

- total Greenland variation: $-1000\text{km}^3$
- total Canadian Shield variation: $+1000/+2000\text{km}^3$

- disturbing signal $+1000\text{km}^3$ → error: 1.2%
- disturbing signal $+2000\text{km}^3$ → error: 2.3%

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Simulation studies

Study 2: noise-free simulation ("patchy pattern")

- #simulated point masses: 24
- total variation: -1000 km³
- #observations: 1338
- #modelled point masses: 234

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Simulation studies

Study 2: noise-free simulation

**LSA solution**
-1000.5 km$^3$  \( \lambda = 3.0 \cdot 10^{-44} \)
error: 0.05%

**GA solution**
-1009.5 km$^3$  \( \lambda = 1.7 \cdot 10^{-44} \)
error: 0.95%
GRACE analysis setup

- Gravity field series: CSR (RL04), April 2002–March 2009 (no filtering/smoothing of SHC)
- #Observations: 1338
- #Modelled point masses: 234

GRACE results

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GRACE results

LSA estimate against regularization parameter

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GRACE results

LSA estimate against regularization parameter
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LSA estimate against regularization parameter
GRACE results

LSA solution

\[-299 \pm 6 \text{km}^3 \text{ yr}^{-1}\]

\[\lambda = 1.0 \cdot 10^{-43}\]

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GRACE results

**LSA solution**

-299 ± 6 km$^3$ yr$^{-1}$

$\lambda = 1.0 \cdot 10^{-43}$

**GA solution**

-299 ± 6 km$^3$ yr$^{-1}$

$\lambda = 6.6 \cdot 10^{-45}$
## Greenland ice-mass loss

<table>
<thead>
<tr>
<th>Period</th>
<th># Years</th>
<th>LSA (km$^3$ yr$^{-1}$)</th>
<th>$\lambda_{LSA}$</th>
<th>GA (km$^3$ yr$^{-1}$)</th>
<th>$\lambda_{GA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>04/2002 – 03/2009</td>
<td>7</td>
<td>-299 ± 6</td>
<td>$1.0 \cdot 10^{-43}$</td>
<td>-299 ± 6</td>
<td>$6.6 \cdot 10^{-45}$</td>
</tr>
<tr>
<td>08/2002 – 07/2008</td>
<td>6</td>
<td>-294 ± 6</td>
<td>$1.0 \cdot 10^{-43}$</td>
<td>-294 ± 6</td>
<td>$9.7 \cdot 10^{-45}$</td>
</tr>
<tr>
<td>04/2002 – 03/2007</td>
<td>5</td>
<td>-254 ± 6</td>
<td>$3.0 \cdot 10^{-43}$</td>
<td>-256 ± 5</td>
<td>$1.3 \cdot 10^{-44}$</td>
</tr>
</tbody>
</table>

Compatible mass-change rates

Impact of different regularization parameters compensated by slightly differing mass-change patterns

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Conclusions

- Point-mass modelling is a highly viable method to derive mass variations on the earth’s surface from time-variable GRACE gravity fields.

- Applied to ice-mass balance over Greenland, it is able to recover both deglaciation magnitudes and geometries.

- Least-squares adjustment (in combination with the L-curve criterion) and GAs provide comparable results; both techniques are equivalent as far as solution quality issues are concerned.

- With regard to computational costs, LSA outperforms GAs.

- Analysis of CSR RL04 gravity fields from April 2002 to March 2009 shows a Greenland ice-mass decline of \(-299\pm6\) km\(^3\) yr\(^{-1}\) (GIA neglected).

- Our results point to ice-mass decline along the whole Greenland coast line; they confirm dominating deglaciation in the south-east and north-west.
Outlook

- methodology refinement: consideration of disturbing signals (e.g., leakage, land hydrology)
- spatial locations of point-mass changes as additional unknown parameters (highly non-linear optimization problem)
- hybrid GA implementation: combination with a local optimizer (e.g., downhill simplex method)
- application of the method on other regions such as Alaska and Patagonia